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(Dated: September 29, 2018)

In Bell experiments, one problem is to achieve high enough photodetection to ensure that there is no possibility of describing the results via a local hidden-variable model. Using the Clauser-Horne inequality and a two-photon non-maximally entangled state, a photodetection efficiency higher than 0.67 is necessary. Here we discuss atom-photon Bell experiments. We show that, assuming perfect detection efficiency of the atom, it is possible to perform a loophole-free atom-photon Bell experiment whenever the photodetection efficiency exceeds 0.50.

Forty-three years after Bell's original paper [1], which contains what has been described as "the most profound discovery of science" [2] or, at least, "one of the greatest discoveries of modern science" [3], there is no experiment testing (the impossibility of) local realism without invoking supplementary assumptions. All reported Bell experiments, for instance [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20], suffer from so-called "loopholes." It has even been argued that, "As more time elapses without a loophole-free violation of local realism, greater should be our confidence on the validity of this principle [local realism]" [21]. Beyond this challenge, quantum information gives us new reasons for performing loophole-free Bell experiments. There is a link between a true ("loophole-free") violation of a Bell inequality and the security of a family of quantum communication protocols [22, 23, 24]. Specifically, there is an intimate connection between the existence of a provably secure key distribution scheme and a true violation of a Bell inequality [25] (even in the case that quantum mechanics ultimately fails).

There are two experimental problems that make supplementary assumptions necessary. The *locality loophole* [26] occurs when the distance between the local measurements is too small to prevent communication between one observer's measurement choice and the result of the other observer's measurement. In short, these two events must be spacelike separated. Massive entangled particles are extremely difficult to separate [19], and high-energy photons are not appropriate due to the lack of efficient polarization analyzers. The best candidates for closing the locality loophole are optical photons, where good polarization analyzers exist and spacelike separation can be achieved [11, 18]. However, thus far, all Bell experiments with photons suffer from the *detection loophole* [27]. The imperfect efficiency of photodetectors makes the results of all these experiments compatible with local realistic models. An overall detection efficiency $\eta > 0.67$ is required for two-photon loophole-free Bell experiments

[28, 29]. Single-photon detectors with more than 0.90 quantum efficiency already exist, but there are other difficulties that reduce the overall efficiency to about 0.30 or less in practice. Other possible loopholes, e.g., those related to the subtraction of background counts, will not be discussed here.

There are some recent proposals as to how to close both the locality and the detection loopholes simultaneously. One is based on the idea of achieving entanglement between two separated atoms by preparing two atom-photon systems and performing a Bell measurement on the photons which swaps the entanglement to the atoms [30, 31]. If we accept that the overall measurement time of the atom is less than 0.5 μs, then the two atoms must be separated at least 150 m [31]. Another proposal is based on Bell inequalities for two-photon systems prepared in hyper-entangled states, in which the minimum required photodetection efficiency is significantly reduced [32].

The most promising proposal is the planned Urbana experiment using two polarization-entangled photons and high-efficiency visible-light photon counters (VLPCs) [33, 34]. The actual measured efficiency of the VLPCs is 0.86 [34]. However, after putting these detectors in a Bell experiment with no less than 60 m separation, and considering the background noise, the effective efficiency could be dangerously close to the minimum required for a loophole-free experiment ($\eta > 0.75$ with a 0.31% background noise [28]).

Here we show that it is possible to close the detection loophole with a photodetection efficiency $\eta > 0.50$ by using a single atom-photon system. Entanglement between the polarization of a single photon and the internal state of a single trapped atom has been observed [31, 35]. Moreover, a violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality [36] with a cadmium atom and a photon has been reported [20]. These experiments, together with new high-efficiency photodetectors, suggest that a loophole-free atom-photon Bell experiment with a sep-

aration of 150 m, a perfect detection efficiency for the atom, and a photodetection efficiency higher than 0.50 is actually feasible.

