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Cooperative Beamforming for the MISO Interference Channel

(Invited Paper)

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Abstract—A distributed beamforming algorithm is proposed for the two-user multiple-input single-output (MISO) interference channel (IFC). The algorithm is iterative and uses as bargaining value the interference that each transmitter generates towards the receiver of the other user. It enables cooperation among the transmitters in order to increase both users' rates by lowering the overall interference. In every iteration, as long as both rates keep on increasing, the transmitters mutually decrease the generated interference. They choose their beamforming vectors distributively, solving the constrained optimization problem of maximizing the useful signal power for a given level of generated interference. The algorithm is equally applicable when the transmitters have either instantaneous or statistical channel state information (CSI). The difference is that the core optimization problem is solved in closed-form for instantaneous CSI, whereas for statistical CSI an efficient solution is found numerically via semidefinite programming. The outcome of the proposed algorithm is approximately Pareto-optimal. Extensive numerical illustrations are provided, comparing the proposed solution to the Nash equilibrium, zero-forcing, Nash bargaining, and maximum sum-rate operating points.

I. INTRODUCTION

The situation when two wireless links operate in the same spectrum, and create mutual interference to one another, is well modeled by the interference channel (IFC). Associated with any IFC there is an achievable rate region, consisting of all pairs of transmission rates R_1 (for link 1) and R_2 (for link 2) that can be achieved, subject to constraints on the power used by the transmitters. The Pareto boundary of the rate region is the part of the outer boundary consisting of rate points, where increasing R_1 necessarily requires decreasing R_2 and vice versa. It is generally desirable to operate at rate points that lie on the Pareto boundary, such as the maximum-sum-rate (SR) point and the Nash bargaining solution (NBS) [1].

In this paper, we consider the two-user multiple-input single-output (MISO) IFC, where the transmitters (TX₁ and TX₂) have multiple antennas and the receivers (RX₁ and RX₂) have a single antenna each. By using beamforming the transmitters are able to steer power in arbitrary directions. On one extreme, when the transmitters do not cooperate, it is natural to act “selfishly” and use the maximum-ratio (MR)

beamforming vector, which maximizes the useful signal power without taking into account the interference generated towards the other receiver. Then, the outcome is the so-called in game-theoretic studies Nash equilibrium (NE), at which none of the users can increase its rate by unilaterally changing its transmit strategy [1]. On the other extreme, the transmitters can be enforced by regulation to act “altruistically” and use the zero-forcing (ZF) beamforming vector, which maximizes the useful signal power without generating any interference towards the other receiver. When both transmitters use the altruistic strategy, we will refer to the corresponding rate pair as the ZF point. In general, both the NE and the ZF points lie far inside the Pareto boundary. Pareto-optimal (PO) operating points can only be achieved by combinations of the two aforementioned extreme strategies [2]–[4]. This is because the maximization of the useful signal power and the minimization of the generated interference are conflicting objectives.

It is evident that the transmitters need to cooperate and agree to mutually decrease the generated interference, in order to achieve larger rates than the NE. Herein, we propose a simple and self-enforcing algorithm for the distributed design of the beamforming vectors, with minimum required channel knowledge. We assume that the transmitters are synchronized and that there are feedback channels from all the receivers to all the transmitters. Each transmitter has CSI only of the direct link to its intended receiver and the crosstalk link to the other receiver. The feedback channels are initially used to provide channel state information (CSI) and in the sequel information about the algorithm evolution.

The proposed algorithm is iterative and uses as bargaining value the level of generated interference. It is natural to initialize the algorithm with the MR transmit strategy. Then, in every iteration, each transmitter will decrease the upper bound on the generated interference and distributively compute a new beamforming vector solving the optimization problem of maximizing the useful power, given the interference bound and the power constraint. Inevitably, in every iteration the optimal value (max useful signal) of each system decreases, since the feasibility set of the optimization is constricted. But, in return, the experienced interference decreases too. The iterations continue as long as the signal-to-interference-plus-noise ratio (SINR), hence the rate, of both users benefits from them. The algorithm can be interpreted as a walk from

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selfish towards altruistic choices of beamforming vectors. The bargaining outcome is approximately PO.

When the signal-to-noise ratio (SNR) is high and the spatial correlation among the direct and crosstalk channels is strong, the ZF point corresponds to larger rates, than the NE, i.e. it is closer to the Pareto boundary. In such a case, the algorithm will converge to a solution faster if it is initialized with the ZF transmit strategy. Then, the interference levels need to be increased in every iteration, to expand the feasibility set, and the walk is from altruistic towards selfish transmit strategies.

