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Parameterization of the MISO IFC Rate Region: The Case of Partial Channel State Information

Johannes Lindblom, Erik G. Larsson and Eduard A. Jorswieck

Abstract—We study the achievable rate region of the multiple-input single-output (MISO) interference channel (IFC), under the assumption that all receivers treat the interference as additive Gaussian noise. We assume the case of two users, and that the channel state information (CSI) is only partially known at the transmitters. Our main result is a characterization of Pareto-optimal transmit strategies, for channel matrices that satisfy a certain technical condition. Numerical examples are provided to illustrate the theoretical results.

Index Terms—Ergodic rate region, interference channel, multiple-input single-output channel, multistream transmission, Pareto optimality.

I. INTRODUCTION

WE are concerned with the scenario where we have two independent but mutually interfering wireless systems operating simultaneously in the same spectral band. System i consists of one base station BS_i that wants to transmit information to a mobile MS_i , $i = 1, 2$. The two mobiles receive a superposition of the signals transmitted from the two base stations. This setup is recognized as an interference channel (IFC) [1]–[3]. The IFC is important because it models the spectrum sharing situation where a number of unrelated senders (base stations) try to communicate information to different receivers (mobile stations) via a common channel. Recently there is a huge interest in understanding IFCs [4], [5]. Finding the capacity region for general IFCs is still an open problem, but various achievable rate regions are known.

We desire to understand what the achievable rate region looks like in the case that the receiver treats interference as noise. In particular, we are interested in the so-called Pareto boundary of the region. This boundary consists of Pareto optimal operating points, which are points where it is impossible to improve the rate of one communication link without simultaneously decreasing the rate of the other link. (See Definition 1 in Section III.) We consider the case when

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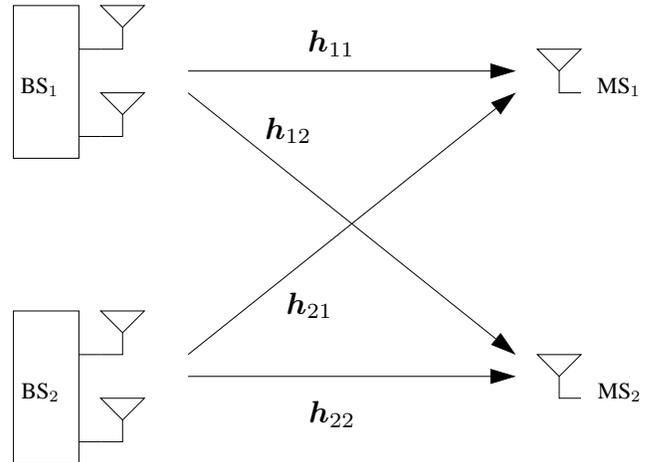


Fig. 1. The two-user MISO interference channel under study (illustrated for $n = 2$ transmit antennas).

BS_1 and BS_2 have n transmit antennas and MS_1 and MS_2 have a single receive antenna each. This setup is a multiple-input single-output (MISO) IFC [6]. See Figure 1.

If the channel state information (CSI) is completely known at the transmitters, then single-stream beamforming is optimal for the IFC [7]. By contrast, if perfect CSI is unavailable, then in general one must use multi-stream beamforming [8]. That is, the transmitters BS_i should send message vectors $\mathbf{x}_i \sim \mathcal{CN}(\mathbf{0}, \Psi_i)$, i.e., \mathbf{x}_i is zero-mean complex Gaussian with covariance matrix Ψ_i . The rank of Ψ_i , say N_i , is at least one. Clearly, there is a conflict situation associated with the choices of Ψ_1, Ψ_2 , since a covariance matrix Ψ_1 which is good for the link $BS_1 \rightarrow MS_1$ may generate substantial interference for MS_2 and vice versa.

Our main result in this paper is a set of necessary conditions for transmit strategies to be Pareto optimal for the MISO IFC. The underlying assumption is that the channel vectors are zero-mean Gaussian with known covariance matrices. That is, the transmitter has only statistical channel knowledge. The covariance matrices studied here must satisfy a certain condition (see Proposition 1). This condition implies that the channel covariance matrices must be rank deficient, which corresponds to the case of a small angular spread, e.g., see [9] or Chapter 7 in [10].

This work extends our work in [11] where a corresponding parameterization was presented for the case of complete CSI, i.e., the channel vectors were perfectly known at the transmitters. This paper also extends our work in [12] where we treated the case of partial CSI, but with the restriction that the transmitters perform single-stream transmission, i.e., $N_i = 1$.

