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Topics in Spectrum Sensing for Cognitive Radio

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Abstract

Cognitive radio is a new concept of reusing licensed spectrum in an unlicensed manner. Cognitive radio is motivated by recent measurements of spectrum utilization, showing unused resources in frequency, time and space. The spectrum must be sensed to detect primary user signals, in order to allow cognitive radios in a primary system. In this thesis we study some topics in spectrum sensing for cognitive radio.

The fundamental problem of spectrum sensing is to discriminate samples that contain only noise from samples that contain a very weak signal embedded in noise. We derive detectors that exploit known structures of the signal, for the cases of an OFDM modulated signal and an orthogonal space-time block coded signal. We derive optimal detectors, in the Neyman-Pearson sense, for a few different cases when all parameters are known. Moreover we study detection when the parameters, such as noise variance, are unknown. We propose solutions the problem of unknown parameters.

We also study system aspects of cognitive radio. More specifically, we investigate spectrum reuse of geographical spectrum holes in a frequency planned primary network. System performance is measured in terms of the achievable rate for the cognitive radio system. Simulation results show that a substantial sum-rate could be achieved if the cognitive radios communicate over small distances. However, the spectrum hole gets saturated quite fast, due to interference caused by the cognitive radios.

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Contents

Al	bstra	nct		\mathbf{V}
A	ckno	wledgi	ments	vii
Ι	Int	roduc	tion	1
\mathbf{A}	Ove	erview	of Spectrum Sensing for Cognitive Radio	3
	1	Mode	1	5
	2	Energ	y Detection	6
	3	Funda	amental Limits on Detection	7
	4	Featu	re Detection	8
		4.1	Cyclostationarity	8
		4.2	Autocorrelation	10
		4.3	Covariance Matrix Eigenvalues	11
	5 Cooperative Detection		erative Detection	12
		5.1	Soft Combining	13
		5.2	Hard Combining	14
	6	Contr	ibutions of the Thesis	15
	Refe	erences		19

II Included Papers

A	A B Ano	ayesian Approach to Spectrum Sensing, Denoising and maly Detection	25
	1	Introduction	27
	2	Problem Formulation	28
	3	Optimal detector	30
	4	Detector for Unknown ρ, σ	31
		4.1 Estimation of ρ, σ using prior knowledge	31
		4.2 Elimination of σ via marginalization	32
	5	Detector Approximations	33
	6	Numerical Results	34
	7	Concluding Remarks	37
	Refe	rences	39
в	On t eter	the Optimal <i>K</i> -term Approximation of a Sparse Param- Vector MMSE Estimate	41
	1	Introduction	43
	2	Model	44
	3	MMSE Estimation and Approximation	46
	4	Optimal One-Term Approximation, $K = 1 \dots \dots \dots$	48
	5	Optimal Approximation with Multiple Terms, $K > 1$	50
	6	Numerical Results	51
	7	Concluding Remarks	53
	Refe	rences	55

\mathbf{C}	Opt nals	imal aı in Kn	nd Near-Optimal Spectrum Sensing of OFDM Sigown and Unknown Noise	57
	1	Introd	uction	59
		1.1	Contributions	61
	2	Model		61
	3	Detect	ion with known σ_n^2 and σ_s^2	62
		3.1	Optimal detector for the general unsynchronized case	65
		3.2	Special cases and benchmarks	66
	4	Detect	ion with unknown σ_n^2 and σ_s^2	69
		4.1	Benchmarks	74
	5	Compa	arisons	75
		5.1	Known σ_n^2 and σ_s^2	76
		5.2	Unknown σ_n^2 and σ_s^2	78
	6	Conclu	ding Remarks	81
	А	Proper	ties of \mathbf{Q}_{τ}	82
		A.1	Efficient Computation of $\mathbf{x}^{H} \left(\mathbf{Q}_{\tau}^{-1} - \frac{1}{\sigma_{n}^{2}} \mathbf{I} \right) \mathbf{x} \dots$	82
		A.2	Efficient Computation of det (\mathbf{Q}_{τ})	83
	В	Statist	ics of R_i	84
		B.1	Moments of \overline{R}_i	85
		B.2	Moments of \widetilde{R}_i	87
		B.3	Derivation of $\operatorname{Cov}\left[\overline{R}_{i}, \widetilde{R}_{i}\right]$	88
	Refe	rences .		91
D	Spec Sign	ctrum als wi	Sensing of Orthogonal Space-Time Block Coded th Multiple Receive Antennas	93
	1	Introd	uction	95
	2	Model	and Problem Formulation	96
	3	Signal	Detection	99
		3.1	Optimal Genie Detection	99