The proposed algorithm can be equally used when the transmitters have either instantaneous CSI (i.e., perfect knowledge of the channel vectors) or statistical CSI (i.e., knowledge of the channel distributions). In the former case, the achievable region is comprised by instantaneous rates, whereas in the latter by ergodic rates. We propose a generic formulation of the beamforming problem as constrained optimization, which is common for both CSI cases. For instantaneous CSI, we solve the optimization in closed-form, using the parameterization in [2]. For statistical CSI, we find an efficient numerical solution via semidefinite programming (SDP), as described in [4].

In this paragraph, we summarize some known bargaining algorithms and cooperative beamforming solutions. In [6], the authors presented a bargaining algorithm, similar in spirit to the one we propose herein. Their algorithm requires instantaneous CSI, starts with the MR strategy, and exploits the parametrization in [2]. In every iteration, a portion of the ZF strategy is added to the previously computed strategy until a stopping criterion is met. This algorithm converges to an operating point which is better than the NE. Compared to [6], the main contributions of our paper are the use of the generated interference level as bargaining value and the fact that the proposed algorithm works for both instantaneous and statistical CSI. In [7], the authors presented a solution to the cooperative beamforming problem in the case of instantaneous CSI, using the notion of virtual SINR. The aforementioned algorithms were also extended in the context of multicell MIMO channels in [8].

We compare the outcome of our algorithm to the NE, ZF, NBS, and SR points, for different cases of CSI, SNR values and spatial correlation levels. Also, we provide exemplary illustrations of the bargaining trajectory and discuss the complexity of the algorithm.

Notation: $\text{Tr}\{\cdot\}$, $\text{rank}\{\cdot\}$, $\mathcal{R}\{\cdot\}$, and $\mathcal{N}\{\cdot\}$ denote the trace, rank, range, and nullspace, respectively, of a matrix. $\Pi_{\mathbf{Z}} \triangleq \mathbf{Z}(\mathbf{Z}^H \mathbf{Z})^{-1} \mathbf{Z}^H$ is the orthogonal projection onto the column space of $\mathbf{Z}$, while $\Pi_{\mathbf{Z}}^\perp \triangleq \mathbf{I} - \Pi_{\mathbf{Z}}$ is the orthogonal projection onto the orthogonal complement of the column space of $\mathbf{Z}$ and $\mathbf{I}$ is the identity matrix. $\mathbb{E}\{\cdot\}$ is the expectation operator.

II. PRELIMINARIES

A. System Model

We assume that transmission consists of scalar coding followed by beamforming¹ and that all propagation channels are

frequency-flat. The matched-filtered symbol-sampled complex baseband data received by RX_i is modeled as²

$$y_i = \mathbf{h}_{ii}^H \mathbf{w}_i s_i + \mathbf{h}_{ji}^H \mathbf{w}_j s_j + e_i, \quad j \neq i, \quad i, j \in \{1, 2\}, \quad (1)$$

where $s_i \sim \mathcal{CN}(0, 1)$ and $\mathbf{w}_i \in \mathbb{C}^n$ are the transmitted symbol and the beamforming vector, respectively, employed by TX_i . Also, $e_i \sim \mathcal{CN}(0, \sigma_i^2)$ models the receiver noise. The (conjugated) channel vector between TX_i and RX_j is modeled as $\mathbf{h}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{ij})$. We denote $r_{ij} \triangleq \text{rank}\{\mathbf{Q}_{ij}\}$. In the case of instantaneous CSI, TX_i accurately knows the channel realizations $\mathbf{h}_{ii}$ and $\mathbf{h}_{ij}$, whereas for statistical CSI it only knows the channel covariance matrices $\mathbf{Q}_{ii}$ and $\mathbf{Q}_{ij}$.

The transmission power is bounded due to regulatory and hardware constraints, such as battery and amplifiers. Without loss of generality, we set this bound to 1. Hence, the set of feasible beamforming vectors is

$$\mathcal{W} \triangleq \{\mathbf{w} \in \mathbb{C}^n \mid \|\mathbf{w}\|^2 \leq 1\}. \quad (2)$$

Note that the set $\mathcal{W}$ is convex. In what follows, a specific choice of $\mathbf{w}_i \in \mathcal{W}$ is denoted as a *transmit strategy* of TX_i .