Consider an atom and a photon brought to distant locations. Suppose A and a are two choices for the observable measured on the atom, and B and b two choices for the observable measured on the photon. Each of these observables can only take the values -1 or 1. In this scenario, a Bell inequality is a necessary constraint imposed by local realistic theories on the values of a linear combination of probabilities that can be measured in four different experimental setups: (A, B), denoting that A is measured on the first particle and B on the second, (A, b), (a, B), and (a, b).

First, we calculate the minimum detection efficiencies η_A and η_B of the atom and the photon detectors, respectively, required for a loophole-free Bell experiment based on the CHSH inequality

$$|\langle AB \rangle + \langle Ab \rangle + \langle aB \rangle - \langle ab \rangle| \le 2. \tag{1}$$

This inequality involves *classical* expectation values of products of measurement results, and is valid for any local hidden-variable model with results between -1 and $+1$ in the case of perfect detectors. Quantum expectation values do not obey (1), and the maximum quantum violation is achieved at the value of $2\sqrt{2}$ on the left-hand side [37].

In a non-ideal experiment, the expectations are usually calculated by conditioning on coincidence. In that case, when $\eta_A = \eta_B = \eta$, it is well known [38, 39] that the CHSH inequality (1) must be modified to

$$|\langle AB \rangle_{\text{coinc}} + \langle Ab \rangle_{\text{coinc}} + \langle aB \rangle_{\text{coinc}} - \langle ab \rangle_{\text{coinc}}| \le \frac{4}{\eta} - 2. \tag{2}$$

This inequality still involves classical conditional expectation values. Inserting quantum conditional expectation values, the maximum of the left-hand side is still $2\sqrt{2}$. Therefore, (2) is violated only if $\eta > 2(\sqrt{2} - 1) \approx 0.83$.

Now we want to modify (2) so that it applies to the case when $\eta_A \ne \eta_B$. To do this we will write, e.g., $A = 0$ when a measurement result is lost due to inefficiency. Thus, assuming that the rate of non-detections is independent of the measurement settings, we have

$$\eta_A = P(A \ne 0) = P(a \ne 0), \tag{3a}$$
$$\eta_B = P(B \ne 0) = P(b \ne 0). \tag{3b}$$

These are theoretical probabilities which are difficult to extract from experiment unless *every* experimental run is taken into account, even those where no detection occurs at either site. Assuming that detections at one site are independent of detections at the other, we have

$$P(A = B = 0) = (1 - \eta_A)(1 - \eta_B). \tag{4}$$

It is now simple to prove [38] that

$$|\langle AB \rangle + \langle Ab \rangle + \langle aB \rangle - \langle ab \rangle| \le 2 - 2P(A = B = 0). \tag{5}$$

Furthermore, the probability of a coincidence is $\eta_A \eta_B$ in this case, and the conditional expectations can be written, e.g.,

$$\langle AB \rangle_{\text{coinc}} = \frac{1}{\eta_A \eta_B} \langle AB \rangle. \tag{6}$$

Thus, (5) can be written

$$|\langle AB \rangle_{\text{coinc}} + \langle Ab \rangle_{\text{coinc}} + \langle aB \rangle_{\text{coinc}} - \langle ab \rangle_{\text{coinc}}|$$
$$\le \frac{2\eta_A + 2\eta_B - 2\eta_A \eta_B}{\eta_A \eta_B} = \frac{2}{\eta_A} + \frac{2}{\eta_B} - 2. \tag{7}$$

This inequality is a generalization of (2). Inserting the maximum quantum value $2\sqrt{2}$ in the left-hand side yields a bound for η_B as a function of η_A. In brief, inequality (7) is violated only if

$$\eta_B > \frac{\eta_A}{(\sqrt{2} + 1)\eta_A - 1}. \tag{8}$$

In the special case when $\eta_A = 1$, there is a violation only if $\eta_B > 1/\sqrt{2} \approx 0.71$.

