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## Retardation-enhanced van der Waals force between thin metal films

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We recently investigated the van der Waals force between thin metal films. Under certain conditions this force decrease with separation to a fractional power. In the present work we use optical data of metals and the zero-temperature Lifshitz formalism to demonstrate a retardation effect. The retarded attraction between thin metal films may be larger than the nonretarded attraction. This property is related to a comparatively weak retardation dependence of the energy that originates from the transverse magnetic modes. At separations where the transverse electric modes give a significant contribution, the net effect can actually be an increased attraction. This effect vanishes with increasing film thickness and with increasing dissipation.

### I. INTRODUCTION

The possibility of measuring changes in the zero-point energies of vacuum fluctuations has recently received a great deal of attention. One of the most intriguing effects is the Casimir attraction<sup>1</sup> between neutral conducting surfaces. Lamoreaux<sup>2,3</sup> measured the force between a sphere and a half space in the 0.6–6  $\mu\text{m}$  range. The major contribution to this force was an electric force that vanished with separation as  $1/d$ . The experimental data points were fitted to the best  $1/d$  dependence and that part of the interaction was subtracted off. In this way the coefficients  $\alpha$  and  $\beta$  were determined for a force law  $F(r) = \alpha/r^\beta$ , with an estimated accuracy of 5%. This force has been shown to agree rather well with the zero-temperature Casimir force between a gold sphere and a gold half space.<sup>4–6</sup> Thermal corrections to the Casimir force between gold surfaces were recently investigated.<sup>7</sup>

As discussed by Barash,<sup>8</sup> the van der Waals (vdW) force between thin metal films may have half-integer separation dependence. The origin of this unusual separation dependence is the normal modes, which are two-dimensional longitudinal plasmons. When two films, each with a thickness  $\delta$  and a density  $n$ , are brought together, the modes on the two films interfere. The total change in the zero-point energy of these modes has a  $d^{-5/2}$  dependence:

$$E^{vdW} \approx -0.012562e\hbar \sqrt{n\delta}/(\sqrt{md^{5/2}}). \quad (1)$$

Another system with this separation dependence is a pair of quantum wells.<sup>9</sup> We recently used optical data to calculate the free energy of attraction between thin metal films<sup>10</sup> for separations less than 0.1  $\mu\text{m}$ . A metal film cannot be much thicker than 100  $\text{\AA}$  to be considered as thin. In the present work we investigate retardation effects on the interaction between metal films. The effect of retardation is usually a lowering of the attraction. In the weakly retarded limit a very unusual thing may occur. There is a possibility that the total effect of including retardation in this system can be an increased interaction. If the attraction is interpreted as a long range retarded van der Waals energy, this is quite counterintuitive. It becomes understandable if the attraction is viewed as a change in the energy of the electromagnetic fields in the

presence of surfaces. The phenomenon is related to a very weak dependence on retardation of the energy contribution that originates from the transverse magnetic (TM) modes. Compared to the nonretarded energy, the effect of retardation on these modes is a minor decrease in attraction. The attraction originating from the transverse electric (TE) modes vanishes if retardation is neglected. As these modes begin to give measurable contributions the effect of retardation may actually be an increased interaction. This effect decreases with increasing film thickness and with increasing dissipation. This theoretically interesting possibility has to our knowledge never previously been reported. In Sec. II we briefly summarize how Lifshitz theory<sup>11–13</sup> for the retarded van der Waals energy between two half spaces can be adjusted to the present system. In Sec. III our numerical results are presented. Finally, in Sec. IV we summarize our results. It should be pointed out that thermal effects are expected to influence the interaction already at room temperature. To be able to observe pure retardation enhanced interaction one has to perform experiments at low temperatures. Furthermore, the metallic films should be deposited on transparent substrates, otherwise the substrate will influence the interaction.

### II. THEORY

Zhou and Spruch<sup>14</sup> considered the retarded vdW interaction between dielectric and perfectly conducting films and also between atoms and such films using a quantized surface mode technique. We adopt their notation. We will consider two films with dielectric functions  $\epsilon_{1,2}$  and thicknesses  $\delta_{1,2}$  separated by a distance  $d$ . The films are assumed to be deposited on a third medium with dielectric function  $\epsilon_3$ . The space between the films is filled with a fourth medium with dielectric function  $\epsilon_4$ .  $c$  is the speed of light in vacuum. The retarded van der Waals energy  $E$  between the films can be written in the following way:

$$E(d) = \frac{\hbar}{4\pi^2} \int_0^\infty \int_0^\infty d\omega dq q \{ \ln[G^{TE}(q, i\omega)] + \ln[G^{TM}(q, i\omega)] \}, \quad (2)$$

where

TABLE I. Drude model parameters.

