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http://dx.doi.org/10.1109/TWC.2009.0801371

Postprint available at: Linköping University Electronic Press

http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-52178
Parameterization of the MISO IFC Rate Region: The Case of Partial Channel State Information

Johannes Lindblom, Erik G. Larsson and Eduard A. Jorswieck

Abstract—We study the achievable rate region of the multiple-input single-output (MISO) interference channel (IFC), under the assumption that all receivers treat the interference as additive Gaussian noise. We assume the case of two users, and that the channel state information (CSI) is only partially known at the transmitters. Our main result is a characterization of Pareto-optimal transmit strategies, for channel matrices that satisfy a certain technical condition. Numerical examples are provided to illustrate the theoretical results.

Index Terms—Ergodic rate region, interference channel, multiple-input single-output channel, multistream transmission, Pareto optimality.

I. INTRODUCTION

We are concerned with the scenario where we have two independent but mutually interfering wireless systems operating simultaneously in the same spectral band. System \(i\) consists of one base station \(BS_i\) that wants to transmit information to a mobile \(MS_i\), \(i = 1, 2\). The two mobiles receive a superposition of the signals transmitted from the two base stations. This setup is recognized as an interference channel (IFC) [1]–[3]. The IFC is important because it models the spectrum sharing situation where a number of unrelated senders (base stations) try to communicate information to different receivers (mobile stations) via a common channel. Recently there is a huge interest in understanding IFCs [4], [5]. Finding the capacity region for general IFCs is still an open problem, but various achievable rate regions are known.

We desire to understand what the achievable rate region looks like in the case that the receiver treats interference as noise. In particular, we are interested in the so-called Pareto boundary of the region. This boundary consists of Pareto optimal operating points, which are points where it is impossible to improve the rate of one communication link without simultaneously decreasing the rate of the other link. (See Definition 1 in Section III.) We consider the case when \(BS_1\) and \(BS_2\) have \(n\) transmit antennas and \(MS_1\) and \(MS_2\) have a single receive antenna each. This setup is a multiple-input single-output (MISO) IFC [6]. See Figure 1.

If the channel state information (CSI) is completely known at the transmitters, then single-stream beamforming is optimal for the IFC [7]. By contrast, if perfect CSI is unavailable, then in general one must use multi-stream beamforming [8]. That is, the transmitters \(BS_i\) should send message vectors \(x_i \sim \mathcal{CN}(0, \Psi_i)\), i.e., \(x_i\) is zero-mean complex Gaussian with covariance matrix \(\Psi_i\). The rank of \(\Psi_i\), say \(N_i\), is at least one. Clearly, there is a conflict situation associated with the choices of \(\Psi_1, \Psi_2\), since a covariance matrix \(\Psi_1\) which is good for the link \(BS_1 \rightarrow MS_1\) may generate substantial interference for \(MS_2\) and vice versa.

Our main result in this paper is a set of necessary conditions for transmit strategies to be Pareto optimal for the MISO IFC. The underlying assumption is that the channel vectors are zero-mean Gaussian with known covariance matrices. That is, the transmitter has only statistical channel knowledge. The covariance matrices studied here must satisfy a certain condition (see Proposition 1). This condition implies that the channel covariance matrices must be rank deficient, which corresponds to the case of a small angular spread, e.g., see [9] or Chapter 7 in [10].

This work extends our work in [11] where a corresponding parameterization was presented for the case of complete CSI, i.e., the channel vectors were perfectly known at the transmitters. This paper also extends our work in [12] where we treated the case of partial CSI, but with the restriction that the transmitters perform single-stream transmission, i.e., \(N_i = 1\).
II. SYSTEM MODEL