		3.2	Unknown Parameters, GLRT Approach	100
		3.3	Unknown Parameters, Eigenvalue-Based Detection	102
		3.4	Energy Detection	102
	4	Numer	rical Results	103
	5	Conclu	Iding Remarks	105
	Refe	rences		107
\mathbf{E}	Cap	acity (Considerations for Uncoordinated Communication	
	in (¹ 000rn	nhical Spectrum Holes 1	$\mathbf{n}\mathbf{o}$
	in C	Jeogra Introd	phical Spectrum Holes 1	. 09 111
	in G	Geogra	phical Spectrum Holes 1 uction	.09 111
	in C 1 2	Feogra Introd Model	phical Spectrum Holes 1 uction	. 09 111 113
	in C 1 2 3	Geograj Introd Model Simula	phical Spectrum Holes 1 uction	.09 111 113 116
	in C 1 2 3	Geograj Introd Model Simula 3.1	phical Spectrum Holes 1 uction	.09 111 113 116 117
	in C 1 2 3	Geograj Introd Model Simula 3.1 3.2	phical Spectrum Holes 1 uction	.09 1111 113 116 117 119
	in C 1 2 3	Feograj Introd Model Simula 3.1 3.2 Point-1	phical Spectrum Holes 1 uction	.09 111 113 116 117 119 121
	in C 1 2 3 4 5	Geograj Introd Model Simula 3.1 3.2 Point-1 Conclu	phical Spectrum Holes 1 uction	.09 111 113 116 117 119 121 126

Part I

Introduction

Chapter A

Overview of Spectrum Sensing for Cognitive Radio

In the last decades, there has been an enormous increase of wireless communication systems. The usage of frequency bands, or spectrum, is strictly regulated, and allocated to specific communication techniques. The vast majority of frequency bands are allocated to licensed users, which are also steered by standards. There are a number of organizations working on standards for frequency allocation, for example the International Telecommunication Union (ITU), the European Telecommunications Standards Institute (ETSI) and the European Conference of Postal and Telecommunications Administrations (CEPT).

Spectrum is a scarce resource, and licensed spectrum is intended to be used only by the spectrum owners. Various measurements of spectrum utilization, have shown unused resources in frequency, time and space [3, 4]. Cognitive radio is a new concept of reusing licensed spectrum in an unlicensed manner [1, 2]. The unused resources are often referred to as spectrum holes or white spaces. These spectrum holes could be reused by cognitive radios, sometimes called secondary users. There might be geographical positions where some frequency bands are allocated to a primary user system, but not currently used. These geographical spectrum holes could be employed by secondary users as shown in Figure 1. There might also be certain time intervals for which the primary system does not use the licensed spectrum, as shown in Figure 2. These time domain spectrum holes could also potentially be employed by secondary users.



Figure 1: Example of geographical spectrum holes.

The introduction of cognitive radios will inevitably create increased interference and thus degrade the quality of service of the primary system. The impact on the primary system, for example in terms of increased interference, must be kept at a minimal level. To keep the impact at an acceptable level, secondary users must sense the spectrum to detect whether it is available or not. Secondary users must be able to detect very weak primary user signals [5, 6, 7]. Spectrum sensing is one of the most essential components of cognitive radio.

In the following we will present some topics in spectrum sensing for cognitive radio, that have been of great interest in recent research. We will highlight some fundamental problems and present techniques for signal detection. First we set up a model for signal detection. Then we present one of the most basic detectors, the energy detector. We will also give some fundamental limits for detection. Furthermore, we show some examples of feature detectors, that exploit knowledge about the signal to be detected. We show the concept of cooperative detection, and finally we provide a summary of the contributions of the thesis.



Figure 2: Example of time domain spectrum holes.