II. SYSTEM MODEL

We consider the 2-user MISO IFC with n transmit antennas and frequency flat channels. See Figure 1. BS $_i$ transmits a vector \mathbf{x}_i with $\mathbf{x}_i \sim \mathcal{CN}(\mathbf{0}, \Psi_i)$. This vector \mathbf{x}_i may be constructed by superimposing multiple streams as follows:

$$\mathbf{x}_i = \sum_{k=1}^{N_i} \mathbf{w}_i^{(k)} \sqrt{p_i^{(k)}} s_i^{(k)} \quad (1)$$

where $\mathbf{w}_i^{(k)}$ are the eigenvectors of Ψ_i and the powers $p_i^{(k)}$ are the corresponding eigenvalues. In (1), $\{s_i^{(k)}\}_{k=1}^{N_i}$ are i.i.d. Gaussian variables with zero mean and unit variance. Also, $N_i = \text{rank}\{\Psi_i\}$, as before. The matched-filtered, symbol-sampled complex baseband data received at MS $_1$ and MS $_2$ will then be

$$\begin{aligned} y_1 &= \mathbf{h}_{11}^H \mathbf{x}_1 + \mathbf{h}_{21}^H \mathbf{x}_2 + e_1 \\ y_2 &= \mathbf{h}_{22}^H \mathbf{x}_2 + \mathbf{h}_{12}^H \mathbf{x}_1 + e_2 \end{aligned} \quad (2)$$

In (2), \mathbf{h}_{ij} is the $n \times 1$ conjugated channel-vector between BS $_i$ and MS $_j$ and \mathbf{x}_i is the vector transmitted by BS $_i$.¹ We model the channel vectors as $\mathbf{h}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{ij})$. Also, e_1 and e_2 are noise variables which we model as $\mathcal{CN}(0, \sigma^2)$. By $(\cdot)^H$, we denote the conjugate transpose. We further assume that each base station can use the transmit power P . Without loss of generality, we shall take $P = 1$. This gives the power constraint

$$E[\|\mathbf{x}_i\|^2] = \text{Tr}\{\Psi_i\} = \sum_{k=1}^{N_i} p_i^{(k)} \leq 1, \quad i = 1, 2,$$

where $E[\cdot]$ and $\text{Tr}\{\cdot\}$ denote the expectation value and the trace respectively. The signal-to-noise ratio (SNR) will be defined as $1/\sigma^2$.

III. THE ACHIEVABLE RATE REGION

For a given pair of covariance matrices $\{\Psi_1, \Psi_2\}$ and fixed \mathbf{h}_{ij} , the signal-to-noise-and-interference ratio in y_1 is

$$\frac{E[|\mathbf{h}_{11}^H \mathbf{x}_1|^2]}{E[|\mathbf{h}_{21}^H \mathbf{x}_2|^2] + E[|e_1|^2]} = \frac{\mathbf{h}_{11}^H \Psi_1 \mathbf{h}_{11}}{\mathbf{h}_{21}^H \Psi_2 \mathbf{h}_{21} + \sigma^2}.$$

A similar expression holds for y_2 . Hence, following instantaneous rates are achievable:

$$R_1 = \log_2 \left(1 + \frac{\mathbf{h}_{11}^H \Psi_1 \mathbf{h}_{11}}{\mathbf{h}_{21}^H \Psi_2 \mathbf{h}_{21} + \sigma^2} \right) \quad (3)$$

for the link BS $_1 \rightarrow$ MS $_1$, and

$$R_2 = \log_2 \left(1 + \frac{\mathbf{h}_{22}^H \Psi_2 \mathbf{h}_{22}}{\mathbf{h}_{12}^H \Psi_1 \mathbf{h}_{12} + \sigma^2} \right) \quad (4)$$

for BS $_2 \rightarrow$ MS $_2$. For fixed channels $\{\mathbf{h}_{ij}\}$, we define the instantaneous achievable rate region as

$$\mathcal{R} = \bigcup_{\Psi_1, \Psi_2, \text{Tr}\{\Psi_i\} \leq 1} (R_1, R_2),$$

where $\text{Tr}\{\Psi_i\} \leq 1$ is the power constraint.

¹We define \mathbf{h}_{ij} as the conjugate of the channel vectors, as this will simplify notation later.

Since only the statistical distributions of \mathbf{h}_{ij} are known, one important performance measure is the average (expected) rate:

$$\bar{R}_i \triangleq E_{\mathbf{h}_{1i}, \mathbf{h}_{2i}}[R_i], \quad (5)$$

where the expectation is over \mathbf{h}_{ij} , $i, j \in \{1, 2\}$. The corresponding rate region is

$$\bar{\mathcal{R}} = \bigcup_{\Psi_1, \Psi_2, \text{Tr}\{\Psi_i\} \leq 1} (\bar{R}_1, \bar{R}_2).$$

We are interested in providing necessary conditions on Ψ_i for (\bar{R}_1, \bar{R}_2) to lie on the Pareto boundary of $\bar{\mathcal{R}}$. This boundary consists of Pareto optimal points. Pareto optimality is defined as follows:

Definition 1: A rate tuple (\bar{R}_1, \bar{R}_2) is Pareto optimal if there is no other tuple (\bar{Q}_1, \bar{Q}_2) with $(\bar{Q}_1, \bar{Q}_2) \geq (\bar{R}_1, \bar{R}_2)$ and $(\bar{Q}_1, \bar{Q}_2) \neq (\bar{R}_1, \bar{R}_2)$. (The inequality is component-wise.)

IV. NECESSARY CONDITIONS FOR THE PARETO BOUNDARY

We now present our main result.

