Linköping University Post Print

Solving a minimum-power covering problem with overlap constraint for cellular network design

Lei Chen and Di Yuan

N.B.: When citing this work, cite the original article.

Solving a minimum-power Covering Problem with Overlap Constraint for Cellular Network Design

Lei Chen [*]   Di Yuan

Department of Science and Technology, Linköping Institute of Technology,
SE-601 74, Sweden
Tel: +46 11363496, Fax: +46 11363170

Abstract

We consider a type of covering problem in cellular networks. Given the locations of base stations, the problem amounts to determining cell coverage at minimum cost in terms of the power usage. Overlap between adjacent cells is required in order to support handover. The problem we consider is NP-hard. We present integer linear models and study the strengths of their continuous relaxations. Preprocessing is used to reduce problem size and tighten the models. Moreover, we design a tabu search algorithm for finding near-optimal solutions effectively and time-efficiently. We report computational results for both synthesized instances and networks originating from real planning scenarios. The results show that one of the integer models leads to tight bounds, and the tabu search algorithm generates high-quality solutions for large instances in short computing time.

Key words: OR in telecommunications, Cellular networks, Covering, Integer programming, Tabu search

1 Introduction

Mobile cellular communications comprise an important application area of operational research (OR). A well-studied problem type in the area is channel assignment in second generation (2G) networks [1]. A recent development of OR in 2G networks is the integration of channel assignment and base station location [22]. At present, there is a

---
[*] Corresponding author.
   Email addresses: `leich@itn.liu.se` (Lei Chen), `diyua@itn.liu.se` (Di Yuan).

rapid deployment of third generation (3G) and beyond-3G cellular networks, for which the planning process does not involve channel assignment, but other types of design decisions [6,7,12]. One important planning task in 3G and beyond-3G networks is radio coverage. We consider a type of covering problem arising in this application context. Given the locations of base stations and the cell antenna directions, the problem is to determine the coverage area of every cell, such that the target service area is completely covered. The coverage area of a cell is determined by the amount of power allocated to a control signal for broadcasting presence of service. A natural performance metric is the total power allocated to these service-coverage signals. Less power for coverage reduces interference, and increases the power left for transmitting user data. Minimum-power covering tends to minimize also the overlap between cells. Whereas excessive cell overlap should be avoided as it results in high interference, some amount of overlap between a cell and its neighboring cells is required for supporting the handover operation when users move from one cell to another. Moreover, coverage overlap is a necessary condition for the so called soft and softer handover states, in which a user is simultaneously connected to two or more cells. For these reasons, it is vital to incorporate, in addition to coverage, overlap as a constraint for pairs of cells between which users are expected to roam.

The problem we consider comes up once the locations of base stations and antennas are known. Base station location in 3G networks has been studied in a number of references. Mathematical programming models as well as heuristic algorithms have been proposed by, for example, Amaldi et al. [2,3], Matar and Schmeink [16], and Zhang et al. [23], and Zhang et al. [24]. Eisenblätter et al. [5,6] present integer models and solution methods for optimization of both base station location and antenna configuration. The planning objectives considered in these references are network deployment cost and performance. The latter is typically represented by the average cell load. The trade-off between infrastructure cost and profit has been dealt with by integer programming by Kalvenes [15]. In [18], Olinick and Rosenberger approached this trade-off by a stochastic optimization model to take into account demand uncertainty.

Another type of planning problem is topology optimization for interconnecting nodes at various levels of the system hierarchy in the radio access network. Jüttner et al. [11] applied a meta heuristic for optimizing the interconnection between two node types, and a branch and bound method for solving a special case of the problem. A branch and cut algorithm has been developed by Fischetti et al. [8] to solve the problem of optimally forming a multi-level star topology to interconnect several types of nodes.

The research in this paper deals with power optimization for coverage planning. For 3G networks, a common practice of setting power for coverage has been the uniform strategy, i.e., to reserve a roughly constant amount (typically 10-15% of the total cell power)

2

to the coverage signal in all cells. A number of previous studies (e.g., [9,20,21]) have shown, however, that the strategy may be inefficient. Adopting a non-uniform power allocation and optimizing its amount can yield considerable power savings as well as better load balancing between cells. The problem studied in this paper is motivated by these findings. Whereas power saving may not be a crucial aspect in earlier 3G systems implementing power control (that is, the power to serve each user is dynamically adjusted to make the signal quality stay at a desired level), it is of great significance in beyond-3G networks, such as high speed downlink packet access (HSDPA) that targets mobile Internet data. Because HSDPA does not use power control, any power saving on the control channels, in particular the one used for coverage, will make more resource available to serving user traffic and thereby improving data throughput. Indeed, in [10], Geerdes observed that for HSDPA, it is best to allocate as much user power as possible in order to maximize the performance.

Optimization of the power of the coverage signals has been addressed in [20,21]. The objectives are power consumption and load balancing. Joint optimization of power and antenna tilt has been investigated in [9,19]. However, the problems studied in these works do not address sufficient overlap between adjacent cells using explicit constraints. One issue that arises with non-uniform coverage power is handover performance, as well as soft handover consideration in earlier 3G systems. (Soft handover does not apply to HSDPA.) Under uniform power, it is relatively simple to predict handover (and soft handover) performance. More care on handover is needed for non-uniform coverage power, because cell shape becomes more irregular, and because power minimization has a tendency of shrinking cell size. As overlap is a necessary condition for handover and soft handover, its treatment in coverage planning, which is a key aspect of our work, becomes essential.

The work in the paper falls into the research domain of OR applications. The paper targets modeling and solving a type of covering problem that is of relevance in mobile telecommunications, and the approach is to develop mathematical programming models and solution algorithms. Similar to many classical set covering problems, the problem considered in this paper is NP-hard. We formulate the problem using integer linear models, and study the strengths of their continuous relaxations. Preprocessing techniques are used for reducing problem size and tightening the models. To deal with large-scale problem instances, we develop a tabu search algorithm, aimed at finding near-optimal solutions time-efficiently. We develop a neighborhood that effectively deals with the constraints of coverage and cell overlap. We apply the integer linear models and the tabu search algorithm to synthesized network instances and instances originating from real planning cases for large cities. The experiments show that the models can be used to compute optimal or near-optimal solutions for instances of up to moderate size. For large-scale instances, the tabu search algorithm is able to provide

3

high-quality solutions using short computing time.

In a network planning context, the optimization problem we consider may have to be solved many times for one network. In particular, the overlap levels are expected to be set initially by engineering expertise in network planning. Simulation can be used to assess in detail the handover performance of the optimized coverage pattern. Based on the results of simulation, the overlap parameters in the problem may have to be adjusted. Thus it is vital to be able to obtain time-efficiently optimal or near-optimal solutions to coverage planning.

The remainder of the paper is organized as follows. In Section 2 we discuss the application context and introduce some notation. The optimization problem is formalized in Section 3. In Section 4 we present two integer linear models, and compare the strengths of their continuous relaxations. Section 5 addresses preprocessing. The tabu search algorithm is detailed in Section 6. We report experimental results in Section 7. Conclusions are provided in Section 8.

