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Wake field generation and nonlinear evolution in a magnetized electron-positron-ion plasma

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The nonlinear propagation of a circularly polarized electromagnetic (CPEM) wave in a strongly magnetized electron-positron-ion plasma is investigated. Two coupled equations describing the interaction between a high-frequency CPEM wave and the low-frequency electrostatic wake field are derived. It is found that the generation of the wake fields partly depends on the presence of the ion species and the external magnetic field. The wake field generation in turn leads to deceleration and frequency down conversion of the electromagnetic pulse. © 2008 American Institute of Physics.
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I. INTRODUCTION

Electron-positron (pair) plasmas are composed of electrons and positrons, which have the same mass and opposite charge. Such pair plasmas appeared in the early Universe,^{1,2} and are frequently encountered in bipolar outflows (jets) in active-galactic nuclei,^{3,4} in microquasars,⁵ in pulsar magnetospheres,^{6–10} in magnetars,¹¹ in cosmological gamma ray fireballs,¹² in solar flares,¹³ and at the center of our galaxy.¹⁴ Multiterawatt and petawatt laser pulses interacting with solid density matters can create pair plasmas.^{15–18} Colliding electromagnetic pulses also generate pairs from vacuum.¹⁹ The pair plasmas at the surface of fast rotating neutron stars and magnetars are held in strong magnetic fields, while superstrong magnetic fields can be created in intense laser-plasma interaction experiments. Accordingly, the understanding of collective phenomena in strongly magnetized pair plasmas has been a topic of significant interest.^{20–23} Specifically, it is to be noted that in a magnetized pair plasma, we have new wave modes whose counterparts do not exist in an electron-ion magnetoplasma.²⁴ Furthermore, both astrophysical and laboratory plasmas contain a fraction of ions besides the electrons and positrons. In such environments, the linear and nonlinear wave propagation characteristics of the electrostatic and electromagnetic modes are modified.^{25–33}

In this paper, we consider the nonlinear propagation of magnetic field-aligned circularly polarized electromagnetic (CPEM) waves^{34,35} in a strongly magnetized electron-positron-ion ($e-p-i$) plasma. It is shown that the ponderomotive force of the CPEM waves can excite electrostatic plasma wakefields. Our manuscript is organized in the following fashion. In Sec. II we discuss the linear dispersion relation,

the group velocity, and the group dispersion for the magnetic field-aligned CPEM waves in an $e-p-i$ magnetoplasma. Section III considers the nonlinear interactions between finite amplitude CPEM waves and electrostatic plasma oscillations (EPOs), and presents the coupled equations. Nonlinearities associated with the electron and positron mass variations, as well as with the CPEM ponderomotive force, are incorporated in the dynamical equations for the modulated CPEM waves and the driven EPOs. Section IV presents the conservation laws for nonlinearly coupled CPEM and EPO waves, and discusses the excitation of wakefields. Section V contains a brief summary of our investigation.

II. THE LINEAR DISPERSION RELATION

We consider a magnetized quasineutral $e-p-i$ plasma. At equilibrium, we have $n_{e0} = n_{i0} + n_{p0}$, where n_{j0} is the unperturbed number density of the particle species j (j equals e for the electrons, i for the ions, and p for the positrons). The external magnetic field is $\mathbf{B}_0 = B_0 \hat{z}$, where B_0 is the strength of the magnetic field and \hat{z} is the unit vector along the z axis in a Cartesian coordinate system. For a right-hand circularly polarized electromagnetic (RCPEM) wave propagating along \hat{z} with frequency $\omega \gg \omega_{pi} = (4\pi n_{i0} e^2 / m_i)^{1/2}$, we have the dispersion relation^{35,36} for the $e-p-i$ plasma with immobile ions,

$$\frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega(\omega + \omega_{ce})} - \frac{\omega_{pp}^2}{\omega(\omega + \omega_{cp})}, \quad (1)$$

where k is the wave number, c is the speed of light in vacuum, $\omega_{pj} = (4\pi n_{j0} e^2 / m)^{1/2}$ with $j = e, p$ is the plasma frequency, $\omega_{ce} = -eB_0 / mc = -\omega_{cp}$ is the electron gyrofrequency, e is the magnitude of the electron charge, and m is the electron/positron mass [for left-hand circular polarization $\omega + \omega_{cp,e}$ is replaced by $\omega - \omega_{cp,e}$ (Refs. 34 and 37)]. In the limit of a strong background magnetic field, i.e., $\omega \ll |\omega_{ce}|$, we can simplify Eq. (1) to obtain