Metal	$\omega_p/10^{16}$ (rad/s)	$\eta/10^{14}$ (rad/s)
Au <sub>I</sub>	1.245	3.221
Au <sub>II</sub>	1.367	0.532
Cu <sub>I</sub>	1.432	1.718
Cu <sub>II</sub>	2.025	0.653
Al	1.73	0.704

$$G^{TE/TM}(q, i\omega) = 1 - Q_1^{TE/TM} Q_2^{TE/TM}, \quad (3)$$

$$Q_i^{TE/TM} = \frac{\Delta_{i3}^{TE/TM} - \Delta_{i4}^{TE/TM} e^{-2\gamma_i \delta_i}}{1 - \Delta_{i3}^{TE/TM} \Delta_{i4}^{TE/TM} e^{-2\gamma_i \delta_i}} e^{-2\gamma_i d}, \quad i=1,2, \quad (4)$$

$$\Delta_{ij}^{TE} = \frac{\gamma_i - \gamma_j}{\gamma_i + \gamma_j}, \quad \Delta_{ij}^{TM} = \frac{\epsilon_j \gamma_i - \epsilon_i \gamma_j}{\epsilon_j \gamma_i + \epsilon_i \gamma_j}, \quad \gamma_i = \sqrt{q^2 + \epsilon_i \omega^2 / c^2}. \quad (5)$$

The dielectric function for imaginary frequencies is obtained from tabulated optical constants,  $\epsilon'' = 2nk$ , and the use of a Kramers-Kronig relation according to

$$\epsilon(i\omega) = 1 + \frac{2}{\pi} \int_0^\infty dx \frac{x \epsilon''(x)}{x^2 + \omega^2}. \quad (6)$$

For a real metal the dielectric function at small momenta and low frequencies can be modeled with the simple Drude expression

$$\epsilon(\omega) = 1 - \omega_p^2 / [\omega(\omega + i\eta)]. \quad (7)$$

The dielectric properties of the metals are taken from Ref. 15. Our numerical procedure has been described elsewhere.<sup>5,10</sup> Since optical data are not tabulated at low enough energies extrapolations must be used. We extrapolate below the lowest tabulated energies with the Drude model [Eq. (7)] according to Table I. For gold we have used two extrapolations. The first<sup>5</sup> (Au<sub>I</sub>) gives agreement with the experimental static resistivity and overlap with experimental data at low energies. The second<sup>6</sup> (Au<sub>II</sub>) gives a reasonable plasma frequency and overlap with experimental data at low energies. Similarly, two extrapolations have been used for copper. The first extrapolation (Cu<sub>I</sub>) is based on additional data given in Ref. 16. The second extrapolation (Cu<sub>II</sub>) and the extrapolation used for Al were found with the same arguments as for Au<sub>I</sub>. In general one needs the full momentum dependence of the dielectric functions. One cannot get this dependence from the optical data. Harris and Griffin<sup>17</sup> showed that appropriate account of dispersion has to be taken when the separation becomes comparable with the Thomas-Fermi wavelength. This is not relevant in our case. For large separations only the small momentum range contributes and in the separation range of interest here we do not need the momentum dependence. Still, a detailed investigation of the effects of including electron gas dispersion<sup>17-19</sup> (nonlocal effects) would be interesting. Furthermore, the role of boundary conditions<sup>20</sup> is an interesting subject that needs further investigation.

