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# Quasi-Maximum-Likelihood Multiple-Symbol Differential Detection for the Time-Varying Rayleigh Fading Channel

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**Abstract**

The maximum-likelihood multiple-symbol differential detector (ML-MSDD) has better bit-error-rate (BER) performance than many other detectors for differential modulation. Unfortunately, the computational complexity of ML-MSDD quickly becomes prohibitive as the observation window size grows. While low-complexity MSDD algorithms for the time-invariant Rayleigh fading channel have been considered before, there is a need for low-complexity MSDD algorithms for general time-varying Rayleigh fading channels. In this paper, a polynomial-time complexity approach called semi-definite relaxation (SDR) is employed to achieve differential detection with near maximum-likelihood (ML) performance. The proposed SDR quasi-maximum-likelihood (QML) multiple-symbol differential detection (SDR-QML-MSDD) is efficient in that its complexity is polynomial in the observation window size, even in the worst case, while it exhibits almost the same performance as ML-MSDD does.

## I. INTRODUCTION

Multiple-symbol differential detection (MSDD) has been discussed as a way of improving performance relative to the standard differential detector for differential phase-shift keying (DPSK). One concern with MSDD is detection complexity at the receiver. Generally, performance improves when increasing the length of the observation window $N$. However brute-force MSDD quickly becomes computationally unfeasible as the *observation window size $N$* grows [1].

Some suboptimum approaches to MSDD tolerate a certain performance loss to achieve reduced complexity. Among them, [2] proposed an ML-MSDD algorithm with a complexity of $O(N \log_2 N)$ per block of $N$ symbols. However it only works well for the additive white Gaussian noise (AWGN) channel and the time-*invariant* Rayleigh fading channel. Another method is decision-feedback differential detection (DF-DD) [3] which is attractive because its complexity is linear in $N$ and the method is able to work over the more general time-varying Rayleigh fading

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channel, however, it achieves suboptimum performance. Reference [4] proposed to use the sphere decoder (SD) for multiple-symbol differential detection (hereafter referred to as SD-MSDD) and it was shown that it could approach the performance of ML-MSDD. However, the complexity of the sphere decoder (SD) is exponential in $N$ under relevant circumstances [5].

ML-MSDD essentially requires solving an integer-constrained least-squares problem of the same type as encountered in multiple-input multiple-output (MIMO) detection [6]. In this letter, we propose a detection approach for MSDD based on semi-definite relaxation (SDR) [7]. The resulting method has polynomial complexity in $N$ and good performance.

## II. SYSTEM MODEL

Let $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ and $\mathcal{E}\{\cdot\}$ denote (componentwise) complex conjugate, transpose, Hermitian transpose, and expectation, respectively. We assume a flat Rayleigh fading channel where the received signals are corrupted by additive noise and fading. For differential QPSK (DQPSK) modulation, at time $\tau$, two bits are Gray mapped to a quaternary data-carrying differential symbol $v_\tau$ which is taken from the QPSK signal constellation set $\mathcal{V} \triangleq \{e^{j2\pi m/4}, \ m = 0, 1, ..., 3\}$. Let the initial transmitted symbol be $\tilde{s}_0 = e^{j\pi/4}$. The transmitted symbol $\tilde{s}_\tau$ is encoded by the differential encoding relation $\tilde{s}_\tau = v_\tau \tilde{s}_{\tau-1}$. At the receiver, the received sample at time $\tau$ is

$$r_\tau = h_\tau \tilde{s}_\tau + n_\tau \tag{1}$$

where $h_\tau$ is the fading coefficient at time $\tau$ and $n_\tau \sim N(0, \sigma^2)$ is a sample of AWGN with variance $\sigma^2$. For differential detection, $h_\tau$ is unknown to both the transmitter and the receiver.

With MSDD, the receiver uses blocks of $N$ consecutively received samples $\boldsymbol{r}_t \triangleq [r_{Nt-N+1}, ..., r_{Nt}]^T$ to perform ML detection of $N$ transmitted symbols $\tilde{\boldsymbol{s}}_t \triangleq [\tilde{s}_{Nt-N+1}, ..., \tilde{s}_{Nt}]^T$. $N$ is referred to as the observation window size. The ML detector for MSDD is given by [4]

$$\tilde{\boldsymbol{s}}^{ML} = \arg\min_{\tilde{\boldsymbol{s}}} \{\boldsymbol{r}^H \boldsymbol{\Theta}_{rr}^{-1} \boldsymbol{r}\}. \tag{2}$$

where $\boldsymbol{\Theta}_{rr} \triangleq \mathcal{E}\{\boldsymbol{r}\boldsymbol{r}^H | \tilde{\boldsymbol{s}}\} \in \mathbb{C}^{N \times N}, \tilde{\boldsymbol{s}} \in \mathbb{C}^{N \times 1}$.