Although the CHSH inequality and the Clauser-Horne (CH) inequality are equivalent in the ideal case [40], the CH inequality is violated by quantum mechanics as soon as $\eta = \eta_A = \eta_B > \frac{2}{3} \approx 0.67$ [28, 29]. That is, even when $0.67 < \eta < 0.83$. We therefore expect to be able to lower the above 0.71 bound using the CH inequality in an atom-photon experiment. The CH inequality can be written

$$P(A = B = 1) + P(A = b = 1) + P(a = B = 1)$$
$$- P(a = b = 1) - P(A = 1) - P(B = 1) \le 0, \tag{9}$$

where $P(A = 1)$ is the probability that $A = 1$ without a corresponding detection being required at the other site. This inequality has the same status as (1), and relates classical probabilities from a local hidden-variable model. The quantum probabilities do not obey the CH inequality and the maximum quantum value of the left-hand side is $\sqrt{2} - 1$. However, as we shall see, the probabilities in the CH inequality scale differently with the efficiency, so the maximum quantum value does not coincide with the minimum efficiency for which there is violation of the inequality.

Indeed, if

$$3\eta_A \eta_B - \eta_A - \eta_B > 0, \tag{10}$$

there are quantum states and local observables violating (9). For instance, we can use the state [29]

$$|\psi\rangle = C \{ [1 - 2\cos(\theta)] |0_a 0_b\rangle$$
$$+ \sin(\theta) (|0_a 1_b\rangle + |1_a 0_b\rangle) \}, \tag{11}$$

and the local observables A and B, defined from the local observables a and b, respectively, by

$$\begin{pmatrix} |0_A\rangle \\ |1_A\rangle \end{pmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{pmatrix} |0_a\rangle \\ |1_a\rangle \end{pmatrix}, \tag{12a}$$

$$\begin{pmatrix} |0_B\rangle \\ |1_B\rangle \end{pmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{pmatrix} |0_b\rangle \\ |1_b\rangle \end{pmatrix}. \tag{12b}$$

Taking the efficiencies into account, and using $\epsilon = \tan\frac{\theta}{2}$ and $K = \sin^2\theta$, we arrive at the *quantum* probabilities

$$P(A=B=1) = K\eta_A\eta_B > 0, \qquad (13a)$$
$$P(A=b=1) = K\eta_A\eta_B, \qquad (13b)$$
$$P(a=B=1) = K\eta_A\eta_B, \qquad (13c)$$
$$P(a=b=1) = 0, \qquad (13d)$$
$$P(A=1) = K\eta_A(1 + \epsilon^2), \qquad (13e)$$
$$P(B=1) = K\eta_B(1 + \epsilon^2). \qquad (13f)$$

For this state and these observables, the left-hand side of the CH inequality (9) is

$$K\left[3\eta_A\eta_B - \eta_A - \eta_B - \epsilon^2\left(\eta_A + \eta_B\right)\right]. \qquad (14)$$

The interesting point is that, when $3\eta_A\eta_B - \eta_A - \eta_B > 0$, we simply choose $\epsilon > 0$ such that $3\eta_A\eta_B - \eta_A - \eta_B > \epsilon^2\left(\eta_A + \eta_B\right)$ (we need to choose $\epsilon \neq 0$ otherwise $K = 0$). Using this value of ϵ, or rather $\theta = 2\arctan\epsilon$, we can construct a quantum state $|\psi\rangle$ and four local observables A, a, B, and b such that the CH inequality (9) is violated.

We continue by proving that a violation can be obtained *only if* (10) is satisfied. We again use the notation $A = 0$ to denote when a measurement result is lost due to inefficiency. We condition on detection on one side, and write the conditional probabilities as, e.g.,

$$P_{\text{detect}}(A=1) = \frac{1}{\eta_A}P(A=1). \qquad (15)$$

We also, again, assume that detections at one site are independent of detections at the other. Then, the probabilities must satisfy the following trivial inequalities:

$$P(A=B=1) \leq \eta_A\eta_B \min_{X=A,B} P_{\text{detect}}(X=1), \qquad (16)$$

$$P(A=b=1) - P(A=1)$$
$$\leq (\eta_B - 1)\eta_A \min_{X=A,B} P_{\text{detect}}(X=1), \qquad (17)$$

$$P(a=B=1) - P(B=1)$$
$$\leq (\eta_A - 1)\eta_B \min_{X=A,B} P_{\text{detect}}(X=1), \qquad (18)$$

and

$$-P(a=b=1) \leq 0. \qquad (19)$$

Therefore, *only* assuming that detections at one site are independent of detections at the other, the left-hand side of the CH inequality (9) must obey

$$P(A=B=1) + P(A=b=1) + P(a=B=1)$$
$$-P(a=b=1) - P(A=1) - P(B=1)$$
$$\leq (3\eta_A\eta_B - \eta_A - \eta_B) \min_{X=A,B} P_{\text{detect}}(X=1). \qquad (20)$$