Proposition 1: Suppose that the channel covariance matrices satisfy the condition $\text{span}\{\mathbf{Q}_{ii}\} \not\subseteq \text{span}\{\mathbf{Q}_{ij}\}$, for $i, j \in \{1, 2\}$, $i \neq j$.² Then, any transmit covariance matrix Ψ_i that corresponds to a rate point (\bar{R}_i, \bar{R}_j) on the Pareto boundary, again for $i, j \in \{1, 2\}$, $i \neq j$, satisfies

- $\text{span}\{\Psi_i\} \subseteq \text{span}\{\mathbf{Q}_{ii}, \mathbf{Q}_{ij}\}$ and
- $\text{Tr}\{\Psi_i\} = 1$, that is, at the boundary, both base stations use full power.

In order to prove Proposition 1 we first state the following two lemmas, which deal with properties of \bar{R}_i . The Lemmas are stated for \bar{R}_1 ; similar results hold for \bar{R}_2 . Proofs of the lemmas can be found in the Appendix. To present the lemmas we need the notation

$$K_{ij} \triangleq \text{rank}\{\mathbf{Q}_{ij}^{1/2} \Psi_i \mathbf{Q}_{ij}^{1/2}\}, \quad \text{and} \quad (6)$$

$$\lambda_k^{ij} \triangleq \text{the } k\text{-th eigenvalue of } \mathbf{Q}_{ij}^{1/2} \Psi_i \mathbf{Q}_{ij}^{1/2}. \quad (7)$$

Lemma 1: The expected value of R_1 can be expressed as

$$\begin{aligned} \bar{R}_1 &= \int_0^\infty \cdots \int_0^\infty \prod_{i=1}^{K_{11}} e^{-x_i} \prod_{j=1}^{K_{21}} e^{-y_j} \times \\ &\times \log_2 \left(1 + \frac{\sum_{i=1}^{K_{11}} \lambda_i^{11} x_i}{\sum_{j=1}^{K_{21}} \lambda_j^{21} y_j + \sigma^2} \right) dx_1 \cdots dx_{K_{11}} dy_1 \cdots dy_{K_{21}}. \end{aligned} \quad (8)$$

Lemma 2: \bar{R}_1 (see (8)) is monotonously increasing with λ_i^{11} , $i = 1, \dots, K_{11}$, for fixed λ_j^{21} , $j = 1, \dots, K_{21}$. Also \bar{R}_1 is monotonously decreasing with λ_j^{21} , $j = 1, \dots, K_{21}$, for fixed λ_i^{11} , $i = 1, \dots, K_{11}$. (Here λ_k^{ij} and K_{ij} are defined in (6) and (7), respectively.)

Proof of Proposition 1a): We give the proof for Ψ_1 ; the proof for Ψ_2 goes in a similar manner. The main idea of this proof is as follows: Assume that transmitter 1 uses a covariance matrix, Φ_1 , which partly lies in the orthogonal complement of $\text{span}\{\mathbf{Q}_{11}, \mathbf{Q}_{12}\}$. Then the parts of Φ_1 , which

²This condition was missing in the statement of Proposition 1 in [12]. (Reference [12] dealt with the single-stream case only.)

are in the orthogonal complement of $\text{span}\{\mathbf{Q}_{11}, \mathbf{Q}_{12}\}$, will not affect the rates (\bar{R}_1, \bar{R}_2) , and hence transmit power is wasted.

The proof is by contradiction, so to proceed, suppose the statement in the proposition is false. Then there exists a Ψ_1 , $\text{Tr}\{\Psi_1\} \leq 1$, which corresponds to a rate point on the boundary but for which $\text{span}\{\Psi_1\} \not\subseteq \text{span}\{\mathbf{Q}_{11}, \mathbf{Q}_{12}\}$. The idea is now this: Given such a Ψ_1 , we first construct a covariance matrix Ψ'_1 that has $\text{Tr}\{\Psi'_1\} < 1$ and which achieves the same rates as Ψ_1 . We then use the power saved when going from Ψ_1 to Ψ'_1 to construct another matrix Ψ''_1 , with $\text{Tr}\{\Psi''_1\} \leq 1$ and which improves one of the rates (\bar{R}_1) but leaves the other (\bar{R}_2) unchanged. Hence, Ψ_1 cannot correspond to a point on the boundary, and we have a contradiction.

To construct Ψ'_1 , we start by letting $\{\mathbf{u}_i\}$ be an ON-basis for $\text{span}\{\mathbf{Q}_{11}, \mathbf{Q}_{12}\}^\perp$. We denote by \mathbf{I} the identity matrix and define $\Pi_X^\perp \triangleq \mathbf{I} - \mathbf{X}(\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H$ to be the orthogonal projection onto the orthogonal complement of the column space of \mathbf{X} . Since $\Pi_{\text{span}\{\mathbf{Q}_{11}, \mathbf{Q}_{12}\}}^\perp \Psi_1 \Pi_{\text{span}\{\mathbf{Q}_{11}, \mathbf{Q}_{12}\}}^\perp$ is positive semidefinite it follows that

$$\Pi_{\text{span}\{\mathbf{Q}_{11}, \mathbf{Q}_{12}\}}^\perp \Psi_1 \Pi_{\text{span}\{\mathbf{Q}_{11}, \mathbf{Q}_{12}\}}^\perp = \sum_{i=1}^K \beta_i \mathbf{u}_i \mathbf{u}_i^H,$$

where $K \triangleq \text{rank}\{\{\mathbf{Q}_{11}, \mathbf{Q}_{12}\}^\perp\}$, all β_i are non-negative and $\beta_i > 0$ for some i , say $i = i'$ and $\mathbf{u}_{i'} \in \text{span}\{\mathbf{Q}_{11}, \mathbf{Q}_{12}\}^\perp$. Now, let

$$\Psi'_1 \triangleq \Psi_1 - [\mathbf{u}_{i'}^H \Psi_1 \mathbf{u}_{i'}] \mathbf{u}_{i'} \mathbf{u}_{i'}^H. \quad (9)$$