2 Definitions

A cellular network consists of a set of radio base stations. A base station has one or several (typically three) cells, each has its own radio antenna. Every cell uses a broadcast channel, referred to as the common pilot channel, to announce the presence of the cell and its service. A mobile terminal is able to access the network only if it can detect at least one pilot channel signal, of which the ratio between the received signal strength and the interference is above a threshold. If a mobile terminal is covered by multiple cells, the common pilot channel facilitates cell selection. Typically, the terminal is attached to the cell having the strongest pilot channel signal. Factors that determine the strength of a received pilot signal include transmit power, attenuation factor between the cell antenna and the mobile phone, as well as antenna gain factors. From a network planning standpoint, the parameter that can be used for adjusting cell coverage is the amount of transmit power allocated to the common pilot channel.

Let $\mathcal{C} = \{1, \ldots, C\}$ denote the set of cells. The service area is represented by a large set of test points, denoted by $\mathcal{J} = \{1, \ldots, J\}$. The (minimum) amount of power required to cover test point $j \in \mathcal{J}$ by cell $i \in \mathcal{C}$ is denoted by P_{ij}. The value depends on, among other factors, the signal propagation condition. We refer to [19,20] for the derivation of the power parameter. We use $\mathcal{J}_i$ to denote the set of test points that can be potentially covered by cell i, and let $J_i = |\mathcal{J}_i|$. To present the mathematical models, it is useful to denote the same information from the viewpoint of test points: We define $\mathcal{C}_j$ as the set

4

of potentially covering cells of j, and let $C_j = |\mathcal{C}_j|$.

To facilitate handover, coverage overlap requirement is defined for cells being geographically adjacent. The overlap requirement is of relevance for any two adjacent cells where some amount of handover between the two cells is expected to take place. Let $\mathcal{J}_{ih} = \mathcal{J}_i \cap \mathcal{J}_h$, and $J_{ih} = |\mathcal{J}_{ih}|$. For two adjacent cells i and h, the overlap requirement specifies, within the area given by $\mathcal{J}_{ih}$, the minimum amount to be covered by both cells. The overlap is measured in the number of test points. We denote by d_{ih} the minimum number of points that must be covered by both i and h. As signal attenuation increases at least quadratically in distance, minimization of power subject to overlap of two cells means that overlap tends to occur somewhere around the middle of the two base stations. Overlap is not needed if two cells are adjacent but do not have users moving between (e.g., due to obstacles). Generally speaking, the overlap requirement is specified by a set $\mathcal{D}$ containing unordered pairs of cells, and a positive integer d_{ih} for each $(i, h) \in \mathcal{D}$. In Figure 1, some of the concepts that have been introduced thus far are illustrated.
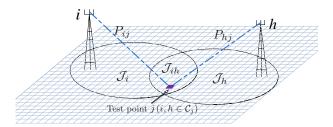

Figure 1. An illustration of some of the concepts in the optimization framework.

Denote by $F_i : \mathcal{J}_i \mapsto \{1, \ldots, J_i\}$ a bijection such that the sequence $P_{i,F_i(1)}, P_{i,F_i(2)}, \ldots,$ $P_{i,F_i(J_i)}$ is monotonously non-decreasing. Ties, if any, are broken arbitrarily. In other words, we sort $P_{ij}, j \in \mathcal{J}_i$ in ascending order. For convenience, we introduce $i(j)$ as a short-hand notation when i and $F_i(j)$ are together used as subscripts. Hence $P_{i(j)}$ is not the power required to cover test point j, but that for covering the jth test point in the sorted sequence. We use F_i' to denote the inverse of F_i, i.e., $F_i'(j)$ gives the position of test point $j \in \mathcal{J}_i$ in the sorted power sequence of cell i.

3 The Optimization Problem

The optimization problem we intend to solve amounts to minimizing the power needed to cover the service area and to satisfy the overlap requirements. We refer to the problem as minimum-power covering with overlap (MPCO). A formal definition is given below.

[**MPCO**] Find a minimum-sum vector $\mathbf{p} = (p_1, p_2, \ldots, p_C)$, where $p_i \in [P_{i(1)}, P_{i(J_i)}]$,

5

$i \in \mathcal{C}$, such that for all $j \in \mathcal{J}$, set $\{i \in \mathcal{C}_j : p_i \geq P_{ij}\} \neq \emptyset$, and for all $(i, h) \in \mathcal{D}$, $|\{j \in \mathcal{J}_{ih} : p_i \geq P_{ij} \wedge p_h \geq P_{hj}\}| \geq d_{ih}$.

Problem MPCO differs from classical minimum-cost set covering [4] in several aspects. First, for every cell, the covering elements (power levels) have a structure. Specifically, the number of ground set elements (test points) covered by a cell grows successively by cell power. Second, since every cell has to be assigned one of the possible power levels, there is a generalized upper bound (multiple choice) constraint per cell. The third difference is the presence of the overlap constraints. Moreover, the pair-wise cell overlap constraints in MPCO differ from the constraints in the set multicover problem. MPCO is however similar to many covering problems in terms of NP-hardness.

Proposition 1 *MPCO is NP-hard.*

We provide a sketch of a proof that reduces minimum-cost set covering to MPCO where $\mathcal{D} = \emptyset$. In the reduction, every covering element corresponds to a cell, and each ground set element of the set covering problem is a test point of MPCO. The MPCO instance has C additional dummy test points. Every cell has two possible power levels. The first level covers a dummy test point that can be covered by the cell only. The second level covers the ground set elements in the set covering instance. The power of the first level is same in all cells. The second power levels are the covering costs in set covering. The two instances have equivalent sets of feasible solutions. For any two peering solutions, the objective function values differ by a constant, and the proposition follows.

4   Mathematical Models

We present two integer linear models for MPCO. We define the following sets of variables for the first model of MPCO (denoted by **F**1). Note that the definition of the x-variables follows from the observation that we can restrict the domain of the power of cell i ($i \in \mathcal{C}$) to the discrete set $\{P_{i(1)}, P_{i(2)}, \ldots, P_{i(J_i)}\}$. This approach leads to a stronger LP relaxation than representing power using continuous variables.

$$x_{ij} = \begin{cases} 1 \text{ if the power of cell } i \text{ equals } P_{ij}, \\ 0 \text{ otherwise.} \end{cases}$$

$$s_j^{ih} = \begin{cases} 1 \text{ if test point } j \text{ is covered by both cell } i \text{ and cell } h, \\ 0 \text{ otherwise.} \end{cases}$$

$$[\mathbf{F}1] \quad \min \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{J}_i} P_{ij} x_{ij}$$

$$\text{s. t.} \sum_{i \in \mathcal{C}_j} \sum_{k=F_i'(j)}^{J_i} x_{i(k)} \geq 1, \quad j \in \mathcal{J}, \tag{1}$$

$$\sum_{j \in \mathcal{J}_i} x_{ij} = 1, \quad i \in \mathcal{C}, \tag{2}$$

$$\sum_{k=F_i'(j)}^{J_i} x_{i(k)} + \sum_{k=F_h'(j)}^{J_h} x_{h(k)} = s_j^{ih} + 1, \quad (i,h) \in \mathcal{D}, j \in \mathcal{J}_{ih} : \mathcal{C}_j = \{i, h\}, \tag{3}$$

$$\sum_{k=F_i'(j)}^{J_i} x_{i(k)} + \sum_{k=F_h'(j)}^{J_h} x_{h(k)} \geq 2s_j^{ih}, \quad (i,h) \in \mathcal{D}, j \in \mathcal{J}_{ih} : \{i, h\} \subset \mathcal{C}_j, \tag{4}$$

$$\sum_{j \in \mathcal{J}_{ih}} s_j^{ih} \geq d_{ih}, \quad (i,h) \in \mathcal{D}, \tag{5}$$

$$x_{ij} \in \{0, 1\}, \quad i \in \mathcal{C}, j \in \mathcal{J}_i, \tag{6}$$

$$s_j^{ih} \in \{0, 1\}, \quad (i,h) \in \mathcal{D}, j \in \mathcal{J}_{ih}. \tag{7}$$