1 Model

As a preliminary, we set up the model for signal detection. Consider a timecontinuous signal $\mathcal{X}(t)$. We wish to express the time-continuous signal in a discrete vector representation over a finite time interval. In general, the signal can be expressed by a basis expansion

$$\mathcal{X}(t) = \sum_{i=1}^{N} x_i \phi_i(t),$$

where $\phi_i(t)$ are basis functions. Then the signal $\mathcal{X}(t)$ can be represented by the vector $\mathbf{x} \triangleq (x_1 x_2 \dots x_N)^T$. The basis representation could be for example a Fourier series, where the basis functions are complex exponential functions, or conventional uniform sampling, where the basis functions are sinc functions. In the sequel we assume that all signals are represented in a basis by a vector, for example by sampling the time-continuous signal. Assume that \mathbf{y} is a received vector of length N, that consists of a signal plus noise. That is

$$\mathbf{y} = \mathbf{x} + \mathbf{w},$$

where **x** is a signal vector, and **w** is a noise vector. The noise **w** is assumed to be i.i.d. zero-mean circularly symmetric complex Gaussian with variance σ^2 . That is, $\mathbf{w} \sim CN(\mathbf{0}, \sigma^2 \mathbf{I})$.

We wish to detect whether there is a signal present or not. That is, we want to discriminate between the following two hypotheses:

$$H_0: \mathbf{y} = \mathbf{w},$$

$$H_1: \mathbf{y} = \mathbf{x} + \mathbf{w}.$$
(1)

The optimal Neyman-Pearson test is to compare the log-likelihood ratio to a threshold. That is

$$\Lambda \triangleq \log \left(\frac{P(\mathbf{y}|H_1)}{P(\mathbf{y}|H_0)} \right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta.$$

Clearly, the log-likelihood ratio depends on the distribution of the signal to be detected.

2 Energy Detection

Initially we will present one of the simplest signal models, for which the optimal detector is the energy detector [8]. We assume that the signal to be detected does not have any known structure that could be used for detection. Thus, we assume that the signal is also zero-mean circularly symmetric complex Gaussian $\mathbf{x} \sim CN(\mathbf{0}, \gamma^2 \mathbf{I})$. Then, $\mathbf{y}|H_0 \sim CN(\mathbf{0}, \sigma^2 \mathbf{I})$ and $\mathbf{y}|H_1 \sim CN(\mathbf{0}, (\sigma^2 + \gamma^2)\mathbf{I})$. The log-likelihood ratio is

$$\log\left(\frac{P(\mathbf{y}|H_1)}{P(\mathbf{y}|H_0)}\right) = \log\left(\frac{\frac{1}{\pi^N(\sigma^2 + \gamma^2)^N}\exp(-\frac{\|\mathbf{y}\|^2}{\sigma^2 + \gamma^2})}{\frac{1}{\pi^N\sigma^{2N}}\exp(-\frac{\|\mathbf{y}\|^2}{\sigma^2})}\right)$$

By removing all constants that are independent of the received vector \mathbf{y} , we obtain the optimal Neyman-Pearson test

$$\Lambda_{\mathbf{e}} \triangleq \|\mathbf{y}\|^2 = \sum_{i=0}^{N-1} |y_i|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\mathbf{e}}.$$
 (2)

Hence, the optimal detector, in the Neyman-Pearson sense, is in this case the energy detector also known as radiometer [8]. In essence the energy detector measures the received energy during a finite time interval, and compares it to a predetermined threshold. The performance of the energy detector is well known, cf. [9], and can be written in closed form. The probability of false alarm $P_{\rm FA}$ is given by

$$P_{\rm FA} \triangleq \Pr(\Lambda_{\rm e} > \eta_{\rm e} | H_0) = \Pr(\frac{\Lambda_{\rm e}}{\sigma^2/2} > \frac{\eta_{\rm e}}{\sigma^2/2} | H_0) = 1 - F_{\chi^2_{2N}} \left(\frac{2\eta_{\rm e}}{\sigma^2}\right),$$

where $F_{\chi^2_{2N}}(\cdot)$ denotes the cumulative distribution function of χ^2 -distributed random variable with 2N degrees of freedom. Thus, given a false alarm probability, we can derive the threshold η from

$$\eta_{\rm e} = F_{\chi^2_{2N}}^{-1} \left(1 - P_{\rm FA}\right) \frac{\sigma^2}{2}.$$
 (3)

The probability of detection is given by

$$P_{\rm D} \triangleq \Pr\left(\Lambda_{\rm e} > \eta_{\rm e} | H_1\right) = 1 - F_{\chi^2_{2N}} \left(\frac{2\eta_{\rm e}}{\sigma^2 + \gamma^2}\right).$$

The energy detector is universal in the sense that it can detect any type of signal, and does not require any knowledge about the signal to be detected. On the other hand, for the same reason it does not exploit any potentially available knowledge about the signal. Moreover, the noise power needs to be known to set the decision threshold (3).

