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Ion streaming instability in a quantum dusty magnetoplasma

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It is shown that a relative drift between the ions and the charged dust particles in a magnetized quantum dusty plasma can produce an oscillatory instability in a quantum dust acousticlike wave. The threshold and growth rate of the instability are presented. The result may explain the origin of low-frequency electrostatic fluctuations in semiconductors quantum wells. © 2008 American Institute of Physics. [DOI: 10.1063/1.2909533]

Recently, there has been a great deal of interest in investigating the linear and nonlinear properties of electrostatic¹⁻⁵ and electromagnetic waves⁶⁻⁸ in dense quantum plasmas. The latter, which are ubiquitous in compact astrophysical bodies⁹ (e.g., the interiors of white dwarf stars, magnetars and supernovae) as well as in micro- and nanoscale objects¹⁰ (e.g., nanowires, ultrasmall semiconductor devices), have an extremely high electron number density. Electron tunneling caused by the quantum Bohm force¹¹ plays then a very important role for phenomena occurring at the nanoscales.

Large amplitude electrostatic waves in an unmagnetized quantum plasma can be excited by two-stream¹² and parametric instabilities.¹³ Our objective here is to investigate the stability properties of low-frequency (in comparison with the ion gyrofrequency), long wavelength (in comparison with the ion gyroradius) electrostatic quantum dust acoustic waves in a uniform dense magnetoplasma. The latter is composed of electrons, ions, and charged dust particles. At equilibrium, we have¹⁴ $Z_i e n_{i0} = e n_{e0} - q_d n_{d0}$, where Z_i is the ion charge state, e is the magnitude of the electron charge, q_d is the dust charge ($q_d = -e Z_{d0}$ for negatively charged dust grains and $e Z_{d0}$ for positively charged dust grains, where Z_{d0} is the number of charges on dust grains), and n_{j0} is the unperturbed particle number density (j is e for the electrons, i for ions, and d for dust grains). We suppose that the micrometer (or nanometer) sized dust grains are spherical, and that they can be treated like point charges. Charged dust impurities may

coexist with the electrons and ions in semiconductor quantum wells, as well as in astrophysical radiative environments and in micromechanical systems.

Let us consider a uniform electron-ion-dust plasma in an external magnetic field $B_0 \hat{z}$, where B_0 is the strength of the magnetic field and \hat{z} is the unit vector along the z axis in a Cartesian coordinate system. In the presence of the equilibrium electric field \mathbf{E}_0 , the electrons and ions will have a drift $\mathbf{u}_{e0,i0} = (c/B_0) \mathbf{E}_0 \times \hat{z}$, while the dust grain velocity is $\mathbf{u}_{d0} = (q_d/m_d \nu_{dn}) \mathbf{E} (\ll \mathbf{u}_{e0,i0})$, if the electron (ion) gyrofrequency is much larger than the electron-neutral (ion-neutral) collision frequency, and the dust gyrofrequency is much smaller than the dust-neutral collision frequency ν_{dn} . Here, c is the speed of light in vacuum and m_d is the dust grain mass. Let us now investigate the stability of our equilibrium against low-frequency (in comparison with the ion gyrofrequency $\omega_{ci} = Z_i e B_0 / m_i c$, where m_i is the ion mass), long wavelength (in comparison with the ion thermal gyroradius) electrostatic perturbations whose electric field is $\mathbf{E} = -\nabla \phi(\mathbf{r}, t)$, where ϕ is the electrostatic potential. For $|(\partial_t + \mathbf{u}_{e0} \cdot \nabla)^2 n_{e1}| \ll \omega_{ce}^2 n_{e1}$ and $V_{Fe}^2 |\partial_z^2 n_{e1}| \ll (\hbar^2 / 4m_e^2) |\nabla^4 n_{e1}|$, the electron number density perturbation $n_{e1} (\ll n_{e0})$ is deduced from¹⁵

$$\nabla^2 n_{e1} + \frac{4m_e n_{e0}}{\hbar^2} \phi = 0, \quad (1)$$

where $\omega_{ce} = e B_0 / m_e c$ is the electron gyrofrequency, m_e is the electron mass, $V_{Fe} = (k_B T_{Fe} / m_e)^{1/2}$, k_B is the Boltzmann constant, T_{Fe} is the Fermi electron temperature, and \hbar is Planck's constant divided by 2π . Equation (1) represents a balance between the electrostatic and quantum forces.