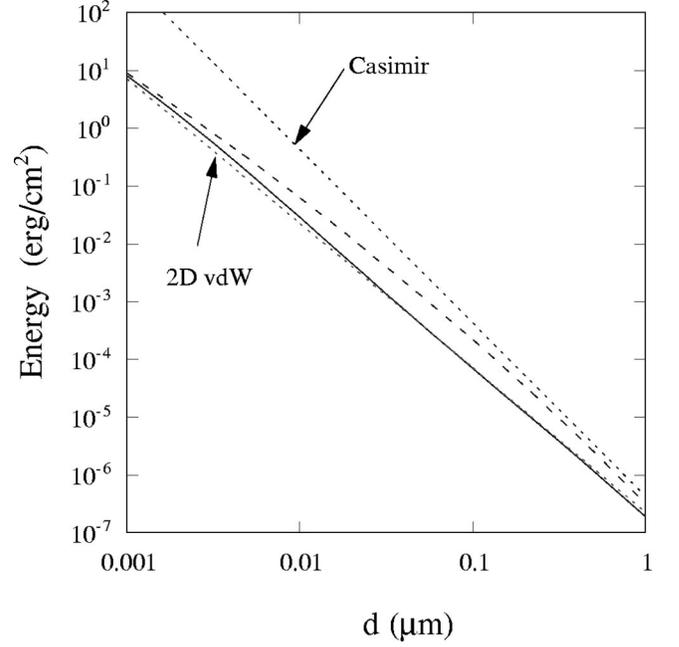


FIG. 1. The van der Waals energy between two systems is examined: for a pair of 20 Å gold films (solid) and a pair of gold half spaces (dashed). These curves have been supplemented with two asymptotes: the Casimir asymptote and the two-dimensional (2D) vdW asymptote.

### III. NUMERICAL RESULTS

The van der Waals interaction between thin metal films can have half-integer separation dependence in agreement with Eq. (1). In this section we will discuss how retardation effects influence this result.

The retarded van der Waals energy between two systems is examined in Fig. 1 for a pair of 20 Å gold films and a pair of gold half spaces. These curves have been supplemented with two asymptotes: the Casimir asymptote and the two-dimensional vdW asymptote according to Eq. (1). The interaction between thin metal films has fractional separation dependence in a certain separation range. At large enough separation the interaction becomes equal to the result found by Casimir<sup>1</sup> for two planar metal half spaces:

$$E^{Casimir} = -\hbar \tilde{c} \pi^2 / (720d^3). \quad (8)$$

In Fig. 2 we examine how retardation effects influence the attraction between 20 Å Cu<sub>I</sub> films. The result is presented as the ratio between different energy components and the non-retarded energy. The components considered are the TM, the TE, and the total retarded energy. When retardation is neglected the TE modes give no contribution. In the separation range where the TE modes give a substantial contribution the net attraction can in this case be larger than the nonretarded attraction. This result is, of course, related to the very weak influence of retardation on the energy contributions that originates from the TM modes. The contribution from TE modes is further partly responsible for the good agreement between retarded and nonretarded energies; i.e., they compensate for some of the decrease of the contributions from TM modes.

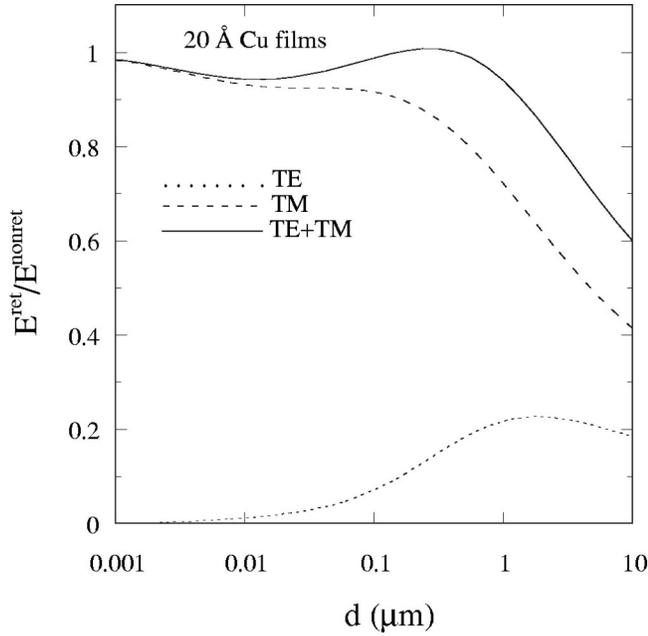


FIG. 2. The van der Waals energy of attraction between 20 Å  $\text{Cu}_7$  films. The result is presented as the ratio between different energy components and the nonretarded energy.