B. Instantaneous CSI

When the transmitters perfectly know the channel vectors and the receivers treat interference as noise, the achievable *instantaneous* rate (in bits/channel use) for link i is [2]

$$R_i(\mathbf{w}_i, \mathbf{w}_j) = \log_2 \left(1 + \frac{|\mathbf{h}_{ii}^H \mathbf{w}_i|^2}{|\mathbf{h}_{ji}^H \mathbf{w}_j|^2 + \sigma_i^2} \right). \quad (3)$$

It is evident that the rate on each link depends on the choice of both beamforming vectors. We define the power that RX_i receives from TX_j as

$$p_{ji}(\mathbf{w}_j) \triangleq |\mathbf{h}_{ji}^H \mathbf{w}_j|^2 = \mathbf{w}_j^H \mathbf{h}_{ji} \mathbf{h}_{ji}^H \mathbf{w}_j. \quad (4)$$

Then, we can write (3) as

$$R_i(\mathbf{w}_i, \mathbf{w}_j) = \log_2 \left(1 + \frac{p_{ii}(\mathbf{w}_i)}{p_{ji}(\mathbf{w}_j) + \sigma_i^2} \right), \quad (5)$$

which is monotonously increasing with the useful signal power $p_{ii}(\mathbf{w}_i)$ for fixed received interference power $p_{ji}(\mathbf{w}_j)$ and monotonously decreasing with $p_{ji}(\mathbf{w}_j)$ for fixed $p_{ii}(\mathbf{w}_i)$.

The main goal of the bargaining algorithm we introduce in Section III is to agree on a PO solution. Hence, we restrict our attention to the beamforming vectors which are candidates to achieve PO points. From [2], we know that the PO beamforming vectors use full power and that they are linear combinations of the MR and ZF strategies

$$\mathbf{w}_i^{\text{PO}}(\lambda_i) = \frac{\lambda_i \mathbf{w}_i^{\text{MR}} + (1 - \lambda_i) \mathbf{w}_i^{\text{ZF}}}{\|\lambda_i \mathbf{w}_i^{\text{MR}} + (1 - \lambda_i) \mathbf{w}_i^{\text{ZF}}\|} \quad (6)$$

for $\lambda_i \in [0, 1]$, where

$$\mathbf{w}_i^{\text{MR}} = \frac{\mathbf{h}_{ii}}{\|\mathbf{h}_{ii}\|} \quad \text{and} \quad \mathbf{w}_i^{\text{ZF}} = \frac{\Pi_{\mathbf{h}_{ij}}^\perp \mathbf{h}_{ii}}{\|\Pi_{\mathbf{h}_{ij}}^\perp \mathbf{h}_{ii}\|}. \quad (7)$$

The outcome when both transmitters use their MR strategies is the NE. When both use their ZF we refer to the ZF point.

¹This is optimal in the case of instantaneous CSI, but not necessarily for statistical CSI, see [5].

²Whenever an expression is valid for both systems, it is denoted once with respect to system i and the index $j \neq i$ refers to the other system.

C. Statistical CSI

When the transmitters only have statistical knowledge of the channels, it is natural to design the achievable beamforming vectors with respect to the *ergodic* rates, which are obtained by averaging over the channel realizations. From [3], we have³

$$R_i(\mathbf{w}_i, \mathbf{w}_j) \triangleq \mathbb{E} \left\{ \log_2 \left(1 + \frac{|\mathbf{h}_{ji}^H \mathbf{w}_i|^2}{|\mathbf{h}_{ji}^H \mathbf{w}_j|^2 + \sigma_i^2} \right) \right\} \quad (8)$$

$$= \frac{p_{ii}(\mathbf{w}_i)}{\ln 2} \frac{f_i(p_{ii}(\mathbf{w}_i)) - f_i(p_{ji}(\mathbf{w}_j))}{p_{ii}(\mathbf{w}_i) - p_{ji}(\mathbf{w}_j)},$$

where

$$f_i(x) \triangleq e^{\sigma_i^2/x} \int_{\sigma_i^2/x}^{\infty} \frac{e^{-t}}{t} dt. \quad (9)$$

In (8), $p_{ji}(\mathbf{w}_j)$ denotes the *average* power that RX_i receives from TX_j

$$p_{ji}(\mathbf{w}_j) = \mathbb{E} \{ \mathbf{w}_j^H \mathbf{h}_{ji} \mathbf{h}_{ji}^H \mathbf{w}_j \} = \mathbf{w}_j^H \mathbf{Q}_{ji} \mathbf{w}_j. \quad (10)$$

Note that the final terms in both (4) and (10) are convex homogeneous quadratics. The difference is that the parameter (channel) matrix in (4) is rank-1 by definition, whereas in (10) it can have any rank.

