A possible unification of the Copenhagen and the Bohm interpretations using local realism

Jan-Åke Larsson

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A POSSIBLE UNIFICATION OF THE COPENHAGEN AND THE BOHM INTERPRETATIONS USING LOCAL REALISM

Jan-Åke Larsson

Matematiska Institutionen, Linköpings Universitet
SE-581 83 Linköping, Sweden
E-mail: jalar@mai.liu.se

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It is well-known in the physics community that the Copenhagen interpretation of quantum mechanics is very different from the Bohm interpretation. Usually, a local realistic model is thought to be even further from these two, as in its purest form it cannot even yield the probabilities from quantum mechanics by the Bell theorem. Nevertheless, by utilizing the "efficiency loophole" such a model can mimic the quantum probabilities, and more importantly, in this paper it is shown that it is possible to interpret this latter kind of local realistic model such that it contains *elements of reality* as found in the Bohm interpretation, while retaining the *complementarity* present in the Copenhagen interpretation.

Key words: Copenhagen interpretation, Bohm interpretation, Local Realism, Bell inequality.

1. INTRODUCTION

The Copenhagen interpretation [1-4] and the Bohm interpretation [5-7] are fundamentally different. The former speaks about "wave function" and measurement setup as description of Nature, in which a fundamental property is *complementarity*; a measurement setup to measure the position of a system makes the very concept of momentum for the same system meaningless. The latter, on the other hand, is a *realistic* description where the two concepts position and momentum are valid simultaneously, and the quantum properties arise from a "quantum potential" which is affected by the measurement setup. It would seem that these interpretations are too far apart for it to be possible to unite

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the two, except for the underlying quantum formalism. In this paper an attempt is made to find an interpretation that contains elements of both, and to do this we will need to look into the probability theory of the two interpretations to see in which way the two differ. The probability theory of local realistic models will also be looked into, and it will be shown that such models where effects of lowered detector efficiency are included may be viewed as containing elements from both the above interpretations (a discussion of lowered visibility is omitted for reasons of brevity). Although there are many subtle issues to be discussed in this context, the present discussion will have to be limited to probability theory, to allow a transparent presentation.

Probabilities and expectation values are obtained in quantum mechanics in a fashion fundamentally different from standard Kolmogorovian probability theory, and only the basics will be mentioned here, using the spin of an electron as an example. There is of course no problem in extending the treatment. In quantum mechanics, a system is described by a normalized vector $|\psi\rangle$, the "quantum state", in a Hilbert space $\mathcal{H}$, the space of all states. An object that has a certain property α, e.g., "the electron has spin up along the x-axis", corresponds to a vector that lies in the subspace $\mathcal{H}_\alpha$ associated with that property, while an object not having the property corresponds to a vector in the orthogonal complement $\mathcal{H}_\alpha^\perp$ of that subspace. Formally, an object for which $|\psi\rangle$ lies in $\mathcal{H}_\alpha$ has $P(\alpha) = 1$, whereas when $|\psi\rangle$ lies in $\mathcal{H}_\alpha^\perp$, $P(\alpha) = 0$. More generally, the probability of the system having the property α is calculated as the inner product of the vector $|\psi\rangle$ and its projection onto $\mathcal{H}_\alpha$, normally denoted

$$P(\alpha) = \langle\psi|\hat{W}_\alpha|\psi\rangle, \tag{1}$$

where $\hat{W}_\alpha$ is the projection operator onto $\mathcal{H}_\alpha$.

To include measurement results in our quantum description, let us look at our example, where spin up would correspond to the value $a = \hbar/2$ of a certain magnetic moment associated with the electron. This measurement on an electron can yield two results: $A = \pm\hbar/2$, and these values are encoded into the measurement operator

$$\hat{A} = +\frac{\hbar}{2}\hat{W}_\alpha - \frac{\hbar}{2}\hat{W}_\alpha^\perp. \tag{2}$$

More generally, for a set of different outcomes α_i and their corresponding measurement outcomes a_i,

$$\hat{A} = \sum_i a_i \hat{W}_{\alpha_i}. \tag{3}$$

It is therefore important to know the eigenvalues of our measurement operator; they encode the possible measurement results[1]. The expectation value of the measurement results is given by

$$E(A) = \sum_i a_i P(\alpha_i) = \sum_i a_i \langle\psi|\hat{W}_{\alpha_i}|\psi\rangle = \langle\psi|\hat{A}|\psi\rangle. \tag{4}$$