Constraint sets (1) and (2) ensure, respectively, that all test points are covered and that exactly one of the candidate power levels is selected in every cell. Note that (2) applies also to the case where the power parameters of a cell have the same value for multiple test points. These points appear consecutively in the sorted sequence. If the optimal cell power is equal to that for covering these points, then there is always an optimal solution in which the highest index level among them is set to one. Constraints (3) and (4), state the condition that s_j^{ih} can be one only if both cells i and h cover j. Constraint (4) is a general formulation of the condition. If, however, cells i and h are the only two possible covering cells of j, then (3) applies. From the LP relaxation standpoint, (3) is clearly stronger than (4) for test points with two potentially covering cells. For test points in (3), (1) can be removed. In addition, the s-variables in (3) can be eliminated by variable substitution. For clarity, however, we keep these s-variables in the model. The last set of constraints (5) specifies required overlap.

We derive a less straightforward but more effective model $\mathbf{F}2$ by further processing of data. Consider $(i, h) \in \mathcal{D}$ and $\ell \in \{2, \ldots, J_i\}$, for which the corresponding test point $F_i(\ell) \in \mathcal{J}_{ih}$. Suppose that the power of cell i is at level $\ell - 1$ in the sorted power sequence, i.e., the power is $P_{i(\ell-1)}$ and cell i covers test points in $\mathcal{J}_{ih}$ requiring power lower than $P_{i(\ell)}$. Let $\mathcal{J}_{ih}(i_{\ell-1})$ denote this set of test points, i.e., $\mathcal{J}_{ih}(i_{\ell-1}) = \{j \in \mathcal{J}_{ih} : P_{ij} \leq P_{i(\ell-1)}\}$. We define parameter $L(i_{\ell-1}, h) \in \{1, \ldots, J_h\}$ to represent the minimum power of cell h that can meet the coverage and overlap constraints for test points in $\mathcal{J}_{ih}$, provided that cell i uses level $\ell - 1$. The value of $L(i_{\ell-1}, h) = \max\{L^c(i_{\ell-1}, h), L^o(i_{\ell-1}, h)\}$, which are defined as follows.

- *Coverage.* If there exist test points for which i and h are the only two possible covering cells, and not all of them are covered by cell i at level $\ell - 1$, then $L^c(i_{\ell-1}, h)$ is the smallest number in $\{1, \ldots, J_h\}$ for which the power of h, $P_{h(L^c(i_{\ell-1}, h))}$, covers all the remaining test points. Otherwise $L^c(i_{\ell-1}, h) = 1$.
- *Overlap.* If $|\mathcal{J}_{ih}(i_{\ell-1})| < d_{ih}$, then power level $P_{i(\ell-1)}$, as well as all levels below it, are infeasible in cell i. In this case we set $L^o(i_{\ell-1}, h) = J_h + 1$. Suppose $|\mathcal{J}_{ih}(i_{\ell-1})| \geq d_{ih}$. Parameter $L^o(i_{\ell-1}, h)$ is defined as the smallest number in $\{1, \ldots, J_h\}$, such that the corresponding power $P_{h(L^o(i_{\ell-1}, h))}$ covers at least d_{ih} test points.

$$\mathcal{J}_{ih} = \{1, \ldots, 12\}$$
$$C_6 = 2, \, C_j > 2, \, j \neq 6$$
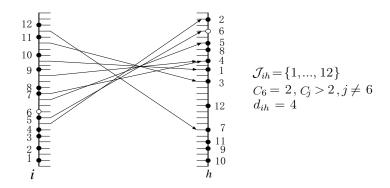$$d_{ih} = 4$$

Figure 2. An illustration of the derivation of the second model.

An illustration of the definition of parameter $L(i_{\ell-1}, h)$ is provided in Figure 2 using a small example. The bars and dots show the candidate power levels and the potential set of test points subject to overlap, respectively. Parameter $L(i_{\ell-1}, h)$ for various ℓ-values is represented by the lines with arrows.

For $(i, h) \in \mathcal{D}$, the domain of possible power levels of cell h is constrained by inequality $x_{i(\ell-1)} \leq \sum_{k=L(i_{\ell-1}, h)}^{J_h} x_{h(k)}$, with the convention that the right-hand side is zero if $L(i_{\ell-1}, h) = J_h + 1$. Next, we observe that the left-hand side of the inequality can be strengthened by taking the sum over all x-variables of cell i for which the power levels are less than or equal to $P_{i(\ell-1)}$, because we have to select exactly one power level for every cell, and $L(i_{\ell_1-1}, h) \geq L(i_{\ell_2-1}, h)$ if $\ell_1 \leq \ell_2$. We arrive at the following model.

$$[\mathbf{F2}] \quad \min \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{J}_i} P_{ij} x_{ij}$$
$$\text{s.t. } (1),$$
$$\sum_{k=1}^{\ell-1} x_{i(k)} \leq \sum_{k=L(i_{\ell-1}, h)}^{J_h} x_{h(k)}, \quad (i, h) \in \mathcal{D}, \ell \in \{2, \ldots, J_i\} : F_i(\ell) \in \mathcal{J}_{ih}, \quad (8)$$
$$x_{ij} \in \{0, 1\}, \quad i \in \mathcal{C}, j \in \mathcal{J}_i.$$

8

The sufficiency of constraints (8) for the overlap requirements can be realized from the following analysis. Consider a solution of **F**2, and suppose it is not feasible to MPCO. Because of (1), the infeasibility must be due to insufficient overlap. Then there exists $(i, h) \in \mathcal{D}$, $\ell \in \{1, \ldots, J_i\}$, and $m \in \{1, \ldots, J_h\}$, for which $x_{i(\ell)} = x_{h(m)} = 1$, $x_{i(k)} = 0$ for all $k > \ell$ and $x_{h(k)} = 0$ for all $k > m$, and $|\{j \in \mathcal{J}_{ih} : P_{ij} \leq P_{i(\ell)}\} \cap \{j \in \mathcal{J}_{ih} : P_{hj} \leq P_{h(m)}\}| < d_{ih}$. Let $\bar{\ell}$ be the first number in the sequence $\ell + 1, \ldots, J_i$, such that $F_i(\bar{\ell}) \in \mathcal{J}_{ih}$. Such $\bar{\ell}$ exists as the overlap requirement for (i, h) is not satisfied. For constraint (8) defined for $\bar{\ell}$, its left-hand side is at least one. Moreover, parameter $L(i_{\bar{\ell}-1}, h) > m$ by definition. Thus the right-hand side of (8) equals zero, i.e., the constraint is violated. Hence the correctness of **F**2.