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$$\frac{k^2 c^2}{\omega^2} = 1 + \frac{\Omega_i^2}{\omega \omega_{cp}}, \quad (2)$$

where we have introduced the notation $\Omega_i = (4\pi n_{i0} e^2 / m)^{1/2}$ and used the equilibrium quasineutrality condition. From Eq. (2) we obtain the group velocity,

$$v_g = \frac{kc^2}{\omega + \Omega_i^2 / 2\omega_{cp}}, \quad (3)$$

and the group velocity dispersion $v_g' = dv_g / dk = (v_g / k)(1 - v_g^2 / c^2)$. In the limit $kc \gg \omega_{pe}^2 / \omega_{cp}$, we obtain

$$v_g \approx c \left[1 - \frac{1}{2} \left(\frac{\Omega_i^2}{2\omega_{cp} kc} \right)^2 \right], \quad (4)$$

$v_g' \approx \Omega_i^2 / 4\omega_{cp}^2 k^3 c$, and we have neglected the root with negative sign of the group velocity. Thus, in this limit the RCPEM waves propagate with a speed close to c .

III. NONLINEAR COUPLED EQUATIONS

Next, we consider a modulated nonlinear CPEM wave (ω, k) , where the ponderomotive force induces slowly ($\partial_t \ll \omega$) varying electrostatic oscillations. We still assume that the motion is fast enough for the ions to be immobile. The evolution equation for the perpendicular vector potential amplitude A in the Coulomb gauge can then be written as³⁸

$$i \left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) A + \frac{1}{2} v_g' \frac{\partial^2 A}{\partial z^2} - \sum_{j=e,p} \frac{\omega \omega_{pj}^2}{2(\omega + \omega_{cj})} \frac{v_g}{kc^2} \times \left[\frac{n_{j1}}{n_{j0}} - \frac{kv_{jz} \omega_{cj}}{\omega(\omega + \omega_{cj})} - \frac{e^2 \omega^3 |A|^2}{m^2 c^4 (\omega + \omega_{cj})^3} \right] A = 0, \quad (5)$$

where the group velocity and dispersion are given by Eqs. (3) and (4), respectively. The electrostatic perturbations are described by the density fluctuation n_{j1} , the longitudinal velocity v_{jz} , and the electrostatic potential ϕ . These variables satisfy the continuity equation

$$\frac{\partial n_{j1}}{\partial t} + n_{j0} \frac{\partial v_{jz}}{\partial z} = 0, \quad (6)$$

the momentum equation

$$m \frac{\partial v_{jz}}{\partial t} = -q_j F_j - q_j \frac{\partial \phi}{\partial z} - \frac{T_j}{n_{j0}} \frac{\partial n_{j1}}{\partial z}, \quad (7)$$

and the Poisson equation

$$\frac{\partial^2 \phi}{\partial z^2} = 4\pi e(n_{e1} - n_{p1}), \quad (8)$$

where $q = -e$ for the electrons, $q = e$ for the positrons, and T_j is the temperature. Here $q_j F_j$ denotes the CPEM ponderomotive force, which can be written as³⁸

$$q_j F_j = \frac{e^2 \omega}{m(\omega + \omega_{cj}) c^2} \left[\frac{\partial}{\partial z} + \frac{k\omega_{cj}}{\omega(\omega + \omega_{cj})} \frac{\partial}{\partial t} \right] |A|^2, \quad (9)$$

where $\omega_{cj} = q_j B_0 / mc$.

Equations (6) and (7) can be combined to obtain

$$\frac{\partial^2 n_{e1}}{\partial t^2} + \frac{n_{e0} e}{m} \frac{\partial^2 \phi}{\partial z^2} = \frac{n_{e0} \omega e^2}{m^2 c^2 (\omega - \omega_{cp})} \times \left[\frac{\partial}{\partial z} - \frac{k\omega_{cp}}{\omega(\omega - \omega_{cp})} \frac{\partial}{\partial t} \right] \frac{\partial |A|^2}{\partial z}, \quad (10)$$