The ergodic rate (8) has the same behavior as the instantaneous rate (3), i.e., it is monotonously increasing (decreasing) with $p_{ii}(\mathbf{w}_i)$ ($p_{ji}(\mathbf{w}_j)$) for fixed $p_{ii}(\mathbf{w}_i)$ ($p_{ji}(\mathbf{w}_j)$) [3]. Also, for points on the Pareto boundary we know that $\mathbf{w}_i \in \mathcal{R}\{\mathbf{Q}_{ii}, \mathbf{Q}_{ij}\}$ [5]. The MR strategy $\mathbf{w}_i^{\text{MR}}$ is the dominant eigenvector of $\mathbf{Q}_{ii}$ [3]. When $\mathcal{R}\{\mathbf{Q}_{ii}\} \not\subseteq \mathcal{R}\{\mathbf{Q}_{ij}\}$, the ZF strategy $\mathbf{w}_i^{\text{ZF}}$ is the dominant eigenvector of $\Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii} \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}}$ and when $\mathcal{R}\{\mathbf{Q}_{ii}\} \subseteq \mathcal{R}\{\mathbf{Q}_{ij}\}$, e.g., when $\mathbf{Q}_{ij}$ is full-rank, then $\mathbf{w}_i^{\text{ZF}} = \mathbf{0}$ [3].

D. Important Operating Points

In the following, we introduce some operating points, which are important in the sense that they lie on the outer boundary of the rate region; see, e.g., [1] and references therein.

Single-user (SU): The points achieved when one transmitter employs its MR strategy while the other refrains from transmission.

Maximum sum-rate (SR): The point where the sum of the rates is maximum. Graphically, it is the point where a line of slope -1 touches the Pareto boundary of the rate region.

Nash bargaining solution (NBS): The outcome of a Nash bargaining is a point $(\bar{R}_1, \bar{R}_2)$ such that $(\bar{R}_1 - R_1^*)(\bar{R}_2 - R_2^*)$ is maximized for some threat point (R_1^*, R_2^*) and $\bar{R}_i \geq R_i^*$. It is natural to use the NE as the threat point, since it is the only reasonable outcome if the systems are not able to agree on a solution. The NBS is only defined on convex utility regions, but we will call the solution to the corresponding optimization problem the NBS.

³We deliberately use the same symbols, as in the case of instantaneous CSI, to denote the rate and the power (R and p , respectively) in order to facilitate in the sequel a uniform treatment of both CSI scenarios.

III. COOPERATIVE BEAMFORMING ALGORITHM

In this section, we elaborate the proposed bargaining algorithm that enables the transmitters to distributively design their beamforming vectors. We assume that there exists a feedback link from every receiver to every transmitter. The receivers use these links to feedback CSI. Each transmitter has CSI only on the links it is affecting. The transmitters are assumed synchronized, but no information (CSI or user data) is exchanged between them.

In the algorithm, we use as bargaining value an upper bound on the interference generated by system i to system j . This bound, denoted c_{ij} , is adjusted in every iteration. During the bargaining, the receivers feed back a one-bit message that tells the transmitters whether the iteration was successful or not, i.e. whether the rates increased or not. We denote l the iteration counter, which also acts as a quantitative measure of the overhead (total number of bits per RX-TX feedback link) and the computational complexity (total number of optimization problems that need to be solved).

A flowchart of the algorithm is illustrated in Fig. 1. The first step of the algorithm is the decision whether the initialization point will be the NE or the ZF point. For this reason, the transmitters send two pilots using their MR and ZF beamforming vectors. The receivers measure the SINR for each transmission and feed back one-bit of information telling the transmitters which strategy yields higher SINR, hence rate. If $R_i(\mathbf{w}_i^{\text{ZF}}, \mathbf{w}_j^{\text{ZF}}) \geq R_i(\mathbf{w}_i^{\text{MR}}, \mathbf{w}_j^{\text{MR}})$ for both systems, the algorithm is initialized with the ZF point, since it is closer than the NE to the Pareto boundary. Hence, the algorithm will require fewer iterations to converge to a solution. If only one system achieves higher rate with the ZF strategies, there is no incentive for the other to accept the ZF point as initial point. The algorithm is then initialized with the NE point.

Then, the algorithm sets the stepsize for updating c_{ij} . As with any iterative algorithm, the best output is obtained for an infinitesimal stepsize. However, this is not practical, so we consider instead a fixed stepsize⁴. We assume that TX_i samples the interval $[0, p_{ij}(\mathbf{w}_i^{\text{MR}})]$ uniformly in $N+1$ points, to allow up to N iterations. This gives the step $\delta_{ij} = \pm p_{ij}(\mathbf{w}_i^{\text{MR}})/N$. The sign of δ_{ij} depends on the initial point. If the algorithm is initialized with the NE, δ_{ij} will be negative (decreasing interference). Otherwise, δ_{ij} will be positive (increasing interference). At iteration l , TX_i updates the interference level as $c_{ij}^l = c_{ij}^{l-1} + \delta_{ij}$ and solves the problem

$$\max_{\mathbf{w}_i \in \mathcal{W}} p_{ii}(\mathbf{w}_i) \quad (11)$$

$$\text{s.t. } p_{ij}(\mathbf{w}_i) \leq c_{ij}^l. \quad (12)$$

The optimal solution of problem (11)–(12) is the beamforming vector which maximizes the useful power given that the generated interference is c_{ij}^l . As long as c_{ij}^l is chosen in the range $[0, p_{ij}(\mathbf{w}_i^{\text{MR}})]$, there always exists a feasible solution to (11)–(12) [4]. The lower and upper end on the interference level correspond to the ZF and MR strategies, respectively.