F2 does not use (2). This constraint set is clearly necessary for the correctness of **F**1. For **F**2, they have impact on the space of feasible solutions, but are redundant at both integer optimum and LP optimum, as stated in the proposition below.

Proposition 2 *Constraint set (2) is redundant at the integer optimum and the LP optimum of* **F**2.

Proof We show the redundancy of (2) for defining the LP optimum only. The same result for integer optimum can be shown similarly. Denote by $\bar{x}_{i(k)}, i \in \mathcal{C}, k \in \{1, \ldots, J_i\}$ an optimal solution to the LP relaxation of **F**2. Suppose that $\sum_{k=1}^{J_i} \bar{x}_{i(k)} > 1$ for some $i \in \mathcal{C}$. We modify this solution as follows. For each cell i with $\sum_{k=1}^{J_i} \bar{x}_{i(k)} > 1$, let ℓ denote the smallest number in $\{1, \ldots, J_i\}$ such that $\bar{x}_{i(\ell)} > 0$, $\bar{x}_{i(k)} = 0, \forall k < \ell$. Thus $\sum_{k=\ell}^{J_i} \bar{x}_{i(k)} > 1$. We change the value of $\bar{x}_{i(\ell)}$ to zero if $\sum_{k>\ell}^{J_i} \bar{x}_{i(k)} \geq 1$; otherwise we set its value to $1 - \sum_{k>\ell}^{J_i} \bar{x}_{i(k)}$. After the modification, $\sum_{k=1}^{J_i} \bar{x}_{i(k)} \leq 1, i \in \mathcal{C}$. That the modification does not affect the feasibility of $\bar{x}$ in (1) is obvious. Since no variable has its value increased, $\bar{x}$ also satisfies (8) for which no value modification has been made to the right-hand side. Consider a constraint of (8) in which the right-hand side has been reduced, and suppose that the variable with value modified is $\bar{x}_{h(m)}$. If $L(i_{\ell-1}, h) > m$, then the constraint remains satisfied. Suppose $L(i_{\ell-1}, h) \leq m$. In this case the right-hand side of (8) equals one, whereas its left-hand side is at most one. Therefore (8) is satisfied. In conclusion, the modified $\bar{x}$ is feasible in **F**2 and has a better objective function value, which leads to a contradiction, and the proposition follows. □

For two cells i and h with overlap requirement, we obtain two groups of constraints of (8) by considering the both ordered pairs (i, h) and (h, i). In **F**2, only one of them is needed. This suffices also for the continuous relaxation, in the sense that including the second group of constraints does not strengthen the relaxation.

Proposition 3 *In the LP relaxation of* **F**2*, a solution satisfying (8), defined for $(i, h) \in \mathcal{D}$, also satisfies the corresponding constraints defined for (h, i).*

9

Proof Consider pair (h, i), $m \in \{2, \ldots, J_h\}$ and $F_h(m) \in \mathcal{J}_{ih}$. Let $\ell = L(h_{m-1}, i) - 1$. Suppose $L(i_{\ell-1}, h) \leq m - 1$. Then both coverage and overlap are satisfied for the set of test point for which i and h are two only two potentially covering cells, if the two power levels are $\ell - 1$ and $m - 1$, respectively. This implies $L(h_{m-1}, i) \leq \ell - 1$, leading to a contradiction. Therefore $L(i_{\ell-1}, h) \geq m$. Utilizing this observation, (8) for pair (i, h), and Proposition 2, we obtain the inequality $\sum_{k=L(h_{m-1}, i)}^{J_i} x_{i(k)} = 1 - \sum_{k=1}^{\ell-1} x_{i(k)} \geq 1 - \sum_{k=m}^{J_h} x_{h(k)} = \sum_{k=1}^{m-1} x_{h(k)}$ This inequality is the constraint of (8) for pair (h, i) and power level m of cell h, and the Proposition follows. $\square$

Model $\mathbf{F}2$ is not only more compact but also stronger than $\mathbf{F}1$ in LP bound. The next proposition establishes this result.

Proposition 4 *The bound provided by the LP relaxation of $\mathbf{F}2$ is greater than or equal to that of the LP relaxation of $\mathbf{F}1$.*

Proof See Appendix A.

Numerically, $\mathbf{F}2$ is typically strictly better than $\mathbf{F}1$ in LP bound. Our experiments in Section 7 show that the difference between the bounds is quite significant.

5 Preprocessing

We can apply preprocessing to MPCO to reduce problem size. First, we use the simple observation that there are typically some test points having only one possible covering cell. Such test points impose lower bounds on cell power. We obtain another lower bound by overlap consideration, since for cell pair $(i, h) \in \mathcal{D}$, the power of each of the two cells must cover at least d_{ih} test points in $\mathcal{J}_{ih}$. Let ℓ_i denote the maximum of the two derived power levels. For both $\mathbf{F}1$ and $\mathbf{F}2$, we can delete all power levels below $\ell_i, i \in \mathcal{C}$, as well as all test points $j \in \mathcal{J}$ for which $|\mathcal{C}_j| = 1$. Moreover, if $|\mathcal{C}_j| > 1$ and j is covered by at least one cell i with power $P_{i(\ell_i)}$, the coverage constraint (1) can be deleted. Removal of the coverage constraint applies also to any test point j with $|\mathcal{C}_j| = 2$, because coverage is imposed by (3) in $\mathbf{F}1$ and (8) in $\mathbf{F}2$.

For $\mathbf{F}1$, consider cell i and any test point j that is covered by power $P_{i(\ell_i)}$ and not deleted in the previous step. Consider another cell h having $j \in \mathcal{J}_h$ and an overlap requirement with i. If $P_{h(\ell_h)}$ covers j, we can delete the s-variable and reduce d_{ih} by one. Otherwise we check if s_j^{ih} is defined by constraint (4). If so, we can strengthen the constraint by replacing it with $s_j^{ih} = \sum_{k=F_h'(j)}^{J_h} x_{h(k)}$. Clearly, these reduction and strengthening tests should be performed for all $(i, h) \in \mathcal{D}$ and test points covered by $P_{i(\ell_i)}$ or $P_{h(\ell_h)}$.

10

6 Tabu Search

To deal with large-scale instances of MPCO, we design a tabu search (TS) algorithm [13], aimed at finding high-quality solutions within short computing time. The basic ingredients of TS are a local search procedure and a memory mechanism. Local search applies a neighborhood structure to iteratively generate and evaluate a sequence of solutions. Replacing the current solution with one in the neighborhood is called a move. The memory mechanism, implemented by means of a tabu list, aims at preventing cycling (i.e., revisiting solutions that have been considered earlier in the search) and thereby avoid getting stuck in local optima. To design an effective and time-efficient TS algorithm for MPCO, the neighborhood and the content of the tabu list should exploit the structure of the coverage and overlap constraints.