3 Fundamental Limits on Detection

Cognitive radios must be able to detect very weak primary user signals [5]. However, there are some fundamental limits for detection in low SNR. For example, to set the decision threshold of the energy detector (3), the noise variance σ^2 must be known. If the knowledge of the noise variance is imperfect, clearly the threshold will be erroneous. It is well known that the performance of the energy detector quickly deteriorates if the noise variance is imperfectly known (cf. [6, 10]). Due to uncertainties in the model assumptions, robust detection is impossible below a certain SNR level, known as the SNR wall [10, 11]. It was shown in [10] that errors in the noise power assumption introduces SNR walls to any moment-based detector. This was further extended in [11] to any model uncertainties, such as assuming perfect white and stationary noise, flat fading, ideal filters and infinite precision A/D converters. These results hold for detectors with imperfect assumptions. However, it is possible to circumvent, or at least mitigate the problem of SNR walls by taking the imperfections into account. For example, it was shown in [11] that noise calibration improves the detector robustness. Exploiting some known features of the signal to be detected can also improve the detector performance and robustness.

4 Feature Detection

If the signal to be detected is perfectly known, the optimal detector is a matched filter (cf. [9]). In practice the signal is never perfectly known, but there is some knowledge about the signal. It is usually known what kind of primary users that are to be detected, and the transmitted signals are to some extent determined by standards and regulations. Thus, some features of the signal to be detected are usually known. In the following, we will describe some detectors exploiting known features of the signal, both to improve performance and to circumvent the problem of model uncertainties, for example imperfectly known noise variance.

4.1 Cyclostationarity

Most man-made signals show periodic patterns related to symbol rate, chip rate, channel code or cyclic prefix, that can be appropriately modeled as a cyclostationary random process [12]. Define the autocorrelation function of the continuous-time stochastic process $\mathcal{X}(t)$ as

$$R(t,T) \triangleq E\left[\mathcal{X}(t+T/2)\mathcal{X}^*(t-T/2)\right].$$

A continuous-time stochastic process $\mathcal{X}(t)$ is said to be almost second-order cyclostationary if its autocorrelation function is almost periodic in t (cf. [12, 13, 14]). Hence, the autocorrelation function R(t, T) is almost periodic, and can be expressed by a Fourier series

$$R(t+T/2, t-T/2) = \sum_{\alpha} R^{\alpha}(T)e^{j2\pi\alpha t},$$

where the sum is over integer multiples of fundamental frequencies and their sums and differences. That is, if there are multiple sources of periodicity, the autocorrelation function contains sums of periodic functions with possibly incommensurate periods. That is, the periodicity can for example be caused by the symbol rate which yields a fundamental period T_1 , and the chip rate which yields a fundamental period T_2 , where T_1/T_2 is not necessarily a rational number. The Fourier coefficients depend on the time lag T and are given by

$$R^{\alpha}(T) = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} R(t + T/2, t - T/2) e^{-j2\pi\alpha t} dt.$$
(4)

The limit in (4) allows for multiple incommensurate periods. If there is only one fundamental frequency (and integer multiples thereof), the limit can be omitted. The Fourier coefficients $R^{\alpha}(T)$ are also known as the *cyclic autocorrelation function* with cyclic frequency α . For $\alpha = 0$ it reduces to the conventional autocorrelation function. The process $\mathcal{X}(t)$ is said to be (almost) cyclostationary if there exists an α such that $R^{\alpha}(T) > 0$. The Fourier transform of the cyclic autocorrelation function is

$$S^{\alpha}(f) \triangleq \int_{-\infty}^{\infty} R^{\alpha}(T) e^{-j2\pi fT} dT,$$

and is called the cyclic spectral density function. For $\alpha = 0$ it reduces to the conventional power spectral density function. For $\alpha \neq 0$, $S^{\alpha}(f)$ is the density of correlation between spectral components at the frequencies $f + \alpha/2$ and $f - \alpha/2$. Knowing some of these cyclic characteristics of a signal, one can construct detectors that exploit the cyclostationarity of the signal [15, 13, 14, 16, 17] and benefit from the spectral correlation. Note that the inherent cyclostationarity property appears both in the cyclic autocorrelation function $R^{\alpha}(T)$ and in the cyclic spectral density function $S^{\alpha}(f)$. Thus, detection of the cyclostationarity can be performed both in the time domain, and in the frequency domain.