We consider the 2-user MISO IFC with \( n \) transmit antennas and frequency flat channels. See Figure 1. BS\( i \) transmits a vector \( \mathbf{x}_i \) with \( \mathbf{x}_i \sim \mathcal{CN}(0, \Psi_i) \). This vector \( \mathbf{x}_i \) may be constructed by superimposing multiple streams as follows:

\[
x_i = \sum_{k=1}^{N_i} \mathbf{w}_i^{(k)} \sqrt{p_i^{(k)}} s_i^{(k)}
\]

where \( \mathbf{w}_i^{(k)} \) are the eigenvectors of \( \Psi_i \) and the powers \( p_i^{(k)} \) are the corresponding eigenvalues. In (1), \( \{s_i^{(k)}\}_{k=1}^{N_i} \) are i.i.d. Gaussian variables with zero mean and unit variance. Also, \( N_i = \text{rank}(\Psi_i) \), as before. The matched-filtered, symbol-sampled complex baseband data received at MS\( _1 \) and MS\( _2 \) will then be

\[
\begin{align*}
y_1 &= h_{11}^H \mathbf{x}_1 + h_{12}^H \mathbf{x}_2 + e_1 \\
y_2 &= h_{21}^H \mathbf{x}_1 + h_{22}^H \mathbf{x}_2 + e_2
\end{align*}
\]

In (2), \( h_{ij} \) is the \( n \times 1 \) conjugated channel-vector between BS\( i \) and MS\( j \) and \( \mathbf{x}_i \) is the vector transmitted by BS\( i \).\(^1\) We model the channel vectors as \( h_{ij} \sim \mathcal{CN}(0, \Theta_{ij}) \). Also, \( e_1 \) and \( e_2 \) are noise variables which we model as \( \mathcal{CN}(0, \sigma^2) \). By \( (\cdot)^H \), we denote the conjugate transpose. We further assume that each base station can use the transmit power \( P \). Without loss of generality, we shall take \( P = 1 \). This gives the power constraint

\[
E[\|\mathbf{x}_i\|^2] = \text{Tr}(\Psi_i) = \sum_{k=1}^{N_i} p_i^{(k)} \leq 1, \quad i = 1, 2
\]

where \( E[\cdot] \) and \( \text{Tr}(\cdot) \) denote the expectation value and the trace respectively. The signal-to-noise ratio (SNR) will be defined as \( 1/\sigma^2 \).

III. THE ACHIEVABLE RATE REGION

For a given pair of covariance matrices \( \{\Psi_1, \Psi_2\} \) and fixed \( h_{ij} \), the signal-to-noise-and-interference ratio in \( y_1 \) is

\[
E[\|h_{11}^H \mathbf{x}_1 \|^2] = \frac{h_{11}^H \Psi_1 h_{11}}{h_{21}^H \Psi_2 h_{21} + \sigma^2}
\]

A similar expression holds for \( y_2 \). Hence, following instantaneous rates are achievable:

\[
R_1 = \log_2 \left( 1 + \frac{h_{11}^H \Psi_1 h_{11}}{h_{21}^H \Psi_2 h_{21} + \sigma^2} \right)
\]

for the link BS\( _1 \rightarrow \)MS\( _1 \), and

\[
R_2 = \log_2 \left( 1 + \frac{h_{22}^H \Psi_2 h_{22}}{h_{12}^H \Psi_1 h_{12} + \sigma^2} \right)
\]

for BS\( _2 \rightarrow \)MS\( _2 \). For fixed channels \( \{h_{ij}\} \), we define the instantaneous achievable rate region as

\[
\mathcal{R} = \bigcup_{\Psi_1, \Psi_2, \text{Tr}(\Psi_i) \leq 1} (R_1, R_2)
\]

where \( \text{Tr}(\Psi_i) \leq 1 \) is the power constraint.

Since only the statistical distributions of \( h_{ij} \) are known, one important performance measure is the average (expected) rate:

\[
\bar{R}_i \triangleq E[h_{ij}, h_{k} | R_i],
\]

where the expectation is over \( h_{ij}, i, j \in \{1, 2\} \). The corresponding rate region is

\[
\mathcal{R} = \bigcup_{\Psi_1, \Psi_2, \text{Tr}(\Psi_i) \leq 1} (\bar{R}_1, \bar{R}_2).
\]

We are interested in providing necessary conditions on \( \Psi_i \) for \((\bar{R}_1, \bar{R}_2)\) to lie on the Pareto boundary of \( \mathcal{R} \). This boundary consists of Pareto optimal points. Pareto optimality is defined as follows:

**Definition 1:** A rate tuple \((\bar{R}_1, \bar{R}_2)\) is Pareto optimal if there is no other tuple \((\bar{Q}_1, \bar{Q}_2)\) with \( \bar{Q}_1, \bar{Q}_2 \geq (\bar{R}_1, \bar{R}_2) \) and \((\bar{Q}_1, \bar{Q}_2) \neq (\bar{R}_1, \bar{R}_2) \). (The inequality is component-wise.)