⁴Also, an adaptive stepsize can easily be incorporated to the algorithm.

Fig. 1. Flowchart describing the proposed algorithm

Furthermore, the bound will be tight at the optimum; hence, the inequality in (12) can be equivalently replaced with equality. We propose a solution to the optimization problem (11)–(12) in Sections III-A and III-B for the case of instantaneous and statistical CSI, respectively. TX_i uses $\mathbf{w}_i^l$ to transmit a pilot. RX_i measures $R_i(\mathbf{w}_i^l, \mathbf{w}_j^l)$ and if it is no smaller than $R_i(\mathbf{w}_i^{l-1}, \mathbf{w}_j^{l-1})$, it feeds back a one-bit message telling the transmitters to continue updating the interference level. As soon as the rate decreases for at least one of the receivers, the algorithm terminates and the transmitters will use the beamforming vectors from the previous iteration.

We claim that the algorithm is self-enforced. Suppose that, in one of the steps, TX_i chooses to cheat by not decreasing the interference level. Then, the rate of system j will decrease and RX_j will feedback a negative bit. According to the last step of the algorithm, the transmitters are expected to choose the beamforming vectors from the previous iteration. If TX_i does not, RX_j will notice and report it to TX_j. Then, TX_j will leave the bargaining and employ its MR beamforming vector instead. That is, if one system tries to cheat, then the cooperation is canceled and the operation falls back to the NE (the so-called threat point, in the context of Nash bargaining).

A. Instantaneous CSI

By inserting the expression (4) in (11)–(12), with inequality changed to equality, we get the problem

$$\max_{\mathbf{w}_i \in \mathcal{W}} |\mathbf{h}_{ii}^H \mathbf{w}_i|^2 \quad (13)$$

$$\text{s.t. } |\mathbf{h}_{ij}^H \mathbf{w}_i|^2 = c_{ij}. \quad (14)$$

Since the objective of the algorithm is to find a PO point, the transmitters are only willing to use beamforming vectors that

are candidates for achieving PO points. Any other beamforming vector will be a waste of power. Using (6) we get

$$\max_{\lambda_i \in [0,1]} |\mathbf{h}_{ii}^H \mathbf{w}_i^{\text{PO}}(\lambda_i)|^2 \quad (15)$$

$$\text{s.t. } |\mathbf{h}_{ij}^H \mathbf{w}_i^{\text{PO}}(\lambda_i)|^2 = c_{ij}. \quad (16)$$

Note that the optimization (15)–(16) is now only with respect to the real scalar λ_i . Furthermore, the power constraint is obsolete, since the PO beamforming vectors use full power. That is, the inequality constraint in (2) is met with equality. Instead, we have a constraint on the range of the weighting factor λ_i . Finally, it is straightforward to see that the objective function (15) is monotonously increasing with λ_i . Thus, we can equivalently rewrite (15)–(16) as

$$\max_{\lambda_i \in [0,1]} \lambda_i \quad (17)$$

$$\text{s.t. } |\mathbf{h}_{ij}^H \mathbf{w}_i^{\text{PO}}(\lambda_i)|^2 = c_{ij}. \quad (18)$$

To simplify notation, we define

$$\alpha_i \triangleq (|\mathbf{h}_{ij}^H \mathbf{h}_{ii}| / \|\mathbf{h}_{ij}\|)^2 \text{ and } \beta_i \triangleq \|\Pi_{\mathbf{h}_{ij}}^\perp \mathbf{h}_{ii}\| / \|\mathbf{h}_{ii}\|. \quad (19)$$

The values (19) are only calculated once per channel realization. For $c_{ij} > 0$ we write (18) as

$$\frac{\lambda_i^2 \alpha_i}{\lambda_i^2 + (1 - \lambda_i)^2 + 2\lambda_i(1 - \lambda_i)\beta_i} = c_{ij} \Leftrightarrow \lambda_i^2(\alpha_i/c_{ij} + 2\beta_i - 2) + \lambda_i(2 - 2\beta_i) - 1 = 0.$$

When $c_{ij} = 0$, the ZF strategy is the optimal solution (i.e., $\lambda_i = 0$). Now, we write (17)–(18) as

$$\max \lambda_i \quad (20)$$

$$\text{s.t. } \lambda_i^2(\alpha_i/c_{ij} + 2\beta_i - 2) + \lambda_i(2 - 2\beta_i) - 1 = 0, \quad (21)$$

$$0 \leq \lambda_i \leq 1. \quad (22)$$

The solution to (20)–(22) is the largest of the two solutions to (21) that satisfies (22).