and

$$\frac{\partial^2 n_{p1}}{\partial t^2} - \frac{n_{p0} e}{m} \frac{\partial^2 \phi}{\partial z^2} = \frac{n_{p0} \omega e^2}{m^2 c^2 (\omega + \omega_{cp})} \times \left[\frac{\partial}{\partial z} + \frac{k\omega_{cp}}{\omega(\omega + \omega_{cp})} \frac{\partial}{\partial t} \right] \frac{\partial |A|^2}{\partial z}, \quad (11)$$

where we have assumed that $|\partial^2 n_{j1} / \partial t^2| \gg V_{Tj}^2 |\partial^2 n_{j1} / \partial z^2|$. Here $V_{Tj} = (T_j / m)^{1/2}$ is the electron/positron thermal speed.

From Eqs. (8), (10), and (11), and introducing $\omega_{pt} = (\omega_{pe}^2 + \omega_{pp}^2)^{1/2}$, we readily obtain

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pt}^2 \right) \frac{\partial \phi}{\partial z} = \frac{e\omega}{mc^2} \left[\frac{(\omega \Omega_i^2 + \omega_{cp} \omega_{pt}^2)}{(\omega^2 - \omega_{cp}^2)} \right] \frac{\partial |A|^2}{\partial z} - \frac{ek\omega_{cp}}{mc^2 (\omega^2 - \omega_{cp}^2)^2} [2\omega \omega_{cp} \Omega_i^2 + (\omega^2 + \omega_{cp}^2) \omega_{pt}^2] \frac{\partial |A|^2}{\partial t}, \quad (12)$$

which reveals that the electrostatic wake potential is created by the CPEM ponderomotive force only if either the ions or the magnetic field is present in an e - p - i plasma. In the absence of ions (i.e., $\Omega_i = 0$), our wake field differs significantly from Eq. (20) of Ref. 39, which neglects the time-derivative part of the ponderomotive force from the outset, and thus has a minus sign in front of the first term in the right-hand side of that equation.

In the limit of low group velocity $v_g \ll c$ the CPEM wave equation reads

$$i \left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) A + \frac{1}{2} v_g' \frac{\partial^2 A}{\partial z^2} + \frac{\omega v_g}{2kc^2 \omega_{cp}} \frac{eA}{m} \frac{\partial^2 \phi}{\partial z^2} + \frac{\omega^4 \omega_{pt}^2 v_g e^2}{2km^2 c^6 \omega_{cp}^4} |A|^2 A = 0. \quad (13)$$

The coupled Eqs. (12) and (13) describe the nonlinear self-interactions of a high-frequency CPEM pulse combined with the generation of a low-frequency electrostatic wakefield in a strongly magnetized electron-positron-ion plasma. They are useful for studying the modulational instability as well as its long-term spatio-temporal evolution.

Next we study the regime $\omega \ll \omega_{cp}$, where $v_g \approx c$. In that case, Eqs. (12) and (13) reduce to

$$\left[\frac{\partial^2}{\partial t^2} + \omega_{pt}^2 \right] \phi = \frac{e|A|^2 \omega^2}{mc^2 \omega_{cp}^2} \left(\Omega_i^2 + \frac{2\omega_{pt}^2 \omega}{\omega_{cp}} \right) \quad (14)$$

and

$$i\left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z}\right)A + \frac{1}{2}v_g' \frac{\partial^2 A}{\partial z^2} + \frac{\omega}{2\omega_{cp}^2} \left[\frac{e}{mc^2} \left(\Omega_i^2 + \frac{2\omega_{pt}^2}{\omega_{cp}} \right) \Phi A - \sum_{e,p} \frac{e^2 \omega^2 \omega_p^2}{m^2 c^4 \omega_{cp}^2} \left(\frac{v_i^2}{c^2} - \frac{2\omega}{\omega_c} \right) |A|^2 A \right] = 0. \quad (15)$$

Those equations are particularly relevant for pulsar and magnetar atmospheres, with their extremely large magnetic fields. In Sec. IV we shall consider some properties of Eqs. (14) and (15).