We shall show that we have for fixed Ψ_2

- i) $\bar{R}'_1 = \bar{R}_1$ (i.e. \bar{R}_1 will be unchanged when Ψ'_1 is used instead of Ψ_1)
- ii) $\bar{R}'_2 = \bar{R}_2$ (i.e. \bar{R}_2 will be unchanged when Ψ'_1 is used instead of Ψ_1)
- iii) $\text{Tr}\{\Psi'_1\} < \text{Tr}\{\Psi_1\} \leq 1$ (i.e. the transmitted power will be strictly decreased when Ψ'_1 is used instead of Ψ_1)
- iv) Ψ'_1 is positive semidefinite (i.e. it is a valid covariance matrix)

where (\bar{R}'_1, \bar{R}'_2) is the rate point associated with (Ψ'_1, Ψ_2) given as

$$\bar{R}'_1 = E_{\mathbf{h}_{11}, \mathbf{h}_{21}} \left[\log_2 \left(1 + \frac{\mathbf{h}_{11}^H \Psi'_1 \mathbf{h}_{11}}{\mathbf{h}_{21}^H \Psi_2 \mathbf{h}_{21} + \sigma^2} \right) \right],$$

$$\bar{R}'_2 = E_{\mathbf{h}_{12}, \mathbf{h}_{22}} \left[\log_2 \left(1 + \frac{\mathbf{h}_{22}^H \Psi_2 \mathbf{h}_{22}}{\mathbf{h}_{12}^H \Psi'_1 \mathbf{h}_{12} + \sigma^2} \right) \right].$$

Item i) follows because $\mathbf{u}_{i'} \perp \mathbf{h}_{11}$ (with probability 1):

$$\mathbf{h}_{11}^H \Psi'_1 \mathbf{h}_{11} = \mathbf{h}_{11}^H (\Psi_1 - [\mathbf{u}_{i'}^H \Psi_1 \mathbf{u}_{i'}] \mathbf{u}_{i'} \mathbf{u}_{i'}^H) \mathbf{h}_{11} = \mathbf{h}_{11}^H \Psi_1 \mathbf{h}_{11}$$

Item ii) follows because $\mathbf{u}_{i'} \perp \mathbf{h}_{12}$ (with probability 1):

$$\mathbf{h}_{12}^H \Psi'_1 \mathbf{h}_{12} = \mathbf{h}_{12}^H (\Psi_1 - [\mathbf{u}_{i'}^H \Psi_1 \mathbf{u}_{i'}] \mathbf{u}_{i'} \mathbf{u}_{i'}^H) \mathbf{h}_{12} = \mathbf{h}_{12}^H \Psi_1 \mathbf{h}_{12}$$

Item iii) follows by

$$\begin{aligned} \text{Tr}\{\Psi'_1\} &= \text{Tr}\{\Psi_1 - [\mathbf{u}_{i'}^H \Psi_1 \mathbf{u}_{i'}] \mathbf{u}_{i'} \mathbf{u}_{i'}^H\} \\ &= \text{Tr}\{\Psi_1\} - \mathbf{u}_{i'}^H \Psi_1 \mathbf{u}_{i'} \text{Tr}\{\mathbf{u}_{i'} \mathbf{u}_{i'}^H\} \\ &= \text{Tr}\{\Psi_1\} - \mathbf{u}_{i'}^H \Psi_1 \mathbf{u}_{i'} \text{Tr}\{\mathbf{u}_{i'} \mathbf{u}_{i'}^H\} \\ &= \text{Tr}\{\Psi_1\} - \mathbf{u}_{i'}^H \Psi_1 \mathbf{u}_{i'} < \text{Tr}\{\Psi_1\} \end{aligned}$$

since $\mathbf{u}_{i'}^H \Psi_1 \mathbf{u}_{i'} > 0$ and $\mathbf{u}_{i'}^H \mathbf{u}_{i'} = 1$.

Item iv) follows since

$$\mathbf{u}_{i'}^H \Psi'_1 \mathbf{u}_{i'} = \mathbf{u}_{i'}^H \Psi_1 \mathbf{u}_{i'} - \mathbf{u}_{i'}^H \Psi_1 \mathbf{u}_{i'} = 0$$

(see (9); note that $\mathbf{u}_{i'}^H \mathbf{u}_{i'} = 1$) and

$$\mathbf{v}^H \Psi'_1 \mathbf{v} = \mathbf{v}^H \Psi_1 \mathbf{v} \geq 0$$

for any $\mathbf{v} \perp \mathbf{u}_{i'}$.