B. Statistical CSI

By inserting (10) in (11)–(12) we get

$$\max_{\mathbf{w}_i \in \mathbb{C}^n} \mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i \quad (23)$$

$$\text{s.t. } \mathbf{w}_i^H \mathbf{Q}_{ij} \mathbf{w}_i \leq c_{ij}, \quad (24)$$

$$\mathbf{w}_i^H \mathbf{w}_i \leq 1. \quad (25)$$

Problem (23)–(25) is a quadratically constrained quadratic program (QCQP). The feasibility set determined by (24)–(25) is convex. However, the optimization is non-convex owing to the form of the objective function. However, it can still be solved optimally and efficiently using semidefinite relaxation. This is because semidefinite relaxation is tight for QCQP problems of the form in (23)–(24), as shown in [9].

We briefly elaborate the procedure, similar to the way we did in [4]. We change the optimization variables to $\mathbf{W}_i \triangleq \mathbf{w}_i \mathbf{w}_i^H$. Note that

$$\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^H \Leftrightarrow \mathbf{W}_i \succeq \mathbf{0} \text{ and } \text{rank}\{\mathbf{W}_i\} = 1. \quad (26)$$

Using (26) and the property that $\text{Tr}\{\mathbf{Y}\mathbf{Z}\} = \text{Tr}\{\mathbf{Z}\mathbf{Y}\}$ for matrices $\mathbf{Y}$, $\mathbf{Z}$ of compatible dimensions, the average power term in (24) can be written as

$$\begin{aligned} \mathbf{w}_i^H \mathbf{Q}_{ij} \mathbf{w}_i &= \text{Tr}\{\mathbf{w}_i^H \mathbf{Q}_{ij} \mathbf{w}_i\} = \text{Tr}\{\mathbf{Q}_{ij} \mathbf{w}_i \mathbf{w}_i^H\} \\ &= \text{Tr}\{\mathbf{Q}_{ij} \mathbf{W}_i\}. \end{aligned} \quad (27)$$

Due to (26) and (27), we equivalently recast (23)–(24) as

$$\max_{\mathbf{W} \in \mathbb{C}^{n \times n}} \text{Tr}\{\mathbf{Q}_{ii} \mathbf{W}_i\} \quad (28)$$

$$\text{s.t.} \quad \text{Tr}\{\mathbf{Q}_{ij} \mathbf{W}_i\} \leq c_{ij}, \quad (29)$$

$$\text{Tr}\{\mathbf{W}_i\} \leq 1, \quad (30)$$

$$\mathbf{W}_i \succeq \mathbf{0}, \quad (31)$$

$$\text{rank}\{\mathbf{W}_i\} = 1. \quad (32)$$

The objective function (28), the constraints (29) and (30) are linear. The cone of positive semidefinite matrices (31) is convex. But the rank constraint (32) is non-convex. Dropping it, the remaining problem (28)–(31) is a semidefinite programming (SDP) problem, which can be solved efficiently. Due to the absence of (32), the SDP problem will not necessarily return rank-1 optimal matrices. We experienced through extensive simulations that it actually *does* yield rank-1 matrices.

IV. NUMERICAL ILLUSTRATIONS

In this section, we present extensive simulation results to evaluate the performance of the algorithm we propose. We focus on the case of statistical CSI, but also provide some results for instantaneous CSI. In Section IV-A, we explain how we generate CSI (i.e., channel covariance matrices or channel vectors) for simulation purposes. In Sections IV-B and IV-C, we compare the outcome of the algorithm to the NE, ZF, SR, and NBS. Furthermore, in Section IV-D, we show exemplary bargaining trajectories. Finally, in Section IV-E, we discuss the overhead and the complexity associated with our algorithm.

Throughout the simulations, we assume that the transmitters use $n = 5$ antennas. We allow our algorithm run up to $N = 20$ iterations. The results reported in Figs. 2–7 and 10, 13 are averages over 100 Monte-Carlo (MC) runs. Figs. 2–7 illustrate the sum of the transmission rates, i.e., $R_1 + R_2$. Figs. 8, 9, 11, and 12 show examples of achievable rate regions, i.e., for a single CSI realization. Figs. 10 and 13 depict the average number of iterations needed till the algorithm terminates, depending on whether the algorithm is initialized with the NE or either of the NE and ZF, respectively.