There has been a huge interest in detection of OFDM signals recently. One reason is that many of the current and future technologies for wireless communication, such as WiFi, WiMAX, LTE and DVB-T, use OFDM signalling. Therefore it is reasonable to assume that cognitive radios must be able to detect OFDM signals. Another reason is that OFDM signals exhibit well known spectral correlation properties [18]. The IEEE 802.22 WRAN standard is intended for cognitive radio-based reuse of spectrum that is allocated to digital TV broadcasts. Cyclostationarity-based detectors for detection of the OFDM-based digital TV-signals for the IEEE 802.22 WRAN standard were proposed e.g. in [19, 20]. Another cyclostationary-based detector of OFDM-signals based on multiple cyclic frequencies was proposed in [21]. We will return to the detection of OFDM signals in Section 4.2.

4.2 Autocorrelation

Many communication signals contain redundancy, introduced for example to facilitate synchronization, by channel coding or to circumvent intersymbol interference. This redundancy occurs as non-zero average autocorrelation at some time lag T. For example, consider an OFDM signal with a cyclic prefix of length N_c and informative data of length N_d . Then, the average autocorrelation of the OFDM signal is non-zero at time lag N_d , owing to the fact that some of the data is repeated in the cyclic prefix of each OFDM signal, although the described detectors are valid for all signals that show a non-zero average autocorrelation at some known time lag. Assume that the signal \mathbf{x} contains $N \triangleq K(N_c + N_d) + N_d$ samples. As a preparation, let

$$r_i \triangleq y_i^* y_{i+N_d}, \ i = 0, \dots, K * (N_c + N_d) - 1$$

Furthermore, we know that if $E[r_i] \neq 0$, then $E[r_{i+k(N_c+N_d)}] \neq 0$, $k = 1, \ldots, K-1$, and analogously if $E[r_i] = 0$, then $E[r_{i+k(N_c+N_d)}] = 0$. That is, r_i and $r_{i+k(N_c+N_d)}$ will have identical statistics, and be independent (since the noise and signals are independent). Thus, we define

$$R_i \triangleq \frac{1}{K} \sum_{k=0}^{K-1} r_{i+k(N_c+N_d)}, \ i = 0, \dots, N_c + N_d - 1.$$

A detector that exploits the autocorrelation property of OFDM signals was proposed in [22]. The detector of [22] uses the test statistic

$$\max_{\tau \in \{0,...,N_c+N_d-1\}} \left| \sum_{i=\tau}^{\tau+N_c-1} r_i \right|.$$
 (5)

The variable τ can be viewed as the synchronization mismatch, or equivalently the time when the first sample is observed. The statistic (5) only takes one OFDM symbol at a time into account. A slight generalization of this test statistic, that uses the whole signal and not only one symbol, is to sum the variables R_i instead of r_i . Then, the test is

$$\Lambda_{\max \operatorname{ac}} \triangleq \max_{\tau \in \{0, \dots, N_c + N_d - 1\}} \left| \sum_{i=\tau}^{\tau + N_c - 1} R_i \right| \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\max \operatorname{ac}}$$

Another autocorrelation-based detector was proposed in [23]. This detector uses the empirical mean of the autocorrelation normalized by the received power, as test statistic. More precisely, the test proposed in [23] is

$$\Lambda_{\overline{\mathrm{ac}}} \triangleq \frac{\frac{1}{N - N_d} \sum_{i=0}^{N - N_d - 1} \operatorname{Re}(r_i)}{\widehat{\beta^2}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\overline{\mathrm{ac}}},$$

where

$$\widehat{\beta^2} \triangleq \frac{1}{2N} \sum_{i=0}^{N-1} |x_i|^2.$$

The detector proposed in [22] requires knowledge about the noise variance to set the decision threshold, while the detector proposed in [23] does not require any knowledge about the noise variance.

4.3 Covariance Matrix Eigenvalues

Assume that the signal \mathbf{x} can be written as

$$\mathbf{x} = \mathbf{Gs},$$

where **G** is an $N \times L$ matrix, and **s** is an $L \times 1$ vector. Furthermore, assume that **x** is highly correlated. That is N > L, and therefore **G** has low rank. This is the case for example in a typical MIMO system [24], or for an OFDM signal. Assume, that the signal **s** is zero-mean Gaussian. More specifically $\mathbf{s} \sim CN(\mathbf{0}, \gamma^2 \mathbf{I})$. Then, the hypothesis test (1) can be written

$$H_0: \mathbf{y} \sim \mathcal{C}N\left(\mathbf{0}, \sigma^2 \mathbf{I}\right), H_1: \mathbf{y} \sim \mathcal{C}N\left(\mathbf{0}, \gamma^2 \mathbf{G}\mathbf{G}^H + \sigma^2 \mathbf{I}\right).$$
(6)