The ion number density perturbation $n_{i1} (\ll n_{i0})$ in the electrostatic field is obtained from the ion continuity equation

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_{i0} \cdot \nabla \right) \left(n_{i1} - \frac{c n_{i0}}{B_0 \omega_{ci}} \nabla_{\perp}^2 \phi \right) + n_{i0} \frac{\partial v_{iz}}{\partial z} = 0, \quad (2)$$

where the magnetic field aligned ion velocity perturbation v_{iz} is obtained from the parallel (to \hat{z}) component of the ion momentum equation

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$$\left(\frac{\partial}{\partial t} + \mathbf{u}_{i0} \cdot \nabla\right) v_{iz} + \frac{Z_i e}{m_i} \frac{\partial \phi}{\partial z} = 0. \quad (3)$$

We note that the second term in parenthesis of the right-hand side of Eq. (2) comes from the divergence of the ion flux involving the ion polarization drift velocity $\mathbf{v}_{ip} = -(c/B_0 \omega_{ci}) \times (\partial_t + \mathbf{u}_{i0} \cdot \nabla) \nabla_{\perp} \phi$, and that Eq. (2) is valid for $v_{in} \ll (\partial_t + \mathbf{u}_{i0} \cdot \nabla) \ll \omega_{ci}$, where v_{in} is the ion-neutral collision frequency. Eliminating v_{iz} from Eq. (2) by using Eq. (3), we have

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_{i0} \cdot \nabla\right)^2 \left(n_{i1} - \frac{cn_{i0}}{B_0 \omega_{ci}} \nabla_{\perp}^2 \phi\right) - \frac{Z_i e n_{i0}}{m_i} \frac{\partial^2 \phi}{\partial z^2} = 0. \quad (4)$$

We assume that the dust grains are unmagnetized and collisionless. This approximation holds for $|\partial_t n_{d1}| \gg \omega_{cd} n_{d1}$, $v_{dn} n_{d1}$, where $\omega_{cd} = Z_{d0} e B_0 / m_d c$ is the dust gyrofrequency and m_d is the dust mass. The dust number density perturbation n_{d1} ($\ll n_{d0}$) is then obtained from the dust continuity equation

$$\frac{\partial n_{d1}}{\partial t} + n_{d0} \nabla \cdot \mathbf{v}_d = 0, \quad (5)$$

where the dust fluid velocity perturbation \mathbf{v}_d is determined from

$$\frac{\partial \mathbf{v}_d}{\partial t} = -\frac{q_d}{m_d} \nabla \phi. \quad (6)$$

Eliminating \mathbf{v}_d from Eq. (5) by using Eq. (6), we obtain

$$\frac{\partial^2 n_{d1}}{\partial t^2} - \frac{q_d n_{d0}}{m_d} \nabla^2 \phi = 0. \quad (7)$$

The equations above are closed with the Poisson equation

$$\nabla^2 \phi = 4\pi(e n_{e1} - Z_i e n_{i1} - q_d n_{d1}). \quad (8)$$

Supposing that ϕ and n_{j1} are proportional to $\exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r})$, where ω and \mathbf{k} are the frequency and the wave vector, respectively, we Fourier transform Eqs. (1), (4), (7), and (8), and combine the resultant equations to obtain the dispersion relation

$$1 + \frac{k_q^4}{k^4} + \frac{\omega_{pi}^2 k_{\perp}^2}{\omega_{ci}^2 k^2} - \frac{\omega_{pi}^2 k_z^2}{(\omega - \mathbf{k} \cdot \mathbf{u}_{i0})^2 k^2} - \frac{\omega_{pd}^2}{\omega^2} = 0, \quad (9)$$

where $k_q = (4m_e^2 \omega_{pe}^2 / \hbar^2)^{1/4} \equiv \lambda_q^{-1}$, $\omega_{pe} = (4\pi n_{e0} e^2 / m_e)^{1/2}$ is the electron plasma frequency, $\omega_{pi} = (4\pi n_{i0} Z_i^2 e^2 / m_i)^{1/2}$ is the ion plasma frequency, and $\omega_d = (4\pi Z_{d0}^2 e^2 n_{d0} / m_d)^{1/2}$ is the dust plasma frequency. We have denoted $k^2 = k_{\perp}^2 + k_z^2$, where \mathbf{k}_{\perp} and k_z are the components of \mathbf{k} across and along $\hat{\mathbf{z}}$.

Two comments are in order. First, in the absence of the DC electric field and dust grains, Eq. (9) yields

$$\omega = \frac{\omega_{pi} k_z k \lambda_q^2}{(1 + k^4 \lambda_q^4 + \omega_{pi}^2 k_{\perp}^2 k^2 \lambda_q^4 / \omega_{ci}^2)^{1/2}}, \quad (10)$$

which is the frequency of our new, low-frequency, obliquely (to $\hat{\mathbf{z}}$) propagating dispersive electrostatic wave in a magnetized quantum plasma. The new feature to the wave is attributed to the non-Boltzmann electron response, given by Eq.