IV. NECESSARY CONDITIONS FOR THE PARETO BOUNDARY

We now present our main result.

**Proposition 1:** Suppose that the channel covariance matrices satisfy the condition \( \text{span}(\Theta_{1i}) \not\subset \text{span}(\Theta_{2i}) \), for \( i, j \in \{1, 2\}, i \neq j \). Then, any transmit covariance matrix \( \Psi_i \) that corresponds to a rate point \((\bar{R}_1, \bar{R}_2)\) on the Pareto boundary, again for \( i, j \in \{1, 2\}, i \neq j \), satisfies

a) \( \text{span}(\Psi_i) \subset \text{span}(\Theta_{1i}, \Theta_{2i}) \) and

b) \( \text{Tr}(\Psi_i) = 1 \), that is, at the boundary, both base stations use full power.

In order to prove Proposition 1 we first state the following two lemmas, which deal with properties of \( R_i \). The Lemmas are stated for \( R_1 \); similar results hold for \( R_2 \). Proofs of the lemmas can be found in the Appendix. To present the lemmas we need the notation

\[
K_{ij} \triangleq \text{rank}(\Theta_{ij}^{1/2} \Psi_i \Theta_{ij}^{1/2}) , \quad \lambda_{ij}^{(k)} \triangleq \text{the } k\text{-th eigenvalue of } \Theta_{ij}^{1/2} \Psi_i \Theta_{ij}^{1/2}.
\]

**Lemma 1:** The expected value of \( R_1 \) can be expressed as

\[
\begin{align*}
\bar{R}_1 &= \int_0^\infty \cdots \int_0^{K_{11}} \int_0^{K_{21}} e^{-x_1} \prod_{j=1}^{K_{21}} e^{-y_j} \times \\
&\times \log_2 \left( 1 + \frac{K_{11} \lambda_{11}^{(1)}}{\sum_{j=1}^{K_{21}} \lambda_{21}^{(j)} y_j + \sigma^2} \right) dx_1 \cdots dx_{K_{11}} dy_1 \cdots dy_{K_{21}}.
\end{align*}
\]

**Lemma 2:** \( \bar{R}_1 \) (see (8)) is monotonically increasing with \( \lambda_{11}^{(j)}, j = 1, \ldots, K_{21} \), for fixed \( \lambda_{21}^{(j)}, j = 1, \ldots, K_{21} \). Also \( \bar{R}_1 \) is monotonically decreasing with \( \lambda_{21}^{(j)}, j = 1, \ldots, K_{21} \), for fixed \( \lambda_{11}^{(j)}, i = 1, \ldots, K_{11} \). (Here \( \lambda_{ij}^{(k)} \) and \( K_{ij} \) are defined in (6) and (7), respectively.)

**Proof of Proposition 1a:** We give the proof for \( \Psi_1 \); the proof for \( \Psi_2 \) goes in a similar manner. The main idea of this proof is as follows: Assume that transmitter 1 uses a covariance matrix, \( \Phi_1 \), which partly lies in the orthogonal complement of \( \text{span}(\Theta_{11}, \Theta_{12}) \). Then the parts of \( \Phi_1 \), which

\footnote{This condition was missing in the statement of Proposition 1 in [12]. (Reference [12] dealt with the single-stream case only.)}
are in the orthogonal complement of span\{Q_{11}, Q_{12}\}, will not affect the rates (R_1, R_2), and hence transmit power is wasted.