## III. MULTIPLE-SYMBOL DIFFERENTIAL DETECTION USING SEMI-DEFINITE PROGRAMMING

To simplify notation, from now on we omit the time index $\tau$ and the block index $t$. We use $x_i$ to denote the $i$th component of the vector $\boldsymbol{x}$. For the derivation of the SDR-MSDD for QPSK signals, it is more convenient to write $\tilde{s}_i = E_v(a_i + b_i \cdot j)$, where $E_v$ is a scalar which normalizes the average energy per symbol to 1 and $a_i, b_i \in \{-1, 1\}$ whose values are determined by the corresponding phases of $\tilde{s}_i$. Since $E_v$ is a constant scalar which would not affect the main point of our results, it is omitted as well in the following description. Let $\text{Diag}(\boldsymbol{x})$ be the diagonal matrix with the elements of the vector $\boldsymbol{x}$ on the main diagonal and let $\boldsymbol{I}_N$ be the $N \times N$ identity matrix. Note that $\boldsymbol{\Theta}_{rr}$ in (2) can be written as $\boldsymbol{\Theta}_{rr} = \text{Diag}(\tilde{s})\boldsymbol{G}\text{Diag}(\tilde{s}^*)$ and $\boldsymbol{G} \triangleq \mathcal{E}\{\boldsymbol{h}\boldsymbol{h}^H\} + \sigma^2 \boldsymbol{I}_N$, $\boldsymbol{G} \in \mathbb{C}^{N \times N}$, and define

$\tilde{Q} \triangleq \text{Diag}(r)\{G^{-1}\}^* \text{Diag}(r)^* \in \mathbb{C}^{N \times N}$. Hence we have

$$\tilde{s}^{ML} = \arg\min_{\tilde{s}} \tilde{s}^H \tilde{Q} \tilde{s} \tag{3}$$

To avoid using complex-valued matrices, the ML detection model (3) can be transformed to real-valued notation by writing

$$\min_{s \in \{-1,+1\}^{2N}} s^T Q s \tag{4}$$

where

$$s = \begin{bmatrix} \Re(\tilde{s}) \\ \Im(\tilde{s}) \end{bmatrix} \in \mathbb{R}^{2N \times 1}, \quad Q = \begin{bmatrix} \Re(\tilde{Q}) & -\Im(\tilde{Q}) \\ \Im(\tilde{Q}) & \Re(\tilde{Q}) \end{bmatrix} \in \mathbb{R}^{2N \times 2N}$$

Then $s$ is a vector of length $2N \times 1$ and its components $s_1, s_2, ..., s_{2N} \in \{-1, +1\}$.

The formulation in (4) is a *Boolean QP* problem. We next derive the semi-definite relaxation detector. Since $s^T Q s = \text{Trace}(s s^T Q)$, problem (4) is equivalent to

$$\min \text{Trace}(SQ)$$
$$\text{s.t. } S = s s^T \in \mathbb{R}^{2N \times 2N}, \ S_{ii} = 1, \ i = 1, ..., 2N. \tag{5}$$

The constraint $S = s s^T$ indicates that $S$ is symmetric, positive semi-definite (PSD) and of rank 1. Problem (5) is a nonconvex optimization problem owing to the constraint $S = s s^T$. However, if the rank-1 constraint is removed from (5), then the following relaxed problem is obtained:

$$\min \text{Trace}(SQ)$$
$$\text{s.t. } S \succeq 0, \ S_{ii} = 1, \ i = 1, ..., 2N. \tag{6}$$

where $S \succeq 0$ means that $S$ is symmetric and PSD. Equation (6) is a semi-definite relaxation of (4) and it is a semi-definite programming (SDP) problem that is convex and which can be efficiently solved. The most common algorithms for solving SDP problems are interior points methods, whose computational complexities are polynomial in the problem size [8].

Due to the removal of the rank-1 constraint, the solution of the SDR problem (6) is a $2N \times 2N$ matrix $S$ which is not a solution to the original problem (5). The final aim of SDR-QML-MSDD is to obtain a vector $\hat{s}$ which solves (5) at least approximately. To achieve this, we perform a so-called randomization procedure to find $s$ from $S$. There are three basic ways to perform randomization: simple rounding quantization, eigenvalue decomposition and Cholesky factorization. We choose to perform the randomization by first computing the Cholesky factorization [7] of $S$: $S = V V^T$. We then generate a vector $u$ which is uniformly distributed over the $2N$-dimensional unit sphere. The components of the vector $\hat{s} = \text{sign}(V u)$ whose values are larger than 0 are set to 1, and the rest are set to $-1$. To obtain a more accurate approximation, we perform the above randomization procedure 20 times and choose the $\hat{s}$ which minimizes the objective function in (4).