We say that a feasible solution of MPCO is non-lowerable, if decreasing the power level of any single cell by one step makes the solution infeasible. The optimum of MPCO is obviously non-lowerable. We define a neighborhood such that the TS algorithm generates a sequence of non-lowerable solutions. For convenience, we denote a solution of MPCO by indices of the sorted power levels of cells, in form of a vector $\mathbf{k} = (k_i, k_2, \ldots, k_i, \ldots, k_C)$, i.e., the power of cell i is $P_{i(k_i)}, i \in \mathcal{C}$.

Given a non-lowerable solution $\mathbf{k}$, we have observed that the strategy of increasing the power of a cell in order to allow for reducing power in some other cells does not lead to an effective neighborhood. Consider increasing the power of cell i. This may enable power reduction in cells of which the current power levels are defined by test points that may be potentially covered by i, i.e., $\{h \in \mathcal{C} : F_k(k_h) \in \mathcal{J}_{ih}\}$. A necessary condition for power reduction in a cell h in this set is to set the power level of cell i to at least $F_i'(F_h(k_h))$. However, the condition is not sufficient, because there may exist overlap constraint between cell h and some other cell, for which the constraint is currently active for test point $F_h(k_h)$. Indeed, cell i may already cover $F_h(k_h)$ at the current level k_i. Hence there is no guarantee that increasing the power of one cell will always lead to a power reduction of some other cell. In our TS algorithm, we define a more effective and convenient move by considering power adjustment in the reverse way. First, we reduce the power of one cell i by one step so the solution becomes infeasible. In step two, the power of one or several cells are increased to re-establish feasibility. Note that infeasibility after the first step is either because of coverage or overlap, but *not* both. In the former case, we increase the power of a cell $h \in \mathcal{C}_{F_i(k_i)}$ to $F_h'(F_i(k_i))$, whereas in the latter case the power of cells, for which the overlap constraint with i is violated, are increased by the minimum necessary amount to give sufficient overlap. There is no explicit restriction on how much power may be increased; this is determined by the amount necessary to restore either coverage or overlap. In addition, several cells may have to perform a jump in their power levels at the same time. This fact contributes to

11

(1) $\mathcal{N}(\mathbf{k}) = \emptyset$;
(2) **for** all $i \in \mathcal{C}$
(3) $\mathbf{k}' = \mathbf{k}$; $k_i' = k_i - 1$; $j = F_i(k_i)$;
(4)   **if** $C_j(\mathbf{k}') = 0$
(5)   **for** all $h \in \mathcal{C}_j \setminus \{i\}$
(6) $k_h' = F_h'(j)$;
(7)   **for** all $m \in \mathcal{C}$
(8) $n = F_m(k_m')$;
(9)   **while** $C_n(\mathbf{k}') \geq 2$ and $(J_{mr}(\mathbf{k}') > d_{mr}$ or $F_r'(n) > k_r', r \in \mathcal{D}_m)$
(10) $k_m' = k_m' - 1$;
(11) $\mathcal{N}(\mathbf{k}) = \mathcal{N}(\mathbf{k}) \cup \{\mathbf{k}'\}$;
(12) $k_h' = k_h$;
(13)   **else**
(14)   **for** all $h \in \mathcal{D}_i : J_{ih}(\mathbf{k}') = d_{ih} - 1$
(15)   **while** $J_{ih}(\mathbf{k}') = d_{ih} - 1$
(16) $k_h' = k_h' + 1$;
(17)   **for** all $m \in \mathcal{C}$
(18) $n = F_m(k_m')$;
(19)   **while** $C_n(\mathbf{k}') \geq 2$ and $(J_{mr}(\mathbf{k}') > d_{mr}$ or $F_r'(n) > k_r', r \in \mathcal{D}_m)$
(20) $k_m' = k_m' - 1$;
(21) $\mathcal{N}(\mathbf{k}) = \mathcal{N}(\mathbf{k}) \cup \{\mathbf{k}'\}$;
(22) **return** $\mathcal{N}(\mathbf{k})$;

Figure 3. The procedure of generating neighboring solutions in TS.

the effectiveness in exploring the solution space. Power increase in step two may enable power reduction in some cells other than i. Thus in the third and last step we decrease the power of these cells, arriving at another non-lowerable solution.

Figure 3 gives a formal description of generating the neighborhood $\mathcal{N}(\mathbf{k})$ of $\mathbf{k}$. Note that the description emphasizes on clarity rather than efficiency of implementation. In the description, $C_j(\mathbf{k}')$ and $J_{ih}(\mathbf{k}')$ denote the number of cells covering test point j and the number of test points covered by both i and h, respectively, in solution vector $\mathbf{k}'$. Moreover, $\mathcal{D}_i$ denotes the set of cells having overlap requirement with i. The first step of a move is carried out in line 3. If this leads to an uncovered test point (line 4), the power of potential cells covering the point is increased (lines 5–6), otherwise power is increased in cells for which the overlap constraint is violated (lines 14–16). The last step of a move to ensure a non-lowerable solution is carried out in lines 7–10 or in lines 17–20. Note that Figure 3 does not specify the order of cells in which power reduction is performed in the last step of a move. Varying the order may result in different solutions. In our implementation, the order is randomized in every iteration.

The second step of a move involves restoring either coverage or overlap. The former may result in multiple neighbors (lines 4–12), whereas the latter (lines 14–21) always defines

12

a single neighbor. Let N_i denote the number of cells that may potentially overlap with cell i. The total number of neighbors, at maximum, is $\sum_{i \in \mathcal{C}} N_i$. In cellular networks, N_i is typically defined by cells that are geographically adjacent to i, and in real-life networks this number tends to be a constant in average. Consequently the size of the TS neighborhood grows linearly in the number of cells.

In every iteration, the TS algorithm generates all the solutions in the neighborhood. Neighboring solutions having tabu status are excluded, except for those yielding a total power better than the best known solution, i.e., the aspiration criterion. Whenever this criterion does not apply, the algorithm moves to the non-tabu solution with minimum total power. The content of the tabu list consists in cell and power indices. Once a move is made, cells and their power levels involved in the move are marked as tabu. A solution has the tabu status if any cell and its power level in the solution are present in the list.

We consider two approaches for obtaining an initial solution in TS. The first approach starts by setting the power levels of all cells to their maximum values. For a randomly chosen cell, the power is reduced to the level such that any further reduction will violate either a coverage or overlap constraint. Next, one of the remaining cells is randomly selected, and its power is decreased as much as possible. This is repeated until all cells have been considered. The second approach is to utilize the LP optimum of model $\mathbf{F}2$. In this approach, the initial solution is constructed by setting the power level of each cell to the highest level for which the corresponding x-variable is strictly positive at the LP optimum. The resulting solution, for which both the coverage and overlap requirements are clearly satisfied, serves as the starting point of TS.

There are several possibilities to refine the basic TS algorithm, such as diversification [13]. We have implemented and tested a diversification scheme. The resulting improvement is however very marginal. The numerical results in the next section show that our TS algorithm is able to find close-optimal solutions without having to incorporate any intricate mechanism for refinement. This is a strength of TS as a meta-heuristic. To obtain satisfactory results from TS, an effective neighborhood is crucial, and the experiments show that our design of the neighborhood structure is successful in this regard.