A. Generating the Channels

We generate the direct and the crosstalk channels in two different ways, to model the scenarios of weak or strong spatial correlation. Specifically, in the case of instantaneous CSI and weak correlation, we generate the channel vectors $\mathbf{h}_{ii}$ and $\mathbf{h}_{ij}$ drawing independent samples from $\mathcal{CN}(\mathbf{0}, \mathbf{I})$. For the scenario of strong correlation, we use the formula

$$\mathbf{h}_{ij} = \mu_i \mathbf{h}_{ii} + \sqrt{1 - \mu_i^2} \tilde{\mathbf{h}}_{ij}, \quad (33)$$

where $\mathbf{h}_{ii}$ and $\tilde{\mathbf{h}}_{ij}$ are drawn from $\mathcal{CN}(\mathbf{0}, \mathbf{I})$, and $\mu_i \in [0, 1]$. A value of μ_i close to 1 refers to the case of strong interference.

In the case of statistical CSI, we construct the covariance matrices, of rank r , randomly as

$$\mathbf{Q} = \sum_{k=1}^r \mathbf{q}_k \mathbf{q}_k^H, \quad (34)$$

where $\mathbf{q}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. For the scenario of weak correlation, we generate the covariance matrices $\mathbf{Q}_{ii}$ and $\mathbf{Q}_{ij}$ independently according to (34). For the scenario of strong correlation, we construct the matrices such that the angle between the eigenvectors of the direct matrix and the eigenvectors of the crosstalk matrix is small. Assuming that $r_{ii} \leq r_{ij}$, we first generate $\mathbf{Q}_{ii}$ as in (34). Then, we construct the vectors $\{\mathbf{q}_{ij,k}\}_k$ that define $\mathbf{Q}_{ij}$ as

$$\begin{cases} \mathbf{q}_{ij,k} = \mu_i \mathbf{q}_{ii,k} + \sqrt{1 - \mu_i^2} \tilde{\mathbf{q}}_{ij,k}, & k \leq r_{ii} \\ \mathbf{q}_{ij,k} = \tilde{\mathbf{q}}_{ij,k}, & k > r_{ii} \end{cases} \quad (35)$$

where $\tilde{\mathbf{q}}_{ij,k} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ and $\mu_i \in [0, 1]$. If $r_{ii} > r_{ij}$, the matrices are constructed the other way around.

B. Statistical CSI

In this section, we provide results for statistical CSI, both for weak and strong spatial correlation. Also, we distinguish among the cases of having full-rank and low-rank covariance matrices. In the low-rank scenario, the covariance matrices of the direct-channels have rank $r_{11} = r_{22} = 2$, and covariance matrices of the cross-talk channels have rank $r_{12} = r_{21} = 4$. For strong correlation, we use $\mu_i = 0.8$.

In Figs. 2 and 5, we study the full-rank scenario. First, we note that the ZF sum rate is equal to 0 since full-rank crosstalk matrices correspond to $\mathbf{w}_i^{\text{ZF}} = \mathbf{0}$. Second, we see that the sum rates for the proposed algorithm, the NBS, and the NE saturate for high SNR. The reason is that when the SNR is high, interference is the main limiting factor. Since the interference cannot become zero, except for the SU-points, there should be a limitation. Third, since there is no interference at the SU points, the corresponding rates will grow unbounded with SNR and the SR will be found at a SU point for high SNR.

In Figs. 3 and 6, we illustrate the low-rank scenario. Here, all points but the NE converge to the same sum rate at high SNR. The difference is that, for strong correlation they converge at higher SNR value than for weak correlation. Also, we see that the rates grow almost linearly with the SNR. In general, there exists a non-trivial zero-forcing point for the case of low-rank matrices. Using this, the noise is the only limitation. When the noise decreases, the rates increase. At low SNR, the ZF starts growing later for strong correlation than for weak correlation.

Furthermore, we evidence that weak correlation (Figs. 2 and 3) gives higher rates for the proposed algorithm, the NE, and the NBS, than strong correlation (Figs. 5 and 6). As a general remark, low SNR means operation in the noise-limited regime and all the rates but the ZF are almost the same.

Concluding, we see that the performance of the proposed algorithm is slightly below the NBS and close to the SR,

except for full-rank matrices and high SNR. Most important, the algorithm performs consistently much better than the NE, which would be the outcome if there was no cooperation.

C. Instantaneous CSI

In Figs. 4 and 7 we report the results for weak and strong correlation ($\mu_i = 0.9$), respectively. We note that the curves behave similarly to the ones in Figs. 3 and 6. The reason for this is that the case of instantaneous CSI can be regarded as a specific instance of the low-rank statistical CSI when all covariance matrices are rank-1.