Let \mathbf{Q} be the covariance matrix of the received vector \mathbf{y} , $\mathbf{Q} \triangleq E[\mathbf{y}\mathbf{y}^H]$. Then, under H_0 , all eigenvalues of \mathbf{Q} are equal to σ^2 . However, under H_1 the eigenvalues of \mathbf{Q} are equal to $\delta_i + \sigma^2$, $i = 0, \ldots, N - 1$, where δ_i are the eigenvalues of $\gamma^2 \mathbf{G} \mathbf{G}^H$. Since \mathbf{G} has low rank, there will be some $\delta_i = 0$. Thus, if we sort the eigenvalues of \mathbf{Q} in descending order, the smallest eigenvalues are equal to σ^2 (when $\delta_i = 0$) and the larger ones are equal to $\delta_i + \sigma^2 > \sigma^2$. Detectors exploiting this property were proposed in [24, 25], and will be briefly described in the following.

Consider M vectors \mathbf{y}_m received in a sequence. Define the sample covariance matrix

$$\widehat{\mathbf{Q}} \triangleq \frac{1}{K} \sum_{m=0}^{M-1} \mathbf{y}_m \mathbf{y}_m^H.$$

Let λ_i , $i = 0, \ldots, N - 1$, be the eigenvalues of $\widehat{\mathbf{Q}}$. There are two eigenvaluebased detectors proposed in [24]. The first detector uses the ratio of the largest eigenvalue to the smallest eigenvalue, and compares it to a threshold. That is, the test statistic of the first proposal of [24] is

$$\frac{\max_i \lambda_i}{\min_j \lambda_j}.$$
(7)

The second detector proposed in [24] uses the ratio of the average eigenvalue to the smallest eigenvalue. That is

$$\frac{\frac{1}{N}\sum_{i=0}^{N-1}\lambda_i}{\min_j\lambda_j}.$$

These eigenvalue-based detectors were shown to perform well when the signal to be detected is highly correlated.

It was shown in [25], that the generalized likelihood ratio (GLR) for the hypothesis test (6) when all parameters (σ^2 , γ^2 and **G**) are completely unknown, is

$$\frac{\frac{1}{N}\sum_{i=0}^{N-1}\lambda_i}{\left(\prod_{j=0}^{N-1}\lambda_j\right)^{1/N}}.$$
(8)

This test is equivalent to the sphericity test of [26]. The sphericity test of [26] decides if the covariance matrix of a multivariate normal distribution is proportional to the identity matrix, or equivalently if all the eigenvalues of the sample covariance matrix are equal or not. The GLR detector (8) and the max/min-ratio detector (7) were compared in [25]. Simulations of a MIMO system where the number of transmit antennas was larger than the number of receive antennas, and the signal was assumed to be Gaussian, showed that the max/min-ratio (7) performs almost as well as the GLR (8).

5 Cooperative Detection

One way of reducing the receiver sensitivity requirements is by using cooperative sensing. The concept of cooperative sensing is to use multiple sensors and combine their measurements to one common decision. This is in essence a way of getting diversity gains.

5.1 Soft Combining

Assume that there are M sensors. Then, the hypothesis test (1) becomes

$$H_0: \mathbf{y}_m = \mathbf{w}_m, \ m = 0, \dots, M - 1,$$

 $H_1: \mathbf{y}_m = \mathbf{x}_m + \mathbf{w}_m, \ m = 0, \dots, M - 1$

Assume that the received signals at all sensors are independent. Let $\mathbf{z} = (\mathbf{y}_0^T \mathbf{y}_1^T \dots \mathbf{y}_{M-1}^T)^T$. Then, the log-likelihood ratio is

$$\Lambda_{\text{coop}} \triangleq \log\left(\frac{P(\mathbf{z}|H_1)}{P(\mathbf{z}|H_0)}\right) = \log\left(\prod_{m=0}^{M-1} \frac{P(\mathbf{y}_m|H_1)}{P(\mathbf{y}_m|H_0)}\right)$$
$$= \sum_{m=0}^{M-1} \log\left(\frac{P(\mathbf{y}_m|H_1)}{P(\mathbf{y}_m|H_0)}\right) = \sum_{m=0}^{M-1} \Lambda^{(m)},$$
(9)

where $\Lambda^{(m)} \triangleq \log \left(\frac{P(\mathbf{y}_m | H_1)}{P(\mathbf{y}_m | H_0)} \right)$ is the log-likelihood ratio for the *m*th sensor. That is, if the received signals for all sensors are independent, the optimal fusion rule is to sum the log-likelihood ratios.