It is of interest to investigate the effects of a change in the film thickness or in the film material. The ratio between the total retarded energy and the nonretarded energy is shown in Fig. 3. The effect of retardation increases with increasing thickness. One should observe that although the results for different metals are quite different they also have common features. The difference in the results for the two extrapolations used for copper is related to the different amount of dissipation. The first extrapolation (solid line), which corresponds to a much larger static resistivity than the second one (dotted line), has a smaller energy ratio. Similarly, Barash showed that half-integer separation dependence cannot be observed if the dissipation is too large.

With increasing plasma frequency the film thickness range where dimensionality effects can be observed obviously becomes smaller. Retardation enhanced attraction cannot be observed in the quantum well structure investigated in Ref. 9. Furthermore, in this low density system thermal effects will already be important<sup>21</sup> at temperatures below 1 K. In other words, a 20 Å metal film with too low or too high a density may never have a ratio larger than 1 K. Thin metal films with low resistivity may have a ratio exceeding one at 0 K if the TE modes give a significant contribution at separations where retardation effects on the TM modes are still quite small.

#### IV. SUMMARY

In this paper we have presented numerical calculations of the retarded van der Waals energy between thin metal films. The effect of including retardation in a calculation is usually a lowering of the energy as compared to when retardation is

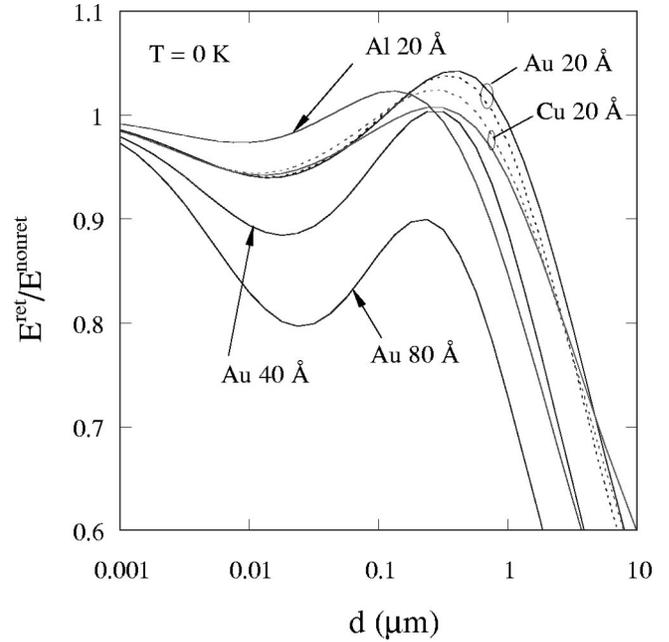


FIG. 3. The zero-temperature attraction between two planar films of different materials and with different thicknesses, presented as the ratio between the retarded and nonretarded energies. The results of using the extrapolations  $\text{Cu}_7$ ,  $\text{Au}_7$ , and  $\text{Al}_7$  is presented as solid curves. The results of using the other two extrapolations are presented as dotted curves.

neglected. The interaction between thin metal films can under very special circumstances be an exception to this general rule of thumb. This anomalous behavior goes away with increasing film thickness and dissipation. The theoretical result is rather sensitive to the particular choice of dielectric function. Optical measurements should therefore be performed on the actual samples used in an experimental setup to allow high accuracy calculations. In the future we intend to investigate this interaction between thin metal films deposited on large transparent spheres. This could possibly be another system exhibiting retardation enhanced interaction.

There has recently been quite a lot of interest in these fractional van der Waals forces and we would finally like to mention some related papers. Tanatar and Das investigated the non dissipative current drag effect between two two-dimensional charged Bose-gas layers.<sup>22</sup> Interactions with fractional power laws were found in this case. The same power law was later found by Boström and Sernelius<sup>23,24</sup> for the nondissipative current drag between quantum wells. Furthermore, Lau Levine, and Pincus,<sup>25</sup> recently reported the theoretical possibility of observing this fractional separation dependence between two-dimensional planar Wigner crystals. As far as we know these fractional power laws have never been experimentally verified. To actually measure retardation enhanced interaction will be a very difficult, but not necessarily impossible, challenge. The metallic films should be deposited on an extremely transparent surface. Even if the substrate is very transparent, any detailed theoretical comparison with experiment requires that one take the substrate into account.

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