D. Bargaining Trajectory

In this section, we give examples of the bargaining trajectory, i.e., the rate points (marked with stars) reached at every iteration of the proposed algorithm. Here, the maximum number of iterations used is $N = 10$. Figs. 8 and 11 illustrate the trajectories for statistical CSI with full-rank covariance matrices and SNR equal to 0 and 10 dB, respectively. The Pareto boundary is calculated using the technique proposed in [4]. Figs. 9 and 12 illustrate the trajectories for instantaneous CSI and SNR equal to 0 and 10 dB, respectively. The Pareto boundary is calculated using the technique proposed in [2].

For statistical CSI and full-rank matrices, the algorithm is always initialized with the NE, since a non-trivial ZF point does not exist. For instantaneous CSI and low SNR it is initialized with the NE point, while for high SNR with the ZF point. We note that the final outcome of the bargaining algorithm is close to the Pareto boundary, but does not necessarily lie on it. On one hand, the final outcome depends on the stepsize of the algorithm. On the other hand, the algorithm terminates when either of the rates stops increasing, i.e., when the tangent of the trajectory stops being positive. More on, the outcome is close to NBS, but generally far from SR. We note that the SR does not imply that both systems have increased their rates compared to NE, while the proposed algorithm and NBS guarantee that both systems get at least their NE rates.

Finally, in all figures we show what the bargaining trajectory would look like if the algorithm went the entire way from one extreme point (NE or ZF) to the other with small steps. Note that for statistical CSI and full-rank matrices the ZF point corresponds to the origin of the rate region.

E. Overhead and Complexity

In this section, we study the number of iterations required for the algorithm to terminate. Specifically, we study the dependence on the SNR and the starting point. In Fig. 10, the algorithm is initialized with the NE and we illustrate the three CSI scenarios (statistical CSI, full- and low-rank, and instantaneous CSI). For each CSI scenario, the solid line corresponds to weak spatial correlation and the dashed line corresponds to strong spatial correlation. At low SNR, we see that the algorithm needs fewer iterations for strong correlation than for weak correlation. As the SNR increases, the number of iterations approaches the threshold N . This is because at

high SNR the ZF point is closer to the Pareto boundary and the transmitters use all available iterations to reach it.

When the systems are interested in minimizing the amount of overhead and complexity, they start at the point that requires the smallest number of iterations. That is, the algorithm starts at the ZF if $R_i(\mathbf{w}_i^{\text{ZF}}, \mathbf{w}_j^{\text{ZF}}) \geq R_i(\mathbf{w}_i^{\text{MR}}, \mathbf{w}_j^{\text{MR}})$ for both systems. We illustrate this in Fig. 13. Compared to Fig. 10, we see that the methods perform equally for low SNR, since this is the noise-limited regime. When the SNR is medium, the number of iterations is increased because both extreme points (NE or ZF) are away from the Pareto boundary. For high SNR, the number of iterations for instantaneous CSI and statistical CSI with low-rank matrices goes to one. This is because this is the interference-limited regime and ZF is optimal. For statistical CSI with full-rank matrices, a non-trivial ZF strategy does not exist. Therefore, the algorithm will always start at NE, as a comparison of Figs. 10 and 13 reveals.

V. CONCLUSIONS

We considered the distributed design of beamforming vectors for the MISO IFC. We proposed a cooperative algorithm that achieves an operating point which is almost Pareto optimal. The final solution is in all cases better than the NE, which would be the outcome if there was no cooperation. The novel element of the proposed algorithm is the use of the generated interference level as bargaining value. The algorithm is equally applicable to the case of instantaneous and statistical CSI. We validated the merit of our algorithm via extensive numerical illustrations.

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Fig. 2. Sum rate; stat. CSI (full-rank), $\mu_i = 0$

Fig. 5. Sum rate; stat. CSI (full-rank), $\mu_i = 0.8$

Fig. 3. Sum rate; stat. CSI (low-rank), $\mu_i = 0$

Fig. 6. Sum rate; stat. CSI (low-rank), $\mu_i = 0.8$

Fig. 4. Sum rate; inst. CSI, $\mu_i = 0$

Fig. 7. Sum rate; inst. CSI, $\mu_i = 0.9$

Fig. 8. Bargaining trajectory; stat. CSI, SNR = 0 dB

Fig. 11. Bargaining trajectory; stat. CSI, SNR = 10 dB

Fig. 9. Bargaining trajectory; inst. CSI, SNR = 0 dB

Fig. 12. Bargaining trajectory; inst. CSI, SNR = 10 dB

Fig. 10. Average number of iterations; algorithm initialization: NE

Fig. 13. Average number of iterations; algorithm initialization: NE or ZF