Next, we construct the matrix Ψ''_1 . For given δ , we define $\Psi''_1 \triangleq \Psi'_1 + \delta \delta^H$ and

$$\bar{R}'_1 \triangleq E_{\mathbf{h}_{11}, \mathbf{h}_{21}} \left[\log_2 \left(1 + \frac{\mathbf{h}_{11}^H \Psi''_1 \mathbf{h}_{11}}{\mathbf{h}_{21}^H \Psi_2 \mathbf{h}_{21} + \sigma^2} \right) \right],$$

$$\bar{R}'_2 \triangleq E_{\mathbf{h}_{12}, \mathbf{h}_{22}} \left[\log_2 \left(1 + \frac{\mathbf{h}_{22}^H \Psi_2 \mathbf{h}_{22}}{\mathbf{h}_{12}^H \Psi''_1 \mathbf{h}_{12} + \sigma^2} \right) \right].$$

We will now show that there exists a δ such that

- v) $\bar{R}'_1 > \bar{R}'_1 = \bar{R}_1$ (i.e. Ψ''_1 will cause an increase in \bar{R}_1 compared to Ψ_1)
- vi) $\bar{R}'_2 = \bar{R}_2$ (i.e. \bar{R}_2 will be unchanged when Ψ''_1 is used instead of Ψ_1)
- vii) $\text{Tr}\{\Psi''_1\} \leq 1$ (i.e. the power constraint is satisfied)

From Lemma 2, we know that increasing one of the λ_i^{11} values leads to an increase of \bar{R}_1 . Item v) is satisfied if at least one of the eigenvalues of $\mathbf{Q}_{11}^{1/2} (\Psi'_1 + \delta \delta^H) \mathbf{Q}_{11}^{1/2}$ is larger than those of $\mathbf{Q}_{11}^{1/2} \Psi_1 \mathbf{Q}_{11}^{1/2}$, and no eigenvalues of $\mathbf{Q}_{11}^{1/2} (\Psi'_1 + \delta \delta^H) \mathbf{Q}_{11}^{1/2}$ are smaller than those of $\mathbf{Q}_{11}^{1/2} \Psi_1 \mathbf{Q}_{11}^{1/2}$. From [13, p. 198] we have

$$\lambda_{\min}(\mathbf{B}) \leq \lambda_k(\mathbf{A} + \mathbf{B}) - \lambda_k(\mathbf{A}) \leq \lambda_{\max}(\mathbf{B})$$

for any Hermitian matrices \mathbf{A} and \mathbf{B} . Hence, for any k , $k = 1, \dots, n$

$$\begin{aligned} \lambda_k(\mathbf{Q}_{11}^{1/2} \Psi'_1 \mathbf{Q}_{11}^{1/2}) + \lambda_{\min}(\mathbf{Q}_{11}^{1/2} \delta \delta^H \mathbf{Q}_{11}^{1/2}) \\ \leq \lambda_k(\mathbf{Q}_{11}^{1/2} (\Psi'_1 + \delta \delta^H) \mathbf{Q}_{11}^{1/2}) \end{aligned}$$

Since $\lambda_{\min}(\mathbf{Q}_{11}^{1/2} \delta \delta^H \mathbf{Q}_{11}^{1/2}) \geq 0$ it follows that

$$\lambda_k(\mathbf{Q}_{11}^{1/2} (\Psi'_1 + \delta \delta^H) \mathbf{Q}_{11}^{1/2}) \geq \lambda_k(\mathbf{Q}_{11}^{1/2} \Psi'_1 \mathbf{Q}_{11}^{1/2}),$$

that is, no eigenvalue will decrease. If we choose δ such that

$$\mathbf{Q}_{11} \delta \neq \mathbf{0}, \quad (10)$$

then we guarantee that

$$\text{Tr}\{\mathbf{Q}_{11}^{1/2} (\Psi'_1 + \delta \delta^H) \mathbf{Q}_{11}^{1/2}\} > \text{Tr}\{\mathbf{Q}_{11}^{1/2} \Psi'_1 \mathbf{Q}_{11}^{1/2}\}.$$

Therefore we also know that at least one of the eigenvalues of $\mathbf{Q}_{11}^{1/2} (\Psi'_1 + \delta \delta^H) \mathbf{Q}_{11}^{1/2}$ will increase compared to the eigenvalues of $\mathbf{Q}_{11}^{1/2} \Psi'_1 \mathbf{Q}_{11}^{1/2}$. Equation (10) says that δ cannot entirely lie in $\text{span}\{\mathbf{Q}_{11}\}^\perp$.

Note that item vi) is satisfied if

$$\delta \in \text{span}\{\mathbf{Q}_{12}\}^\perp. \quad (11)$$

To construct δ we therefore proceed as follows. First choose a $\bar{\delta}$ such that (10) and (11) are satisfied. This can be accomplished by solving the equation system

$$\begin{cases} \mathbf{Q}_{11} \bar{\delta} \neq \mathbf{0} \\ \mathbf{Q}_{12} \bar{\delta} = \mathbf{0} \end{cases}. \quad (12)$$

Then let $\delta = \epsilon \bar{\delta} / \|\bar{\delta}\|$ where $\epsilon > 0$ (to be chosen shortly). Note that we cannot find any solution of (12) if $\text{span}\{\mathbf{Q}_{11}\} \subseteq \text{span}\{\mathbf{Q}_{12}\}$.