The proof is by contradiction, so to proceed, suppose the statement in the proposition is false. Then there exists a \( \Psi_1 \), Tr\{\( \Psi_1 \)\} \leq 1, which corresponds to a rate point on the boundary but for which span\{\( \Psi_1 \)\} \supsetneq span\{Q_{11}, Q_{12}\}. The idea is now this: Given such a \( \Psi_1 \), we first construct a covariance matrix \( \Psi'_1 \) that has Tr\{\( \Psi'_1 \)\} < 1 and which achieves the same rates as \( \Psi_1 \). Then we use the power saved when going from \( \Psi_1 \) to \( \Psi'_1 \) to construct another matrix \( \Psi''_1 \), with Tr\{\( \Psi''_1 \)\} \leq 1 and which improves one of the rates (R_1) but leaves the other (R_2) unchanged. Hence, \( \Psi_1 \) cannot correspond to a point on the boundary, and we have a contradiction.

To construct \( \Psi'_1 \), we start by letting \( \{u_i\} \) be an ON-basis for span\{Q_{11}, Q_{12}\}. We denote by \( I \) the identity matrix and define \( \Pi '_1 \triangleq I - X(X^H X)^{-1} X^H \) to be the orthogonal projection onto the orthogonal complement of the column space of \( X \). Since \( \Pi '_1 \Pi '_1 \Pi '_1 \) is positive semidefinite it follows that

\[
\Pi '_1 \Pi '_1 \Pi '_1 \sum_{i=1}^{K} \beta_i u_i^H u_i^H,
\]

where \( K \triangleq \text{rank}\{Q_{11}, Q_{12}\} \}, \beta_i \) are non-negative and \( \beta_i > 0 \) for some \( i \), say \( i = i' \) and \( u'_i \in \text{span}\{Q_{11}, Q_{12}\} \}

Now, let

\[
\Psi'_1 \triangleq \Psi_1 - [u_i^H \Psi_1 u_i']^T u'_i u_i^H.
\]

We shall show that we have for fixed \( \Psi_2 \)

1. \( R'_1 = R_1 \) (i.e. \( R_1 \) will be unchanged when \( \Psi'_1 \) is used instead of \( \Psi_1 \))
2. \( R'_2 = R_2 \) (i.e. \( R_2 \) will be unchanged when \( \Psi'_1 \) is used instead of \( \Psi_1 \))
3. Tr\{\( \Psi'_1 \)\} < Tr\{\( \Psi_1 \)\} \leq 1 (i.e. the transmitted power will be strictly decreased when \( \Psi'_1 \) is used instead of \( \Psi_1 \))
4. \( \Psi'_1 \) is positive semidefinite (i.e. it is a valid covariance matrix)

where \( (R'_1, R'_2) \) is the rate point associated with \( (\Psi'_1, \Psi_2) \) given as

\[
\begin{align*}
R'_1 &= E_{h_{11}, h_{21}} \left[ \log_2 \left( 1 + \frac{h_{11}^H \Psi'_1 h_{11}}{h_{21}^H \Psi_2 h_{21} + \sigma^2} \right) \right], \\
R'_2 &= E_{h_{12}, h_{22}} \left[ \log_2 \left( 1 + \frac{h_{12}^H \Psi'_1 h_{12}}{h_{22}^H \Psi_2 h_{22} + \sigma^2} \right) \right].
\end{align*}
\]

Item i) follows because \( u_i' \perp h_{11} \) (with probability 1):

\[
h_{11}^H \Psi'_1 h_{11} = h_{11}^H (\Psi_1 - [u_i^H \Psi_1 u_i']^T u'_i u_i^H) h_{11} = h_{11}^H \Psi h_{11}
\]

Item ii) follows because \( u_i' \perp h_{12} \) (with probability 1):

\[
h_{12}^H \Psi'_1 h_{12} = h_{12}^H (\Psi_1 - [u_i^H \Psi_1 u_i']^T u'_i u_i^H) h_{12} = h_{12}^H \Psi h_{12}
\]

Item iii) follows by

\[
\begin{align*}
\text{Tr}\{\Psi'_1\} &= \text{Tr}\{\Psi_1\} - \text{Tr}\{[u_i^H \Psi_1 u_i']^T u'_i u_i^H\} \\
&= \text{Tr}\{\Psi_1\} - \text{Tr}\{[u_i^H \Psi_1 u_i']^T u'_i u_i^H\} \\
&= \text{Tr}\{\Psi_1\} - \text{Tr}\{[u_i^H \Psi_1 u_i']^T u'_i u_i^H\} < \text{Tr}\{\Psi_1\}
\end{align*}
\]

since \( u_i^H \Psi_1 u_i' > 0 \) and \( u_i^H u_i' = 1 \).