Let us compare the description above with Kolmogorovian probability theory, to see the differences. In this description, the properties of the system are represented by a "sample" which encodes a pre-required existence of the properties of the system. This sample is represented as a point λ in a sample space Λ. An object that has a certain property α (e.g., "the electron has spin up along the x-axis") would in this description have a sample lying in Λ_α, the subset of all samples corresponding to this property, whereas a system not having the property would have a sample lying in the complement set $\Lambda_\alpha^{\mathcal{C}}$. In probability theory such a subset is called an "event", and on the collection of all events $\mathcal{F}$ (a collection of sets) we have a probability measure P, allowing us to calculate the probability as

$$P(\alpha) = \int_{\Lambda_\alpha} dP. \tag{5}$$

Again, we introduce measurement results into the formalism. Here, they are encoded into random variables (RV:s) which are functions from the sample space into (e.g.) the real numbers. In our case, the measurement result A is described by a RV (see Fig. 1)

$$A(\lambda) = \begin{cases} +\hbar/2, & \text{if } \lambda \in \Lambda_\alpha, \\ -\hbar/2, & \text{if } \lambda \in \Lambda_\alpha^{\mathcal{C}}. \end{cases} \tag{6}$$

In the general case, A is

$$A(\lambda) = a_i, \text{ if } \lambda \in \Lambda_{\alpha_i}, \tag{7}$$

and, given this, the expectation value is

$$E(A) = \sum_i a_i P(\alpha_i) = \sum_i a_i \int_{\Lambda_{\alpha_i}} dP = \int_\Lambda A(\lambda) dP. \tag{8}$$

The formal differences between quantum mechanics and Kolmogorovian probability theory are visible here; compare (1) with (5) and (4) with (8).

[1]More generally, it is important to know the spectrum of our operator. For simplicity, it is here assumed that $\{a_i\}$ is finite and that all a_i are different.

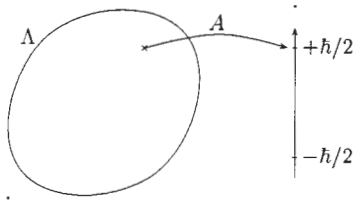

Fig. 1. A random variable (RV) is a function, which maps points in the sample space into (e.g.) the real numbers, or as in our example one of the two points $\pm\hbar/2$.

2. THE COPENHAGEN AND BOHM INTERPRETATIONS

To illuminate the differences between the two mentioned interpretations, the system used as an example above will be extended from the spin of one electron into a system consisting of two electrons with total spin 0, in the spirit of Bohm's version of the EPR paradox as used in the Bell inequality [8-11]. The example contains three devices; a source and two detectors, all separated at a great (space-like) distance. The source emits a pair of electrons in a state of total spin 0, one moving towards each detector. Each detector can be used to measure the spin of an electron along any direction we choose, labeled $\mathbf{a}$ at one detector and $\mathbf{b}$ at the other, while their results will be labeled A and B, resp.. The total spin 0 of the electron pair implies that if $\mathbf{a} = \mathbf{b}$, the spin measurement results obey $A = -B$. Having set the stage, we may now return to the question at hand; the relation between the two interpretations.

The Copenhagen interpretation takes the stand that the studied object has no properties but its quantum description ("wave-function") until we have made a measurement. Granted, the probability of the object having a certain property is given by quantum mechanics, but the property itself does not exist until we measure it. For example, a statement like "This electron has spin up along the x-axis" is very much dependent upon the measurement device used. A measurement device constructed to discern electrons with spin up along the x-axis from other electrons is needed to ascertain any validity of the statement. If the measurement device measures e.g. the spin along the y-axis, the electron *cannot be said to have the property spin up or down along the x-axis.* The statement is neither true nor untrue, it is meaningless, because in this interpretation, spin along perpendicular axes are *complementary*; if one is measured, the other is undefined.

We have a description allowing us to calculate probabilities (1) and expectation values (4), but there is no probability space or probability measure in the Kolmogorovian sense. When the measurement

is made[2], there emerges a probabilistic description: a sample space, a probability measure, and two RVs. Notable is that if we were to make a different measurement, we would get a completely different sample space with a completely different probability measure and completely different RVs. This is where complementarity emerges in the probabilistic description; Λ, P, A, and B change completely for different experimental setups.

I: The Copenhagen interpretation[3]:

Given **a** and **b**, there exists a probabilistic description $(\Lambda, \mathcal{F}, P)$ with

$$A : \Lambda \to \pm\hbar/2, \quad B : \Lambda \to \pm\hbar/2.$$

The description does not exist unless **a** and **b** are given.