7   Computational Experiments

We report experimental results for two groups of networks. The first group contains synthesized networks N1–N5. The second group is formed by three networks R1–R3 originating from real-life planning scenarios. R1 is a small downtown cellular network. Networks R2 and R3, provided by the EU MOMENTUM project [17], represent two

network planning cases for the city of Berlin and Lisbon, respectively. Statistics of the networks are shown in Table 1. For each network, the table displays the numbers of cells, test points, and pairs of cells having overlap requirement. We introduce overlap requirement for any cell pair for which the potential overlap is above a threshold. The threshold is chosen such that the resulting number of cell pairs with overlap requirement is approximately equal to the average number of pairs of cells being geographically adjacent. Column "Service Area" shows the size of the service area. Every test point represents a small square area for which signal propagation is considered uniform. This area size represented by each point is shown in column "Resolution". For each cell, we measure its potential size by the number of test points covered under maximum power. The last three columns present, respectively, the minimum, maximum, and average values of potential coverage over cells.

Table 1
Network statistics.

Network	Cells	Test points	Pairs	Service area [m^2]	Resolution [m^2]	Potential coverage		
	C	J	$|\mathcal{D}|$			Min	Max	Ave.
N1	42	2708	162	2400×2000	40×40	25	287	156
N2	70	5029	248	2880×2800	40×40	21	366	194
N3	140	9409	548	4000×4000	40×40	20	535	278
N4	203	19088	954	5600×5600	40×40	15	992	504
N5	255	21678	1250	6000×6000	40×40	17	972	495
R1	60	1375	202	1280×1800	40×40	3	68	36
R2	148	22500	568	7500×7500	50×50	29	1439	734
R3	140	62500	442	5000×5000	20×20	57	5538	2798

For each network, we consider two levels of overlap, defined by setting $d_{ih}/J_{ih} = 10\%$ and 20%, respectively, for all pairs $(i, h) \in \mathcal{D}$, leading to a total of 16 instances. In the subsequent text, subscripts are added to network names to indicate overlap level.

We first solve MPCO by applying CPLEX 10.1 [14] to the two integer linear models $\mathbf{F}1$ and $\mathbf{F}2$, with a time limit of one hour. In addition to examining to what extent MPCO can be solved to optimality using the two models, we numerically study the strengths of their continuous relaxations. These experiments have been conducted on a computer with a dual core CPU at 2.4 GHz and 7 GB RAM.

In the next set of experiments, we apply our TS algorithm. TS stops either because a maximum iteration limit of 2000 is reached, or if the best solution found is not improved

in 300 consecutive iterations, whichever becomes satisfied first. The length of the tabu list equals 25. We remark that this particular length value is not crucial to algorithm performance. The TS algorithm is run on a notebook with a dual core CPU at 2.0 GHz.

Table 2

Computational results of $\mathbf{F}1$ and $\mathbf{F}2$. (Objective values have unit Watt, and times are in seconds.)

Network	$\mathbf{F}1$				$\mathbf{F}2$			
	LP		ILP		LP		ILP	
	Value	Time	Value	Time	Value	Time	Value	Time
$N1_{10}$	31.1	1.1	[33.7, 35.4]	limit	35.4	0.2	35.4	0.9
$N2_{10}$	54.9	4.6	[57.2, 58.3]	limit	58.0	0.5	58.1	2.3
$N3_{10}$	110.3	17.5	[116.6, 122.7]	limit	120.2	1.4	120.2	9.3
$N4_{10}$	174.7	150.8	[180.9, -]	limit	203.2	8.2	203.6	279.0
$N5_{10}$	215.7	79.8	[226.2, -]	limit	239.4	6.1	240.2	236.0
$R1_{10}$	38.8	0.1	44.1	4.8	44.1	0.1	44.1	0.1
$R2_{10}$	125.1	2337.5	[125.7, -]	limit	162.5	56.3	162.6	785.5
$R3_{10}$	–	–	–	–	–	–	–	–
$N1_{20}$	31.5	1.1	[34.8, 36.9]	limit	36.8	0.2	36.8	0.7
$N2_{20}$	55.7	3.2	[59.1, 60.6]	limit	60.2	0.4	60.2	1.2
$N3_{20}$	112.3	12.8	[119.7, 126.2]	limit	120.2	1.3	123.7	6.1
$N4_{20}$	178.4	236.7	[189.6, -]	limit	215.7	9.0	216.1	147.2
$N5_{20}$	220.1	145.0	[233.3, 3620.4]	limit	249.5	5.6	250.1	80.0
$R1_{20}$	39.7	0.1	45.3	3.09	45.3	0.1	45.3	0.1
$R2_{20}$	126.7	148.0	[130.3, -]	limit	179.7	18.9	179.9	583.9
$R3_{20}$	–	–	–	–	–	–	–	–

In Table 2 we summarize the results of solving $\mathbf{F}1$ and $\mathbf{F}2$. For each instance, the table displays the objective function values obtained from solving the integer models and their LP relaxations, and the solution times. The objective function value is given in Watt (W). We use "limit" to denote that the time limit is reached. If a model can not be solved to integer optimality within the time limit, we present the interval formed by the best lower bound and the best integer solution found. We use "–" to denote that no solution is obtained because of excessive memory requirement. Note that the models of R3 have approximately three times more numbers of variables and constraints than those

15

of R2. For R3, the solver process acquired almost 98% of the RAM (without producing any result), before the process got killed by the operating system.

$\mathbf{F}1$ is clearly outperformed by $\mathbf{F}2$. The former leads to integer optimum for one network only, and for some instances it does not enable any feasible solution at all within the time limit. The latter delivers integer optimum for all but one network. We note that for network R1, for which optimum is found using both models, the solution time of $\mathbf{F}2$ is orders of magnitude shorter. Moreover, the continuous relaxation of $\mathbf{F}2$ yields much sharper bounds than that of $\mathbf{F}1$. The tight bounds of $\mathbf{F}2$ contribute to its performance in obtaining integer optimum, and in some cases the integrality gap of $\mathbf{F}2$ is zero. In conclusion, exploiting the problem structure results in significant improvement in solving MPCO efficiently. However, for network R3 (the city of Lisbon), none of the models is able to deliver solutions. This fact justifies the use of heuristics for large-scale instances of MPCO. A further observation is that the solution time (of both models) tends to decrease when the overlap requirement grows from 10% to 20%. This is because higher overlap requirement restricts the set of feasible power settings more stringently, resulting in a higher number of power levels that are discarded due to infeasibility.

In Table 3 we report two sets of results for the TS algorithm. The table shows the solution value (in Watt), computing time (excluding that needed for the initial solutions), the number of iterations, and the relative gap to optimum. When initial solutions are derived by solving LP, "*" denotes cases where the LP optimum is also integer optimal. The sign "–" denotes that results are not available either because integer optimum or LP optimum is not known. We observe a maximum gap of 3.4%. For the two types of initial solutions, the average gaps are 1.4% and 0.6%, respectively. The computational effort required by the algorithm is moderate. Thus the proposed TS algorithm is a promising approach to deal with large-scale instances. Starting from the LP solution does enable TS to find better results. However, as this scheme requires the LP solution prior to running TS, the overall computing time is longer.