The left-hand side of (20) can be positive only if $3\eta_A\eta_B - \eta_A - \eta_B > 0$, so the CH inequality (9) can only be violated if this is the case.

Summing up, the CH inequality (9) is violated if and only if inequality (10) is satisfied, or

$$\eta_B > \frac{\eta_A}{3\eta_A - 1}. \qquad (21)$$

Note especially that the CH inequality (9) is violated if and only if

$$\eta_B > \frac{1}{2}, \quad \text{when } \eta_A = 1, \qquad (22)$$
$$\eta_B > \frac{2}{3}, \quad \text{when } \eta_B = \eta_A. \qquad (23)$$

Therefore, closing the detection loophole with an atom-photon system using the CH inequality, and assuming perfect detection efficiency of the atom, requires a minimum photodetection efficiency of 0.50 (vs $\eta_B = 0.67$ for the photon-photon case [28, 29]).

Choosing $|\psi\rangle$ to be a maximally entangled state and assuming that $\eta_A = 1$, the minimum η_B required for a loophole-free Bell experiment coincides with that previously calculated for the CHSH inequality.

So far, we have assumed that only the pairs prepared in the entangled state contribute to the counting rates, and that local measurements are perfect. In order to take into account deviations from that ideal case, we now introduce *background noise* (see Fig. 1.), as in [28], and we have numerically found the maximum affordable background
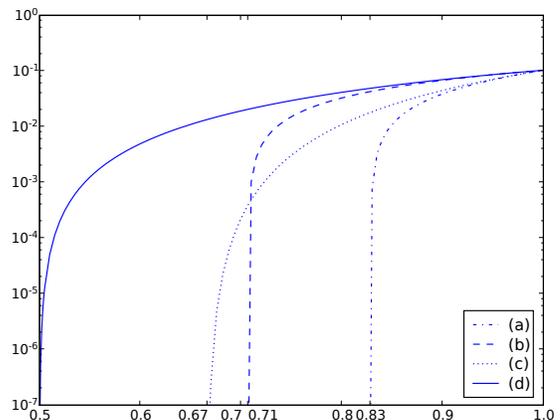

FIG. 1: Maximum affordable background noise for a loophole-free Bell experiment as a function of the photodetection efficiency η_B. (a) using the CHSH inequality with $\eta_A = \eta_B$, (b) using the CHSH inequality with $\eta_A = 1$, (c) using the CH inequality with $\eta_A = \eta_B$, and (d) using the CH inequality with $\eta_A = 1$. The cases (a) and (c) are appropriate in a photon-photon experiment and the cases (b) and (d) are appropriate in an atom-photon experiment.

noise for a loophole-free Bell experiment as a function of the photodetection efficiency, in four relevant cases, combining the usage of the CH or the CHSH inequalities with a photon-photon or an atom-photon experiment. In the case of two photons, our calculations agree with those in [28]. The detailed calculations will be presented elsewhere.

The main result can be summarized as follows: Using an atom-photon system, and assuming perfect detection efficiency of the atom (as is usually the case in actual experiments), the minimum photodetection efficiency required for a loophole-free Bell experiment can be lowered to 0.50 in the absence of noise (vs $\eta > 0.67$ for the photon-photon case), and to 0.58 for a 0.31% background noise (vs $\eta > 0.75$ for the photon-photon case). This result suggests a new approach for performing a loophole-free Bell experiment.

Note added in proof.—After submitting this manuscript, we have become aware that some results presented here have been independently derived by Brunner *et al.* [41].

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