In the Bohm interpretation, on the other hand, an object indeed possesses properties which have well-defined values; *elements of reality*. In our example, the statement "This electron has spin up along a certain axis" is either true or untrue; it is always a meaningful statement. These properties follow the laws of ordinary mechanics with the modification that the behavior is affected by an additional potential, a quantum potential originating in the quantum description. It permeates the whole system, causing it to behave in classically unexpected ways, and the potential is much dependent on the measurement setup we have chosen; for different setups, the potential is different. Two different measurement devices designed to measure spin along different axes would yield different quantum potentials, causing the object's properties to behave differently. This in itself is not strange, but what may seem strange is that changes in the measurement setup affects the quantum potential instantaneously throughout the system, and thus,

[2]If preferred, one could say that the probabilistic description appears when the measurement device is chosen.

[3]Another way of describing this is known as *quantum probability* (see e.g. [12]), which would in the notation presented here be

I': Quantum probability

There always exist probabilistic descriptions $(\Lambda(\mathbf{a},\mathbf{b}), \mathcal{F}(\mathbf{a},\mathbf{b}), P_{\Lambda(\mathbf{a},\mathbf{b})})$ for all setups with

$$A(\mathbf{a},\mathbf{b}) : \Lambda(\mathbf{a},\mathbf{b}) \to \pm\hbar/2, \quad B(\mathbf{a},\mathbf{b}) : \Lambda(\mathbf{a},\mathbf{b}) \to \pm\hbar/2.$$

A probabilistic description always exists, but different experimental settings are difficult to compare, because there is no common probability space to compare the results in. Furthermore, the description changes abruptly even for small changes of the parameters **a** and **b**. In this paper the concept of realism is taken to contain less dramatic changes.

a change of the setting **a** changes the quantum potential at the second site instantaneously, so that the result B is affected.

The Bohm interpretation says that the object does have a definite value of the property, but it is part of an ensemble in which the values may be different; probabilities and expectations are given by (1) and (4)[4]. Here the probabilistic description always exists, but the Bohm interpretation is nonlocal as the result at one detector (e.g., B) is affected by both settings **a** and **b**.

II: The Bohm interpretation:

There always exists a probabilistic description $(\Lambda, \mathcal{F}, P)$ with

$$A(\mathbf{a}, \mathbf{b}) : \Lambda \to \pm\hbar/2, \quad B(\mathbf{a}, \mathbf{b}) : \Lambda \to \pm\hbar/2.$$

In this way the two key pieces of these two interpretations are exhibited: Complementarity in the Copenhagen interpretation by the change in the probabilistic description; Realism in the Bohm interpretation by the permanent existence of one such description.

3. LOCAL REALISM

Both the above interpretations contain a seeming nonlocality, in the Copenhagen interpretation as the collapse of the wave function at measurement, and in the Bohm interpretation as the instantaneous reaction in the quantum potential to changes in the measurement setup. Even though there is no superluminal information or energy transfer induced by this [13], the very hint of a superluminal influence has motivated a search for a local realistic model that describes the process in question. It is well known that the Bell inequality [10, 11] proves such a construction impossible in the case of ideal detectors. In the case of nonideal detectors as found in experiments [14-17], such a construction is not excluded [18, 19] but quite possible [20-22] and one may ask where such constructions fit in to the interpretational considerations presented above.

In the case of lowered efficiency, there are in principle three possible "measurement results": $\pm\hbar/2$ and "no detection". The previous description of RVs is modified in this case so that the RV A is defined on a certain *subset* of Λ [19] (see Fig. 2):

$$A : \Lambda_A \to \pm\hbar/2. \tag{9}$$

If one asks what $A(\lambda)$ is, there can be three possible results: $\pm\hbar/2$ if λ is in Λ_A, and "undefined" for other λ:s. The efficiency η is the

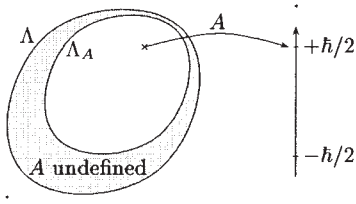

Fig. 2. A random variable which is only defined on a subset Λ_A of the full sample space Λ; here the efficiency is less than 1.

probability of getting a result, e.g., $\eta = P(\Lambda_A)$.