IV. CONSERVATION LAWS

In a coordinate system moving with the group velocity and with suitable normalizations [i.e., $\xi = \omega_{pt}(z - v_g t)/c$, etc.], Eqs. (14) and (15) can be put into the generic forms

$$i \frac{\partial A}{\partial \tau} + \frac{\partial^2 A}{\partial \xi^2} = -A(\Phi - \alpha |A|^2), \quad (16)$$

$$\frac{\partial^2 \Phi}{\partial \xi^2} + \Phi = |A|^2, \quad (17)$$

where α is a constant that determines the relative importance of the self-nonlinearity, and in our case is given by

$$\alpha = \frac{\omega_{pt}^2}{\left(\Omega_i^2 + 2\frac{\omega_{pt}^2}{\omega_{cp}}\right)^2} \left(\omega_{pt}^2 \frac{v_i^2}{c^2} + 2\frac{\Omega_i^2 \omega}{\omega_{cp}} \right). \quad (18)$$

Various aspects of the coupled system of this type have been studied by Refs. 40–42. Following the presentation in Ref. 40, we refer to the low-frequency field Φ as the wakefield. The equation system (16) and (17) can be derived from a variational principle. Introducing the Lagrangian density

$$\mathcal{L} = \frac{1}{2} \left(A^* \frac{\partial A}{\partial \tau} - A \frac{\partial A^*}{\partial \tau} \right) - \left| \frac{\partial A}{\partial \xi} \right|^2 + \frac{1}{2} \alpha |A|^4 + \frac{1}{2} \left(\frac{\partial \Phi}{\partial \xi} \right)^2 - \frac{\Phi^2}{2} + |A|^2 \Phi, \quad (19)$$

where the action functional is $\int \mathcal{L}(\Phi, A, A^*) d\xi d\tau$, we obtain Eqs. (16) and (17) by varying Φ and A^* and minimizing the action as usual. The system has three conservation laws,

$$\frac{d}{d\tau} \int_{-\infty}^{\infty} |A|^2 d\xi = 0, \quad (20)$$

$$\frac{d}{d\tau} \int_{-\infty}^{\infty} \left(\left| \frac{\partial A}{\partial \xi} \right|^2 - \frac{\alpha}{2} |A|^4 - |A|^2 \Phi \right) d\xi = 0, \quad (21)$$

$$\frac{d}{d\tau} \int_{\xi_-}^{\xi_+} \left(\frac{\partial A}{\partial \xi} A^* - \frac{\partial A^*}{\partial \xi} A \right) = W_\Phi |_{\xi_-}^{\xi_+}, \quad (22)$$

where $W_\Phi = (1/2)[\Phi^2 + (\partial\Phi/\partial\xi)^2]$. Within the framework of the WKB approximation, $\int_{-\infty}^{\infty} |A|^2 d\xi$ is proportional to the number of high-frequency quanta N , and thus Eq. (20) de-

scribes the conservation of N . Using a quantum mechanical analog, the process of wake field generation can thus be viewed as a parametric process where a high-frequency quantum with frequency ω decays into a low frequency quantum Ω and another high-frequency quantum with frequency $\omega - \Omega$, conserving the total number of high-frequency quanta. Furthermore, Eq. (21) is the conservation equation for the Hamiltonian. Although Eq. (22) does not look like a conservation law, it actually describes the conservation of energy. To see this, it is convenient to choose the positions ξ_\pm before and after the pulse passage, respectively. For this choice we note that the left-hand side of Eq. (22) is proportional to $d/d\tau \int_{-\infty}^{\infty} \kappa |A_\kappa|^2 d\kappa / \int_{-\infty}^{\infty} |A_\kappa|^2 d\kappa = d\Delta\kappa/d\tau = (1/v_g) \times d\Delta\omega/d\tau$. Furthermore, since N is conserved, $d\Delta\omega/d\tau$ is proportional to the high-frequency energy decay rate. Finally, the difference in W_Φ before and after the pulse passage is proportional to the energy transfer rate to the wake field. Thus we deduce that Eq. (22) implies energy conservation.

V. SUMMARY AND CONCLUSIONS

To summarize, we have considered the nonlinear interactions between magnetic field-aligned CPEM waves and electrostatic plasma oscillations (EPOs) in a pair-ion magnetoplasma. It is found that the EPOs are generated by the ponderomotive force of the CPEM waves only if either the ions or the external magnetic field in a pair-ion plasma are present. The present nonlinear wave-wave interactions provide the possibility of CPEM pulse localization and the generation of wakefields. Localized EM pulses, in association with wakefields, can be identified in observations from astrophysical settings as well as from next generation intense laser-solid density plasma experiments where pairs and ions appear simultaneously.

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