Item *vii)* is satisfied if

$$\begin{aligned} \text{Tr}\{\Psi_1''\} &= \text{Tr}\{\Psi_1'\} + \|\delta\|^2 = \text{Tr}\{\Psi_1'\} + \epsilon^2 \|\bar{\delta}\|^2 \leq 1 \Leftrightarrow \\ \epsilon &\leq \sqrt{1 - \text{Tr}\{\Psi_1'\}}. \end{aligned}$$

Such an ϵ exists, because $\text{Tr}\{\Psi_1'\} < 1$ according to item *iii)*. For example, take $\epsilon = \sqrt{1 - \text{Tr}\{\Psi_1'\}}$. Hence we have a contradiction: (Ψ_1'', Ψ_2) achieves (\bar{R}_1'', \bar{R}_2) where $\bar{R}_1'' > \bar{R}_1$. \square

Proof of Proposition 1b): To show that we must have $\text{Tr}\{\Psi_1\} = 1$ at the Pareto boundary, assume that $\text{Tr}\{\Psi_1\} < 1$. Let $\Psi_1' = \Psi_1 + \delta\delta^H$ where δ is chosen according to the recipe above. Together this shows that if $\text{Tr}\{\Psi_1\} < 1$ then it is possible to choose a new Ψ_1' such that $\text{Tr}\{\Psi_1'\} = 1$, \bar{R}_1 is increased and \bar{R}_2 is unchanged. \square

Note that Proposition 1b) holds only when \mathbf{Q}_{11} lies partly in the null space of \mathbf{Q}_{12} . The single-input single-output (SISO) IFC is a special case when Proposition 1b) does not hold, see, for example Sections III-B and III-C in [14].

V. SPECIAL CASES

Here we present two important special cases. Both treat the case $N_i = 1$, i.e., single-stream beamforming.

$N_i = 1$ and general \mathbf{Q}_{ij} : This case was treated in [12]. From Proposition 1 in [12] we know that any point on the Pareto boundary corresponds to a rate pair where the beamforming vectors are chosen such that $\mathbf{w}_i^{(1)} \in \text{span}\{\mathbf{Q}_{i1}, \mathbf{Q}_{i2}\}$, $i = 1, 2$. This is consistent with our Proposition 1.

$N_i = 1$ and $\text{rank}\{\mathbf{Q}_{ij}\} = 1$: This is the case of a rank-one channel (no angular spread). In this case \mathbf{Q}_{ij} can be written as $\mathbf{Q}_{ij} = \mathbf{q}_{ij}\mathbf{q}_{ij}^H$ where \mathbf{q}_{ij} is an n -vector. Let $\bar{h}_{ij} \sim \mathcal{CN}(0, 1)$. Then we can write

$$\mathbf{h}_{ij} = \bar{h}_{ij}\mathbf{q}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{q}_{ij}\mathbf{q}_{ij}^H) = \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{ij}).$$

This means that the CSI is known up to an unknown scalar constant. The beamforming vectors should then be chosen as:

$$\begin{aligned} \mathbf{w}_1 &= \xi_{11}\mathbf{h}_{11} + \xi_{12}\mathbf{h}_{12} \\ \mathbf{w}_2 &= \xi_{22}\mathbf{h}_{22} + \xi_{21}\mathbf{h}_{21} \end{aligned}$$

for some ξ_{ij} . In this special case, the parameterization in Proposition 1 essentially reduces to Proposition 1 in [11] (where only perfectly informed transmitters were treated). Note that with perfect CSI, single-stream beamforming with $N_i = 1$ is optimal for the MISO IFC [7].

VI. NUMERICAL RESULTS

We illustrate Proposition 1 with two numerical examples. We used two different sets of channel covariance matrices: one set with weak interference between the systems (see Fig. 2), and one set with strong interference (see Fig. 3). For the case with strong interference, the covariance matrices were chosen such that $\text{span}\{\mathbf{Q}_{12}\}$ was close to $\text{span}\{\mathbf{Q}_{11}\}$ and such that $\text{span}\{\mathbf{Q}_{21}\}$ was close to $\text{span}\{\mathbf{Q}_{22}\}$. In both simulations we used $n = 5$ transmit antennas and $\text{rank}\{\mathbf{Q}_{ij}\} = 2$. The matrices Ψ_i were chosen such that $N_i = \text{rank}\{\mathbf{Q}_{i1}, \mathbf{Q}_{i2}\} = 4$.

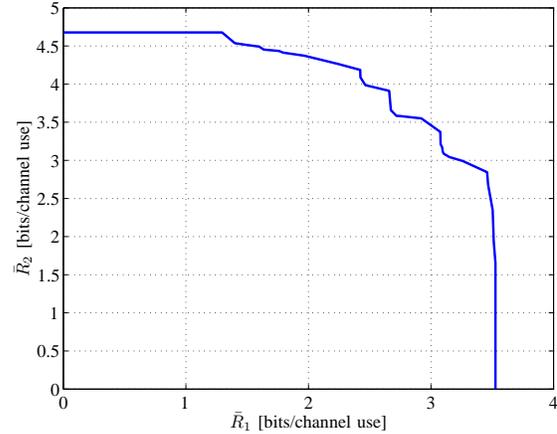


Fig. 2. Rate region for a system with weak interference, SNR=10 dB. (The roughness of the curve is an artifact of the numerical simulation.)