## IV. SIMULATION RESULTS

To verify the effectiveness of SDR detection, we performed a simulation with the same parameter set as was used in [4]. The system uses DQPSK modulation and the channel is the widely used Clarke's fading model with the maximum normalized fading bandwidth $B_f T = 0.03$. The channel also introduces AWGN. A BER versus $E_b/N_0$ performance comparison between SD-MSDD and SDR-QML-MSDD is given in Fig. 1. The Schnorr-Euchner sphere decoder with infinite initial radius was used to obtain the sphere decoding results [4]. The search radius for SD-MSDD was chosen sufficiently large such that ML estimate is found inside the sphere with high probability. The performance of the coherent detector is also plotted as a reference. Two cases have been simulated, $N = 6$ and $N = 10$. The performance improves with increasing $N$, as expected. The performance difference between SD-MSDD and SDR-QML-MSDD is marginal. Note that SD-MSDD finds the maximum-likelihood solution and therefore its performance is a lower bound on the performance of any algorithm. It is clear that SDR-QML-MSDD can approach this lower bound very closely.

Since both SD-MSDD and SDR-QML-MSDD have similar performance, it is interesting to compare their complexity. We measured the complexity by counting the total number of floating point operations (FLOPS) required by the detection process by performing $10^6$ independent simulation trials. Two types of complexity, *average complexity* and *worst-case complexity*, are given in Fig. 2. It can be observed that SD-MSDD generally has less average complexity than the SDR-QML-MSDD and that it becomes more efficient as the $E_b/N_0$ increases. The worst-case complexity is evaluated by picking the 1% upper percentile of the measured FLOPS. This indicates that SD-MSDD has much higher complexity than SDR-QML-MSDD in the low $E_b/N_0$ regime. For SDR-QML-MSDD, there is no big difference between average complexity and worst-case complexity, which implies that the complexity of SDR-QML-MSDD does not vary much between different instances of the problem. Taken together, SDR-QML-MSDD is superior to SD-MSDD in terms of worst-case complexity.

## V. CONCLUSION

By using semi-definite programming methods, multiple-symbol differential detection can be performed with low worst-case complexity and negligible performance loss relative to maximum-likelihood. We proposed a specific algorithm for doing this and demonstrated its effectiveness via simulations on a time-varying Rayleigh fading channel.

## VI. ACKNOWLEDGMENT

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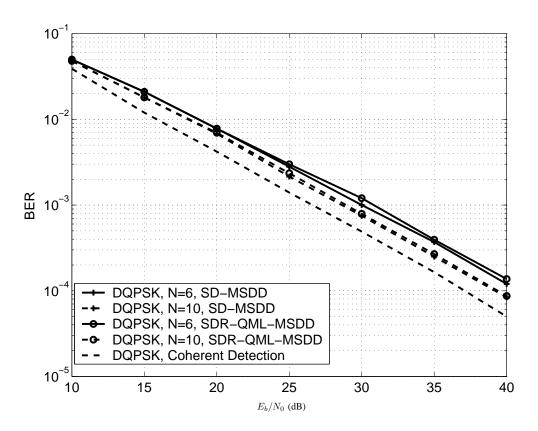
**Figures caption list**

Fig. 1 BER versus $E_b/N_0$ for DQPSK and Rayleigh fading with $B_f T = 0.03$.

Fig. 2 Average and worst-case complexity comparison of SDR-QML-MSDD and SD-MSDD for $N = 10$.

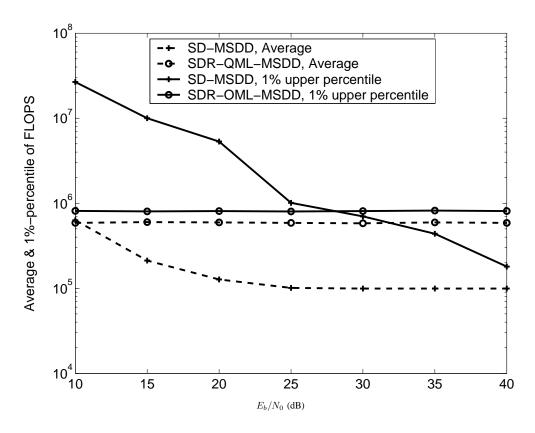Fig. 1.   BER versus $E_b/N_0$ for DQPSK and Rayleigh fading with $B_f T = 0.03$.

Fig. 2.   Average and worst-case complexity comparison of SDR-QML-MSDD and SD-MSDD for $N = 10$.