Item iv) follows since

\[
\begin{align*}
\text{Tr}\{\Psi'_1\} u_i' &= u_i^H \Psi_1 u_i' - u_i^H \Psi_1 u_i' \\
&= 0
\end{align*}
\]

(see 9); note that \( u_i^H u_i' = 1 \) and

\[
\begin{align*}
u_i^H \Psi_1 v &= u_i^H \Psi_1 v \geq 0
\end{align*}
\]

for any \( v' u_i' \).

Next, we construct the matrix \( \Psi''_1 \). For given \( \delta \), we define

\[
\Psi''_1 \triangleq \Psi_1 + \delta \delta^H
\]

\[
\begin{align*}
R''_1 &= E_{h_{11}, h_{21}} \left[ \log_2 \left( 1 + \frac{h_{11}^H \Psi''_1 h_{11}}{h_{21}^H \Psi_2 h_{21} + \sigma^2} \right) \right], \\
R''_2 &= E_{h_{12}, h_{22}} \left[ \log_2 \left( 1 + \frac{h_{12}^H \Psi''_1 h_{12}}{h_{22}^H \Psi_2 h_{22} + \sigma^2} \right) \right].
\end{align*}
\]

We will now show that there exists a \( \delta \) such that

\[
\begin{align*}
v) R''_1 > R'_1 &= R_1 \quad \text{(i.e. } \Psi''_1 \text{ will cause an increase in } R_1 \text{ compared to } \Psi_1) \\
v) R''_2 &\leq R_2 \quad \text{(i.e. } \Psi''_1 \text{ will be unchanged when } \Psi''_1 \text{ is used instead of } \Psi_1) \\
vii) \text{Tr}\{\Psi''_1\} &\leq 1 \quad \text{(i.e. the power constraint is satisfied)}
\end{align*}
\]

From Lemma 2, we know that increasing one of the \( \lambda_{11} \) values leads to an increase of \( R_1 \). Item v) is satisfied if at least one of the eigenvalues of \( Q_{11}^2 (\Psi'_1 + \delta \delta^H) \) is larger than those of \( Q_{11}^2 \Psi'_1 \), and no eigenvalues of \( Q_{11}^2 (\Psi'_1 + \delta \delta^H) \) are smaller than those of \( Q_{11}^2 \Psi_1 Q_{11}^2 \). From [13, p. 198] we have

\[
\lambda_{\min}(B) \leq \lambda_k(A + B) - \lambda_k(A) \leq \lambda_{\max}(B)
\]

for any Hermitian matrices \( A \) and \( B \). Hence, for any \( k = 1, \ldots, n \)

\[
\lambda_k(Q_{11}^2 \Psi'_1 Q_{11}^2) + \lambda_{\min}(Q_{11}^2 \delta \delta^H) Q_{11}^2 \leq \lambda_k(Q_{11}^2 \Psi'_1 + \delta \delta^H) Q_{11}^2
\]

Since \( \lambda_{\min}(Q_{11}^2 \delta \delta^H) Q_{11}^2 \geq 0 \) it follows that

\[
\lambda_k(Q_{11}^2 \Psi'_1 + \delta \delta^H) Q_{11}^2 > \lambda_k(Q_{11}^2 \Psi'_1 Q_{11}^2)
\]

that is, no eigenvalue will decrease. If we choose \( \delta \) such that

\[
\begin{align*}
\Psi''_1 &\neq 0,
\end{align*}
\]

then we guarantee that

\[
\text{Tr}\{Q_{11}^2 (\Psi'_1 + \delta \delta^H) Q_{11}^2\} > \text{Tr}\{Q_{11}^2 \Psi'_1 Q_{11}^2\}
\]

Therefore we also know that at least one of the eigenvalues of \( Q_{11}^2 (\Psi'_1 + \delta \delta^H) Q_{11}^2 \) will increase compared to the eigenvalues of \( Q_{11}^2 \Psi'_1 Q_{11}^2 \). Equation (10) says that \( \delta \) cannot entirely lie in span\{\( Q_{12}\)\}.