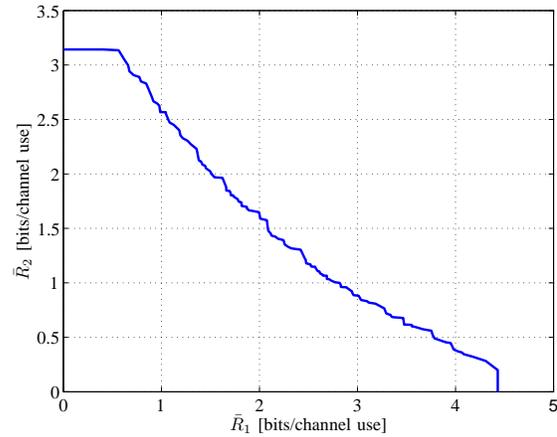


Fig. 3. Rate region for a system with strong interference, which leads to a non-convex region. SNR=10 dB.

For each simulation we generated $4 \cdot 10^9$ pairs of transmit covariance matrices using the parameterization. To be able to do efficient simulations we used Eq. (37) in [15] to evaluate the integrals in (8) in closed form. Figs. 2 and 3 show the results for the cases of weak and strong interference, respectively. One important observation is that the rate region may be either convex or non-convex, even for perfect CSI [11] and SISO [14].

VII. CONCLUSION

The motivation of this paper has been the huge interest in IFCs as a model for spectrum resource conflicts. We have studied the MISO IFC, and especially the case when the CSI is not perfectly known at the transmitter. Our main contribution is a set of necessary conditions on Pareto-optimal transmit strategies, channel matrices which satisfy a certain technical condition. The results in [11] and [12] follow as special cases of this parameterization. The results should be useful for future research on resource allocation and spectrum sharing for situations that are well modeled via the MISO IFC.

APPENDIX

Proof of Lemma 1: First we define

$$\begin{aligned}\alpha_{ij} &\triangleq \mathbf{h}_{ij}^H \Psi_i \mathbf{h}_{ij} = \tilde{\mathbf{h}}_{ij}^H \mathbf{Q}_{ij}^{1/2} \Psi_i \mathbf{Q}_{ij}^{1/2} \tilde{\mathbf{h}}_{ij} \\ &= \tilde{\mathbf{h}}_{ij}^H \mathbf{T}_{ij} \Delta_{ij} \mathbf{T}_{ij}^H \tilde{\mathbf{h}}_{ij} = \bar{\mathbf{h}}_{ij}^H \Delta_{ij} \bar{\mathbf{h}}_{ij}\end{aligned}$$

where \mathbf{T}_{ij} is a unitary matrix, Δ_{ij} is a diagonal matrix that consists of the eigenvalues of $\mathbf{Q}_{ij}^{1/2} \Psi_i \mathbf{Q}_{ij}^{1/2}$. Both $\tilde{\mathbf{h}}_{ij}$ and $\bar{\mathbf{h}}_{ij}$ are complex circularly symmetric Gaussian random vectors with zero mean and the identity matrix as covariance matrix. Now we can write

$$\alpha_{ij} = \sum_{k=1}^{K_{ij}} \lambda_k^{ij} \chi_k^{ij},$$

where K_{ij} and λ_k^{ij} are defined in (6) and (7) respectively. Also $\chi_k^{ij} \sim \exp(1)$, that is χ_k^{ij} is exponentially distributed with parameter 1 and its pdf is $p(\chi_k^{ij}) = e^{-\chi_k^{ij}}$. Note that all χ_k^{ij} are statistically independent. Without loss of generality we can assume that $\lambda_1^{ij} \geq \lambda_2^{ij} \geq \dots \geq \lambda_n^{ij}$. The expected value of R_1 (5) can then be expressed as

$$\begin{aligned}\bar{R}_1 &= E_{\mathbf{h}_{11}, \mathbf{h}_{21}} [R_1] \\ &= E_{\mathbf{h}_{11}, \mathbf{h}_{21}} \left[\log_2 \left(1 + \frac{\mathbf{h}_{11}^H \Psi_1 \mathbf{h}_{11}}{\mathbf{h}_{21}^H \Psi_2 \mathbf{h}_{21} + \sigma^2} \right) \right] \\ &= E_{\alpha_{11}, \alpha_{21}} \left[\log_2 \left(1 + \frac{\alpha_{11}}{\alpha_{21} + \sigma^2} \right) \right] \\ &= E_{x_1 \dots x_{K_{11}}, y_1 \dots y_{K_{21}}} \left[\log_2 \left(1 + \frac{\sum_{i=1}^{K_{11}} \lambda_i^{11} x_i}{\sum_{j=1}^{K_{21}} \lambda_j^{21} y_j + \sigma^2} \right) \right] \\ &= \int_0^\infty \dots \int_0^\infty \prod_{i=1}^{K_{11}} p(x_i) \prod_{j=1}^{K_{21}} p(y_j) \times \\ &\quad \times \log_2 \left(1 + \frac{\sum_{i=1}^{K_{11}} \lambda_i^{11} x_i}{\sum_{j=1}^{K_{21}} \lambda_j^{21} y_j + \sigma^2} \right) \times \\ &\quad \times dx_1 \dots dx_{K_{11}} dy_1 \dots dy_{K_{21}},\end{aligned}$$

where all variables x_i , $i = 1, \dots, K_{11}$ and y_j , $j = 1, \dots, K_{21}$ are independent since $\bar{\mathbf{h}}_{11}$ and $\bar{\mathbf{h}}_{21}$ are independent. By inserting the pdf's we arrive at (8). Here, x_k and y_k are shorthands for χ_k^{11} and χ_k^{21} , respectively. The expression for \bar{R}_2 is similar to that for \bar{R}_1 . \square