Consider the case when the noise vectors \mathbf{w}_m are independent, such that $\mathbf{w}_m \sim \mathcal{C}N(\mathbf{0}, \sigma_m^2 \mathbf{I})$, and the signal vectors \mathbf{x}_m are independent, such that $\mathbf{x}_m \sim \mathcal{C}N(\mathbf{0}, \gamma_m^2 \mathbf{I})$. Then, the log-likelihood ratio (9) is written

$$\Lambda_{\rm ce} = \sum_{m=0}^{M-1} \log \left(\frac{\frac{1}{\pi^N (\sigma_m^2 + \gamma_m^2)^N} \exp(-\frac{\|\mathbf{y}_m\|^2}{\sigma_m^2 + \gamma_m^2})}{\frac{1}{\pi^N \sigma_m^{2N}} \exp(-\frac{\|\mathbf{y}_m\|^2}{\sigma_m^2})} \right)$$

Removing all constants that are independent of \mathbf{z} yields

$$\Lambda_{\rm ce} = \sum_{m=0}^{M-1} \|\mathbf{y}_m\|^2 \frac{\gamma_m^2}{\sigma_m^2 \left(\sigma_m^2 + \gamma_m^2\right)}.$$
 (10)

The statistic $||\mathbf{y}_m||^2$ is the soft decision from an energy detector at the *m*th sensor, as shown in (2). Thus, the optimal cooperative detection scheme is to use energy detection for the individual sensors, and combine the soft decisions by the weighted sum (10). This result was also shown in [27], for the case when $\sigma_m^2 = 1$, and thus γ_m^2 is equivalent to the SNR experienced by the *m*th sensor. Clearly, if both the noise power and signal power are equal for all sensors, we can ignore the weight factor and just sum the soft decisions.

The cooperative gain under that assumption was analyzed in [28]. It was shown in [28], that correlation between the sensors severely decreases the cooperation gain. The main source of correlation between users is shadow fading. Multipath fading is uncorrelated at very small distances, on the scale of half a wavelength, and can easily be avoided. Hence the correlation is mainly distance dependent, and the cooperation gains are limited by the distance separation of the cognitive users. From a detection perspective a large distance separation between users is desired. However, if cognitive users should be able to communicate without disturbing the primary system they must be sufficiently near to one another. Thus, there is a distance trade off between detection performance and cognitive communication. The effect of untrusted users was also analyzed. The conclusion of [28] is that if one out of M sensors is untrustworthy, the sensitivity of each individual sensor must be as good as that achieved with M trusted users.

5.2 Hard Combining

So far we have considered optimal cooperative detection. That is, all users transmit soft decisions to a fusion center, which combines the soft values to one common decision. This is equivalent to the case where the fusion center has access to the received data for all sensors, and performs optimal detection based on all data. This requires potentially a huge amount of data to be transmitted to the fusion center. The other extreme case of cooperative detection is that each sensor takes its own decision, and transmits only a binary value to the fusion center. Then, the fusion center combines the hard decisions to one common decision.

In the following we will describe the AND, OR, and voting rules (cf. [29]) for combining of hard decisions. Assume that the individual statistics $\Lambda^{(m)}$ are quantized to one bit, such that $\Lambda^{(m)} = 0, 1$ is the hard decision from the *m*th sensor. Here, 1 means that a signal is detected and 0 means that the channel is available.

The **AND** rule decides that a signal is detected if *all* sensors have detected a signal. That is, the cooperative test using the AND rule decides on H_1 if

$$\sum_{m=0}^{M-1} \Lambda^{(m)} = M.$$

The **OR** rule decides on signal presence if *any* of the sensors reports signal detection. Hence, for the OR rule the cooperative test decides on H_1 if

$$\sum_{m=0}^{M-1} \Lambda^{(m)} \ge 1.$$

Finally, the **voting** rule decides that a signal is present if at least C of the M sensors have detected a signal, for $1 \leq C \leq M$. The test decides on H_1 if

$$\sum_{m=0}^{M-1} \Lambda^{(m)} \ge C.$$

Taking a majority decision is a special case of the voting rule, for C = M/2. The AND-logic and the OR-logic are clearly also special cases of the voting rule for C = M and C = 1 respectively.