Note that item vi) is satisfied if

\[
\begin{align*}
\delta \in \text{span}\{Q_{12}\} \}.
\end{align*}
\]

To construct \( \delta \) we therefore proceed as follows. First choose a \( \delta \) such that (10) and (11) are satisfied. This can be accomplished by solving the equation system

\[
\begin{cases}
Q_{11} \delta &\neq 0 \\
Q_{12} \delta &\neq 0
\end{cases}
\]
Then let $\delta = \epsilon \delta / \|\delta\|$ where $\epsilon > 0$ (to be chosen shortly). Note that we cannot find any solution of (12) if $\text{span}\{Q_{11}\} \subseteq \text{span}\{Q_{12}\}$.

Item (iii) is satisfied if
\[
\text{Tr}\{\Psi''_i\} = \text{Tr}\{\Psi'_i\} + \|\delta\|^2 = \text{Tr}\{\Psi'_i\} + \epsilon^2 \|\delta\|^2 \leq 1 \Leftrightarrow \\
\epsilon \leq \sqrt{1 - \text{Tr}\{\Psi'_i\}}.
\]

Such an $\epsilon$ exists, because $\text{Tr}\{\Psi'_i\} < 1$ according to item (iii). For example, take $\epsilon = \sqrt{1 - \text{Tr}\{\Psi'_1\}}$. Hence we have a contradiction: $(\Psi''_i, \Psi_2)$ achieves $(\tilde{R}'_1, \tilde{R}_2)$ where $\tilde{R}'_1 > \tilde{R}_1$.

**Proof of Proposition 1b):** To show that we must have $\text{Tr}\{\Psi'_i\} = 1$ at the Pareto boundary, assume that $\text{Tr}\{\Psi'_i\} < 1$. Let $\Psi'_i = \Psi_i + \delta \delta^H$ where $\delta$ is chosen according to the recipe above. Together this shows that if $\text{Tr}\{\Psi'_i\} < 1$ then it is possible to choose a new $\Psi'_i$ such that $\text{Tr}\{\Psi'_i\} = 1$, $\tilde{R}_1$ is increased and $\tilde{R}_2$ is unchanged. Note that Proposition 1b) holds only when $Q_{11}$ lies partly in the null space of $Q_{12}$. The single-input single-output (SISO) IFC is a special case when Proposition 1b) does not hold, see, for example Sections III-B and III-C in [14].

**V. SPECIAL CASES**

Here we present two important special cases. Both treat the case $N_i = 1$, i.e., single-stream beamforming.

**$N_i = 1$ and general $Q_{ij}$:** This case was treated in [12]. From Proposition 1 in [12] we know that any point on the Pareto boundary corresponds to a rate pair where the beamforming vectors are chosen such that $w^{(1)}_i \in \text{span}\{Q_{11}, Q_{12}\}$, $i = 1, 2$. This is consistent with our Proposition 1.

**$N_i = 1$ and $\text{rank}\{Q_{ij}\} = 1$:** This is the case of a rank-one channel (no angular spread). In this case $Q_{ij}$ can be written as
\[
Q_{ij} = q_{ij} q_{ij}^H
\]
where $q_{ij}$ is an $n$-vector. Let $\bar{h}_{ij} \sim \mathcal{CN}(0, 1)$. Then we can write
\[
\bar{h}_{ij} = h_{ij} q_{ij} \sim \mathcal{CN}(0, q_{ij} q_{ij}^H) = \mathcal{CN}(0, Q_{ij}).
\]
This means that the CSI is known up to an unknown scalar constant. The beamforming vectors should then be chosen as:
\[
w_1 = \xi_{11} h_{11} + \xi_{12} h_{12} \\
w_2 = \xi_{22} h_{22} + \xi_{21} h_{21},
\]
for some $\xi_{ij}$. In this special case, the parameterization in Proposition 1 essentially reduces to Proposition 1 in [11] (where only perfectly informed transmitters were treated).