In our example, a local probabilistic model would state that the result at one detector A only depends on the setting $\mathbf{a}$ at that detector and the sample λ, which also means that the definition-set of $A(\mathbf{a})$ only depends on $\mathbf{a}$ ($\Lambda_A(\mathbf{a})$). We have

IIIa: Local realistic interpretation:

There always exists a probabilistic description $(\Lambda, \mathcal{F}, P)$ with

$$A(\mathbf{a}) : \Lambda_A(\mathbf{a}) \to \pm\hbar/2, \quad B(\mathbf{b}) : \Lambda_B(\mathbf{b}) \to \pm\hbar/2.$$

This is a probabilistic description which includes *elements of reality* in the spirit of the Bohm interpretation; *one* probabilistic description of what the result will (or will not) be. However, the ensemble changes upon change of our detector settings, which from the point of view of a measurement is visible as a change in the probabilistic description as in the Copenhagen interpretation; *the sample set of a measurement (Λ_A, say) changes with the setting of* $\mathbf{a}$. This is where *complementarity* arises in this interpretation.

The probability of the event α is the many-experiment limit of the ratio of α-events to the total number of events. Since the undetected events are just that; all we see is events having their sample in $\Lambda_A(\mathbf{a})$, the probability measure to use on the obtained data is then the restriction of the measure P to the set $\Lambda_A(\mathbf{a})$:

$$P_{\Lambda_A(\mathbf{a})}(\alpha) = P\big(\alpha|\Lambda_A(\mathbf{a})\big) = \frac{P\big(\alpha \cap \Lambda_A(\mathbf{a})\big)}{P\big(\Lambda_A(\mathbf{a})\big)}. \tag{10}$$

In a two-particle experiment, i.e., when correlating results from *both* detectors, single events are quite naturally discarded as measurement errors. The remaining results come from a smaller subset

[4]To clarify, these two interpretations agree on the formulas (1) and (4), and when the object is described by a vector that lies entirely in some $\mathcal{H}_{\alpha_i}$, also the Copenhagen interpretation agrees that the relevant property exists.

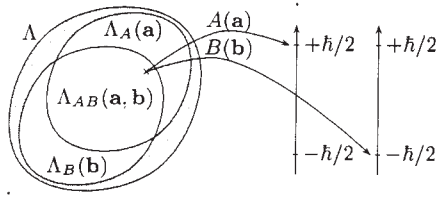

Fig. 3. A two-particle experiment in this description.

in the sample space, the subset where *both* A and B are defined: $\Lambda_{AB}(\mathbf{a},\mathbf{b}) = \Lambda_A(\mathbf{a}) \cap \Lambda_B(\mathbf{b})$ (see Fig. 3). We then have

IIIb: Local realistic interpretation (cont.):

For two-particle experiments, the *product* is

$$A(\mathbf{a})B(\mathbf{b}) : \Lambda_{AB}(\mathbf{a},\mathbf{b}) \rightarrow \pm\hbar^2/4.$$

Our sample space is now restricted to $\Lambda_{AB}(\mathbf{a},\mathbf{b})$, which again changes when the settings change. Our probability measure is also affected by this, changing with the settings. The probability measure to use here is

$$P_{\Lambda_{AB}(\mathbf{a},\mathbf{b})}(\alpha) = P\big(\alpha\big|\Lambda_{AB}(\mathbf{a},\mathbf{b})\big). \tag{11}$$

There is a seeming introduction of nonlocality in IIIb because of the presence of both parameters in the set $\Lambda_{AB}(\mathbf{a},\mathbf{b})$; this is only apparently so, since the result and detection in, say, the A experiment only depends on $\mathbf{a}$. Only because we restrict our resulting data to events where *both* A and B are defined does the apparent nonlocality in IIIb appear. The models that have been constructed to mimic the quantum statistics fit well into this picture [20-22].

4. CONCLUSIONS

In conclusion, local realistic models may yield the correlations by quantum mechanics by using the "efficiency loophole" in the Bell inequality, or more accurately, using a changing ensemble, and an interpretation of this is presented here which is consistent with both the Copenhagen and the Bohm interpretations. It is worth mentioning that the discussion in this paper assumes that the quantum-mechanical predictions are correct, but that the efficiency problem is fundamental rather than due to inexact measurements. There is a widespread belief that the detector efficiency problem is only of minor importance, soon to be solved by our experimental efforts. J. S. Bell puts it as follows:

> " ... if you can demonstrate that quantum mechanics imposes some limit on the degree to which the ideal experiment can be approached, I will be very interested. I will also be very surprised! Experimental colleagues have told me that optical photon counters could be made as efficient as we like if size and expense are unlimited." [23]

There is yet no two-particle experiment which successfully exceeds the current efficiency bound [18, 19], but the subject is currently under very active research, both theoretically and experimentally.

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