(1). Second, we investigate the two-stream instability¹⁶ involving the streaming of ions against charged dust grains. For this purpose, we let $\omega = \mathbf{k} \cdot \mathbf{u}_{i0} + \delta$ in Eq. (9) and assume $\delta \ll |\mathbf{k} \cdot \mathbf{u}_{i0}|$ to obtain

$$1 + k^4 \lambda_q^4 + \frac{k_{\perp}^2 k^2 \lambda_q^4 \omega_{pi}^2}{\omega_{ci}^2} - \frac{k_z^2 k^2 \lambda_q^4 \omega_{pi}^2}{\delta^2} - \frac{k^4 \lambda_q^4 \omega_{pd}^2}{(\mathbf{k} \cdot \mathbf{u}_{i0})^2} \left(1 - \frac{2\delta}{\mathbf{k} \cdot \mathbf{u}_{i0}}\right) = 0. \quad (11)$$

Equation (11) reveals that for

$$k u_{i0} \cos \theta = \frac{k^2 \lambda_q^2 \omega_{pd}}{(1 + k^4 \lambda_q^4 + k_{\perp}^2 k^2 \lambda_q^4 \omega_{pi}^2 / \omega_{ci}^2)^{1/2}}, \quad (12)$$

we have

$$\delta^3 = (k u_{i0} \cos \theta)^3 k_z^2 \omega_{pi}^2 / 2k^2 \omega_{pd}^2, \quad (13)$$

where θ is the angle between \mathbf{k} and \mathbf{u}_{i0} . Solutions of Eq. (13) are

$$\delta = \frac{1 \pm i\sqrt{3}}{2^{4/3}} k u_{i0} \cos \theta (k_z/k)^{2/3} (\omega_{pi}/\omega_{pd})^{2/3}. \quad (14)$$

The growth rate for our purposes reads

$$\gamma = \frac{\sqrt{3} k^2 \lambda_q^2 \omega_{pd} (k_z/k)^{2/3} (Z_i^2 n_{i0} m_d / Z_{d0}^2 n_{d0} m_i)^{1/3}}{2^{4/3} (1 + k^4 \lambda_q^4 + k_{\perp}^2 k^2 \lambda_q^4 \omega_{pi}^2 / \omega_{ci}^2)^{1/2}}. \quad (15)$$

The threshold condition and the growth rate, given by Eqs. (12) and (15), depend on the quantum parameter λ_q . Hence, our new electron response, given by Eq. (1), plays an important role in the ion streaming instability driving low-frequency electrostatic fluctuations having the real part of the frequency

$$\omega_r = \frac{k^2 \lambda_q^2 \omega_{pd}}{(1 + k^4 \lambda_q^4 + k_{\perp}^2 k^2 \lambda_q^4 \omega_{pi}^2 / \omega_{ci}^2)^{1/2}} \left[1 + \frac{1}{2^{4/3}} \left(\frac{k_z \omega_{pi}}{k \omega_{pd}}\right)^{2/3}\right], \quad (16)$$

which represents a quantum dust acoustic wave in a dense magnetoplasma.

As an illustration, we apply our results to a gedanken semiconductor quantum well experiment for verifying our theoretical prediction. The typical parameters representative of semiconductor quantum wells are:¹⁷ $n_{e0} \sim 5 \times 10^{16} \text{ cm}^{-3}$, $n_{i0} \sim 4 \times 10^{16} \text{ cm}^{-3}$, $n_{d0} \sim 10^{13} \text{ cm}^{-3}$, $Z_i = 1$, $Z_{d0} \sim 10^3$, $m_d = 10^{12} m_i$, $B_0 \sim 10^4 \text{ G}$, and $T_{Fe} \sim 85.7 \text{ K}$. Accordingly, we have $\omega_{pe} = 1.26 \times 10^{13} \text{ s}^{-1}$, $\omega_{pi}/\omega_{ci} = 3 \times 10^3$, $\omega_{pd} = 4 \times 10^6 \text{ s}^{-1}$, and $\lambda_q = 2 \times 10^{-7} \text{ cm}$. Taking $k \lambda_q \sim 0.1$, $k_z/k \sim 0.01$, we obtain, from Eqs. (15) and (16), $\gamma \sim \omega_r = 2 \times 10^3 \text{ s}^{-1}$. Thus, within a fraction of milliseconds our quantum dust acoustic wave in a semiconductor quantum well can be excited.

To summarize, we have shown that the free energy stored in ion streams (relative to the charged dust grains) can be coupled to the low-frequency quantum dust acoustic wave in a dense magnetoplasma containing degenerate electrons, ions and dust grains. The knowledge of the threshold condition as well as the growth rate and the real part of the wave frequency, as presented here, is essential for identifying non-

thermal electrostatic fluctuations that may originate in dense dusty magnetoplasmas, such as those in semiconductors quantum wells. The dust acoustic wave in a magnetized semiconductor plasma can be used as a diagnostic tool for inferring the dust charge.

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