Note that with perfect CSI, single-stream beamforming with $N_i = 1$ is optimal for the MISO IFC [7].

**VI. NUMERICAL RESULTS**

We illustrate Proposition 1 with two numerical examples. We used two different sets of channel covariance matrices: one set with weak interference between the systems (see Fig. 2), and one set with strong interference (see Fig. 3). For the case with strong interference, the covariance matrices were chosen such that $\text{span}\{Q_{12}\}$ was close to $\text{span}\{Q_{11}\}$ and such that $\text{span}\{Q_{21}\}$ was close to $\text{span}\{Q_{22}\}$. In both simulations we used $n = 5$ transmit antennas and $\text{rank}\{Q_{ij}\} = 2$. The matrices $\Psi_i$ were chosen such that $N_i = \text{rank}\{Q_{11}, Q_{12}\} = 4$. For each simulation we generated $4 \cdot 10^9$ pairs of transmit covariance matrices using the parameterization. To be able to do efficient simulations we used Eq. (37) in [15] to evaluate the integrals in (8) in closed form. Figs. 2 and 3 show the results for the cases of weak and strong interference, respectively. One important observation is that the rate region may be either convex or non-convex, even for perfect CSI [11] and SISO [14].

**VII. CONCLUSION**

The motivation of this paper has been the huge interest in IFCs as a model for spectrum resource conflicts. We have studied the MISO IFC, and especially the case when the CSI is not perfectly known at the transmitter. Our main contribution is a set of necessary conditions on Pareto-optimal transmit strategies, channel matrices which satisfy a certain technical condition. The results in [11] and [12] follow as special cases of this parameterization. The results should be useful for future research on resource allocation and spectrum sharing for situations that are well modeled via the MISO IFC.
APPENDIX

Proof of Lemma 1: First we define
\[ \alpha_{ij} = \sum_{k=1}^{K_{ij}} \lambda_k^{ij} \]
where \( K_{ij} \) and \( \lambda_k^{ij} \) are defined in (6) and (7) respectively. Also \( \chi_k^{ij} \sim \exp(1) \), that is \( \chi_k^{ij} \) is exponentially distributed with parameter 1 and its pdf is \( e^{-\chi_k^{ij}} \). Note that all \( \chi_k^{ij} \) are statistically independent. Without loss of generality we can assume that \( \lambda_1^{ij} \geq \lambda_2^{ij} \geq \ldots \geq \lambda_{K_{ij}}^{ij} \). The expected value of \( R_1 \) (5) can then be expressed as
\[ \bar{R}_1 = E_{h_{11}, h_{21}}[R_1] \]
\[ = \sum_{i=1}^{K_{11}} \sum_{j=1}^{K_{21}} \int_0^\infty \int_0^\infty p(x_i) p(y_j) \times \log_2 \left( 1 + \frac{h_{11}^{ii} x_i + h_{21}^{jj} y_j + \sigma^2}{\alpha_{21}^{ij}} \right) \times dx_1 \ldots dx_{K_{11}} dy_1 \ldots dy_{K_{21}}, \]
where all \( x_i, y_j \) are non-negative and at least one \( \Delta_i \) is positive. This shows increasing \( R_1^{21} \) for fixed \( \lambda_1^{ij} \). Similarly to show decreasing \( R_1^{21} \) for fixed \( \lambda_1^{ij} \).

\[ R_1 > \int_0^\infty \ldots \int_0^\infty \prod_{i=1}^{K_{11}} p(x_i) \prod_{j=1}^{K_{21}} p(y_j) \times \log_2 \left( 1 + \frac{\sum_{i=1}^{K_{11}} \chi_k^{ij} x_i}{\sum_{j=1}^{K_{21}} (\Delta_j) y_j + \sigma^2} \right) \times dx_1 \ldots dx_{K_{11}} dy_1 \ldots dy_{K_{21}}, \]

where all \( \Delta_j \) are non-negative and at least one \( \Delta_j > 0 \). □

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