In Figure 4 we use the optimal solutions of MPCO to illustrate and compare the two cell overlap levels for network R2 (city of Berlin). The small dots represent the locations of the base station sites. The cells are represented by short lines showing the directions of the cell antennas. For each test point, its color denotes the number of cells covering it. From the figure, we observe that the overlap does have a form that facilitates handover between cells. If two cell antennas are co-located at the same site, the overlap has a beam shape, and its direction is somewhere between the directions of the two antennas. For two geographically adjacent cells of different sites, the overlap appears in some region between the two sites. When the overlap level requirement goes from 10% to 20%, the overlap regions become more continuous, and there is a clear increment in the number of test points being covered by more than two cells.

16

Table 3
Computational results of TS. (Objective values have unit Watt, and times are in seconds.)

Network	Initial solution: heuristic				Initial solution: LP			
	Value	Time	Iter.	Gap (%)	Value	Time	Iter.	Gap (%)
$N1_{10}$	35.6	0.7	354	0.8	*	*	*	*
$N2_{10}$	58.6	2.3	379	0.8	58.2	1.9	305	0.2
$N3_{10}$	122.4	19.6	872	1.8	120.3	15.8	719	0.1
$N4_{10}$	207.4	25.4	382	1.8	204.2	31.2	466	0.3
$N5_{10}$	242.5	69.2	590	1.0	241.8	94.1	780	0.7
$R1_{10}$	44.2	0.6	575	0.1	*	*	*	*
$R2_{10}$	167.3	36.4	596	2.9	163.3	30.4	464	0.4
$R3_{10}$	131.5	291.6	1057	–	–	–	–	–
$N1_{20}$	37.0	0.6	318	0.6	*	*	*	*
$N2_{20}$	60.8	3.6	775	1.0	*	*	*	*
$N3_{20}$	126.5	7.0	351	2.3	123.8	7.1	364	0.1
$N4_{20}$	220.4	20.8	380	2.0	216.4	20.4	362	0.1
$N5_{20}$	252.1	37.8	375	0.8	251.1	57.4	553	0.4
$R1_{20}$	45.4	0.3	314	0.3	*	*	*	*
$R2_{20}$	185.9	19.2	455	3.4	185.4	38.5	931	3.1
$R3_{20}$	141.3	201.6	969	–	–	–	–	–

8 Conclusions

We have studied a type of covering problem with application in cellular network design, where cell overlap is necessary for enabling handover of users between cells. We have developed two integer linear programming models that differ in the level of exploiting problem structure. Comparisons of the two models have been conducted both theoretically and numerically. In addition, we have presented and evaluated a tabu search algorithm. The computational experiments show that our second integer linear model performs very well in delivering sharp lower bounds as well as optimal solutions to instances up to medium size, and the tabu search algorithm is a promising approach for obtaining close-to-optimal solutions using little computing time.

One interesting line of further research is to extend the current work to a version of the
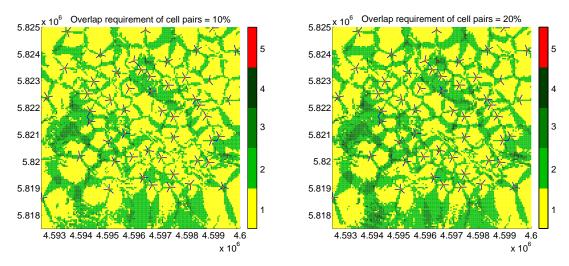
Figure 4. An illustration of cell overlap in the solutions for network R2 (Berlin).

problem where the power is restricted to a relatively small number of possible levels. This problem version becomes of relevance as an approximation of the general problem considered in this paper, or in case a quantization has been performed in calculating the power. If power is confined to a small discrete set, many test points are expected to have the same power value; such test points may be clustered together. Thus one research topic is the development of stronger preprocessing schemes and faster solution approaches that are tailored to this specific problem version. A second topic is to investigate whether or not the power restriction will open up a polynomial time approximation.

Acknowledgments

A   Proof of Proposition 4

To avoid a proof of excessive length, we assume that the bijection F_i is unique for all $i \in \mathcal{C}$. The proof can be extended to the general case, although it would require

18

additional technical details.

Before moving into the proof itself, we sketch the underlying idea. We introduce $\mathcal{J}_{ih}^2$ to denote the set of points in $\mathcal{J}_{ih}$ having i and h as the only two possible covering cells, that is, $\mathcal{J}_{ih}^2 = \{j \in \mathcal{J}_{ih} : \mathcal{C}_j = \{i, h\}\}$. Let the LP optimum of **F**2 be $\bar{x}$, we define, following (3)–(4), $\bar{s}_j^{ih} = \sum_{k=F_i'(j)}^{J_i} \bar{x}_{i(k)} + \sum_{k=F_h'(j)}^{J_h} \bar{x}_{h(k)} - 1, (i, h) \in \mathcal{D}, j \in \mathcal{J}_{ih}^2$, and $\bar{s}_j^{ih} = 1/2(\sum_{k=F_i'(j)}^{J_i} \bar{x}_{i(k)} + \sum_{k=F_h'(j)}^{J_h} \bar{x}_{h(k)}), (i, h) \in \mathcal{D}, j \in \mathcal{J}_{ih} \setminus \mathcal{J}_{ih}^2$. We then show that $\bar{s}$ satisfies the overlap requirement (5). Note that, for each $j \in \mathcal{J}_{ih}$, there is a constraint of (8) defined for $j = F_i(\ell)$, cf. Figure 2. Due to Proposition 2, the left-hand side equals $1 - \sum_{k=\ell}^{J_i} \bar{x}_{i(k)}$, resulting in the following inequality.

$$\sum_{k=\ell}^{J_i} \bar{x}_{i(k)} + \sum_{k=L(i_{\ell-1},h)}^{J_h} \bar{x}_{h(k)} \geq 1 \tag{A.1}$$

Suppose we define all the s-variables by (4). This gives in fact a model that is valid but weaker than **F**1. In this case, we can simply take, among the points in $\mathcal{J}_{ih}$, the first d_{ih} points in the sorted sequence of cell i. For each point, the first term of (A.1) equals one. Take the d_{ih} points defined correspondingly for cell h. Next, observe that each sum of (A.1) for these points appears exactly once in (5), and the proposition follows. The proof is more complicated if (3) is present. In the proof, we first consider the pairs of power levels defined by (A.1) for the remaining $J_{ih} - d_{ih}$ points of cell i. Applying (A.1) directly does not give a proof, because there may exist one or several $\ell' \neq \ell$, with $L(i_{\ell'-1}, h) = L(i_{\ell-1}, h)$, requiring the use of a sum in (5) more than once (cf. Figure 2). However, note that (A.1) remains valid for a power level lower than ℓ for i, or a power level lower than $L(i_{\ell-1}, h)$ for h. We therefore modify the pairs of power levels in this direction, and show the existence of a sequence of modifications, such that each sum of (A.1) for the modified levels is used no more than once in the left-hand side of (5).