Proof of Lemma 2: From (8) we note that

$$\begin{aligned}\bar{R}_1 &< \int_0^\infty \dots \int_0^\infty \prod_{i=1}^{K_{11}} e^{-x_i} \prod_{j=1}^{K_{21}} e^{-y_j} \times \\ &\quad \times \log_2 \left(1 + \frac{\sum_{i=1}^{K_{11}} (\lambda_i^{11} + \Delta_i) x_i}{\sum_{j=1}^{K_{21}} \lambda_j^{21} y_j + \sigma^2} \right) \times \\ &\quad \times dx_1 \dots dx_{K_{11}} dy_1 \dots dy_{K_{21}},\end{aligned}$$

where all Δ_i are non-negative and at least one $\Delta_i > 0$. This shows ‘‘increasing with λ_i^{11} $i = 1, \dots, K_{11}$ for fixed λ_j^{21} $j = 1, \dots, K_{21}$ ’’. Similarly to show ‘‘decreasing with λ_j^{21} $j = 1, \dots, K_{21}$ for fixed λ_i^{11} $i = 1, \dots, K_{11}$ ’’:

$$\begin{aligned}\bar{R}_1 &> \int_0^\infty \dots \int_0^\infty \prod_{i=1}^{K_{11}} e^{-x_i} \prod_{j=1}^{K_{21}} e^{-y_j} \times \\ &\quad \times \log_2 \left(1 + \frac{\sum_{i=1}^{K_{11}} \lambda_i^{11} x_i}{\sum_{j=1}^{K_{21}} (\lambda_j^{21} + \Delta_j) y_j + \sigma^2} \right) \times \\ &\quad \times dx_1 \dots dx_{K_{11}} dy_1 \dots dy_{K_{21}},\end{aligned}$$

where all Δ_j are non-negative and at least one $\Delta_j > 0$. \square

REFERENCES

- [1] R. Ahlswede, ‘‘The capacity region of a channel with two senders and two receivers,’’ *Ann. Probab.*, vol. 2, pp. 805-814, Oct. 1974.
- [2] A. B. Carleial, ‘‘Interference channels,’’ *IEEE Trans. Inf. Theory*, vol. 24, no. 1, pp. 60-70, Jan. 1978.
- [3] T. Han and K. Kobayashi, ‘‘A new achievable rate region for the interference channel,’’ *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49-60, Jan. 1981.
- [4] G. Scutari, D. P. Palomar, and S. Barbarossa, ‘‘Competitive design of multiuser MIMO systems based on game theory: A unified view,’’ *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1089-1103, Sept. 2008.
- [5] V. Cadambe and S. A. Jafar, ‘‘Interference alignment and the degrees of freedom for the K -user interference channel,’’ *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425-3441, Aug. 2008.
- [6] S. Vishwanath and S. A. Jafar, ‘‘On the capacity of vector Gaussian interference channels,’’ *Proc. IEEE Inf. Theory Workshop*, Oct. 2004, pp. 365-369.
- [7] X. Shang and B. Chen, ‘‘Achievable rate region for downlink beamforming in the presence of interference,’’ in *Proc. Forty-First Asilomar Conf. Signals, Syst. Computers*, Pacific Grove, 2007, pp. 1684-1688.
- [8] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, ‘‘Capacity limits of MIMO channels,’’ *IEEE J. Sel. Areas Commun.*, vol. 21, no. 5, pp. 684-702, June 2003.
- [9] A. M. Sayeed, ‘‘Deconstructing multi-antenna fading channels,’’ *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2563-2579, Oct. 2002.
- [10] D. Tse and P. Viswanath, *Fundamentals Wireless Communication*, Cambridge, U.K.: Cambridge Univ. Press, 2005.
- [11] E. A. Jorswieck, E. G. Larsson, and D. Danev, ‘‘Complete characterization of the Pareto boundary for the MISO interference channel,’’ *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 5292-5296, Oct. 2008.
- [12] J. Lindblom, E. G. Larsson, and E. A. Jorswieck, ‘‘Parameterization of the MISO interference channel with transmit beamforming and partial channel state information,’’ in *Proc. Forty-Second Asilomar Conf. Signals, Systems Computers*, Pacific Grove, 2008, pp. 1103-1107.
- [13] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, 1985.
- [14] M. Charafeddine, A. Sezgin, and A. Paulraj, ‘‘Rate region frontiers for n -user interference channel with interference as noise,’’ in *Proc. Forty-Fifth Annual Allerton Conf. Commun., Control, Computing*, Sept. 2007, pp. 1135-1140.
- [15] R. K. Mallik, M. Z. Win, J. W. Shao, M-S. Alouini, and A. J. Goldsmith, ‘‘Channel capacity of adaptive transmission with maximal ratio combining in correlated Rayleigh fading,’’ *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, pp. 1124-1133, July 2004.