6 Contributions of the Thesis

The major part of the thesis concerns topics in spectrum sensing, whereas the last paper studies system aspects of cognitive radio. Brief summaries of the included papers are given in the following.

Paper A: A Bayesian Approach to Spectrum Sensing, Denoising and Anomaly Detection

Published at the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2009.

This paper deals with the problem of discriminating samples that contain only noise from samples that contain a signal embedded in noise. The focus is on the case when the variance of the noise is unknown. We derive the optimal soft decision detector using a Bayesian approach. The complexity of this optimal detector grows exponentially with the number of observations and as a remedy, we propose a number of approximations to it. The problem under study is a fundamental one and it has applications in signal denoising, anomaly detection, and spectrum sensing for cognitive radio. We illustrate the results in the context of the latter.

Paper B: On the Optimal *K*-term Approximation of a Sparse Parameter Vector MMSE Estimate

Published at the IEEE Workshop on Statistical Signal Processing (SSP), 2009.

This paper considers approximations of marginalization sums that arise in Bayesian inference problems. Optimal approximations of such marginalization sums, using a fixed number of terms, are analyzed for a simple model. The model under study is motivated by recent studies of linear regression problems with sparse parameter vectors, and of the problem of discriminating signal-plus-noise samples from noise-only samples. It is shown that for the model under study, if only one term is retained in the marginalization sum, then this term should be the one with the largest a posteriori probability. By contrast, if more than one (but not all) terms are to be retained, then these should generally *not* be the ones corresponding to the components with largest a posteriori probabilities.

Paper C: Optimal and Near-Optimal Spectrum Sensing of OFDM Signals in Known and Unknown Noise

Submitted in parts to the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2010.

We consider spectrum sensing of OFDM signals. The main concerns are the two cases of completely known, or completely unknown, noise power and signal power. For the case of completely known noise power and signal power, we derive the optimal Neyman-Pearson detector from first principles. The optimal detector exploits the inherent correlation of the OFDM signal, incurred by the repetition of data in the cyclic prefix. We compare the optimal detector to the energy detector numerically. We show that the energy detector is near-optimal (within 1 dB SNR) when the noise variance is known. Thus, when the noise power is known, no substantial gain can be achieved by using any other detector than the energy detector.

For the case of completely unknown noise power and signal power, we propose a GLRT detector based on the correlation of the OFDM signal. The proposed detector exploits the known structure of the signal, and does not require any knowledge of the noise power or the signal power. The GLRT detector is compared to other state-of-the-art OFDM detectors, and shown to improve detection performance with 5 dB SNR in relevant cases.

Paper D: Spectrum Sensing of Orthogonal Space-Time Block Coded Signals with Multiple Receive Antennas

Submitted to the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2010.

We consider detection of signals encoded with orthogonal space-time block codes (OSTBC), using multiple receive antennas. Such signals contain redundancy and they have a specific structure, that can be exploited for detection. We derive the optimal detector, in the Neyman-Pearson sense, when all parameters are known. We also consider unknown noise variance, signal variance and channel coefficients. We propose a number of GLRT based detectors for the different cases, that exploit the redundancy structure of the OSTBC signal. We also propose an eigenvalue-based detector for the case when all parameters are unknown. The proposed detectors are compared to the energy detector. We show that when only the noise variance is known, there is no gain in exploiting the structure of the OSTBC. However, when the noise variance is unknown there can be a significant gain.

Paper E: Capacity Considerations for Uncoordinated Communication in Geographical Spectrum Holes

Published in ELSEVIER Physical Communications, 2009.

Cognitive radio is a new concept of reusing licensed spectrum in an unlicensed manner. The motivation for cognitive radio is various measurements of spectrum utilization, that generally show unused resources in frequency, time and space. These "spectrum holes" could be exploited by cognitive radios. Some studies suggest that the spectrum is extremely underutilized, and that these spectrum holes could provide ten times the capacity of all existing wireless devices together. The spectrum could be reused either during time periods where the primary system is not active, or in geographical positions where the primary system is not operating. In this paper, we deal primarily with the concept of geographical reuse, in a frequency-planned primary network. We perform an analysis of the potential for communication in a geographical spectrum hole, and in particular the achievable sum-rate for a secondary network, to some order of magnitude.

Simulation results show that a substantial sum-rate could be achieved if the secondary users communicate over small distances. For a small number of

secondary links, the sum-rate increases linearly with the number of links. However, the spectrum hole gets saturated quite fast, due to interference caused by the secondary users. A spectrum hole may look large, but it disappears as soon as someone starts using it.

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