We start the proof by some preparation. Consider any $(i, h) \in \mathcal{D}$. Let $j_i^1, j_i^2, \ldots, j_i^{J_{ih}}$ be the sequence of test points in $\mathcal{J}_{ih}$, such that their positions in the sorted power sequence of cell i, $F_i'(j_i^1), F_i'(j_i^2), \ldots, F_i'(j_i^{J_{ih}})$, is monotonously increasing. For cell h, order the same test points in a similar manner, and denote the resulting sequence by $j_h^1, j_h^2, \ldots, j_h^{J_{ih}}$. For any $k \in \{d_{ih} + 1, \ldots, J_{ih}\}$, parameter $L(i_{F_i'(j_i^k)-1}, h)$ defined for constraint (8) clearly corresponds to a power level of cell h for covering one of the points in $\mathcal{J}_{ih}$, i.e., $L(i_{F_i'(j_i^k)-1}, h) = F_h'(j_h^m)$, where m is an integer between d_{ih} and J_{ih}. To simplify the notation, we use $L(j_i^k, h)$ to replace $L(i_{F_i'(j_i^k)-1}, h)$ in the remainder of the proof.

19

We introduce a set $\mathcal{A} = \{(j_i^k, F_h(L(j_i^k, h))), k = d_{ih} + 1, \ldots, J_{ih}\} \subseteq \mathcal{J}_{ih} \times \mathcal{J}_{ih}$. Each element $(j, a_j) \in \mathcal{A}$ is referred to as an association because it associates one test point $j \in \mathcal{J}_{ih}$ to another test point $a_j \in \mathcal{J}_{ih}$. Test points j and a_j may coincide. There are $J_{ih} - d_{ih}$ elements in $\mathcal{A}$. The lemma below states some properties of $\mathcal{A}$.

Lemma 5

(1) *For any j_i^k, $k \in \{d_{ih} + 1 \ldots, J_{ih}\}$, there is exactly one element $(j, a_j) \in \mathcal{A}$ where $j = j_i^k$.*

(2) *For any j_i^k, $k \in \{d_{ih} + 2 \ldots, J_{ih}\}$, and $d_{ih} + 1 \leq t < k$, $F_h'(a_{j_i^t}) \geq F_h'(a_{j_i^k})$.*

(3) *For any $(j, a_j) \in \mathcal{A}$ where $j \in \mathcal{J}_{ih}^2$ and there are m test points in $\mathcal{J}_{ih}^2$ requiring power greater than or equal to P_{ij} in cell i, $F_h'(a_j) \geq F_h'(j_h^{d_{ih}+m})$.*

(4) *For any $k \in \{d_{ih} + 1, \ldots, J_{ih}\}$ for which $j_h^k \in \mathcal{J}_{ih}^2$, if there are m test points in $\mathcal{J}_{ih}^2$ requiring power greater than or equal to P_{h,j_h^k} in cell h, then the number of elements $(j, a_j) \in \mathcal{A}$ where $F_h'(a_j) \geq F_h'(j_h^k)$ is at least m.*

Proof The first property follows immediately from the construction of $\mathcal{A}$. The second property is simply a reformulation of $L(j_i^t, h) \geq L(j_i^k, h)$ for any two points j_i^k and j_i^t where $t < k$. Consider the third claimed property, and assume that its condition is satisfied. Suppose cell i uses a power level strictly lower than $F_i'(j)$. Then there are m test points in $\mathcal{J}_{ih}$ that have to be covered by cell h. Moreover, these points cannot contribute to overlap between cells i and h. Consequently cell h must additionally cover at least d_{ih} other points in $\mathcal{J}_{ih}$ to satisfy the overlap requirement. Hence $L(j, h)$ used to define constraint (8) for point j must correspond to a power level of cell h allowing for covering $d_{ih} + m$ points in $\mathcal{J}_{ih}$, and the result follows.

To prove the last property, we assume, without any loss of generality, that the m test points in $\mathcal{J}_{ih}^2$ requiring power greater than or equal to P_{h,j_h^k} in cell h are $j_h^k, j_h^{k+1}, \ldots,$ $j_h^{k+t-1}, j_h^{k+t}, \ldots, j_h^{k+m-1}$, among which the first t points ($0 \leq t \leq \min\{m, d_{ih}\}$) in this sequence are in the set $\{j_i^1, \ldots, j_i^{d_{ih}}\}$. Parameters $L(j_i^{k+t}, h), \ldots, L(j_i^{k+m-1}, h)$ are all greater than or equal to $F_h'(j_h^k)$, because these parameters must ensure that cell h can cover $j_h^{k+t}, \ldots, j_h^{k+m-1}$. Hence the $m-t$ elements in set $\mathcal{A}$ defined for $j = j_h^{k+t}, \ldots, j = j_h^{k+m-1}$ all satisfy the property. Next, suppose cell i uses level $F_i'(j_i^{d_{ih}})$, and hence covers exactly d_{ih} test points in $\mathcal{J}_{ih}$. If the power level used by cell h is strictly below $F_h'(j_h^k)$, than the overlap is at most $d_{ih} - t$, because $j_h^k, j_h^{k+1}, \ldots, j_h^{k+t-1}$ are covered by cell i but not cell h. We follow the sequence of points $j_i^{d_{ih}+1}, j_i^{d_{ih}+2}, \ldots, j_i^{J_{ih}}$ one by one, until we encounter a test point, say j_i^r, such that $j_i^r \notin \{j_h^{k+t}, \ldots, j_h^{k+m-1}\}$. At this stage, the maximum possible overlap is $d_{ih} - t$, and thus $L(j_i^r, h) \geq F_h'(j_h^k)$. Point j_i^r must exist because both sequences $j_i^1, j_i^2, \ldots, j_i^{J_{ih}}$ and $j_h^1, j_h^2, \ldots, j_h^{J_{ih}}$ are of the same length. We can continue following the sequence, and repeat the same argument for exactly t test points, for which the associations satisfy the property. These t elements, together with the $m - t$ elements discussed earlier, complete the proof. $\square$

We define a second set of association $\mathcal{G} \subseteq \mathcal{J}_{ih} \times \mathcal{J}_{ih}$. The next lemma concerns the existence of $\mathcal{G}$ with the specified properties.

Lemma 6 *There exists a set of associations $\mathcal{G} \subseteq \mathcal{J}_{ih} \times \mathcal{J}_{ih}$ satisfying the following properties.*

    I. *For any j_i^k, $k \in \{d_{ih} + 1 \ldots, J_{ih}\}$ and $j_i^k \in \mathcal{J}_{ih}^2$, there is exactly one element $(j, g_j) \in \mathcal{G}$ where $j = j_i^k$ and $F_h'(g_j) \geq F_h'(j_h^{d_{ih}}) + 1$.*

    II. *For any j_i^k, $k \in \{d_{ih} + 1 \ldots, J_{ih}\}$, $F_h'(a_{j_i^k}) \geq F_h'(g_{j_i^k})$.*

    III. *For any j_h^k, $k \in \{d_{ih} + 1 \ldots, J_{ih} - 1\}$ and $j_h^k \in \mathcal{J}_{ih}^2$, there is at least one element (j, g_j) in $\mathcal{G}$ where $g_j = j_h^k$.*

    IV. *For any j_h^k, $k \in \{d_{ih} + 1 \ldots, J_{ih} - 1\}$, there is at most one element (j, g_j) in $\mathcal{G}$ where $j \in \mathcal{J}_{ih}^2$ and $g_j = j_h^k$.*

Proof We start by setting $\mathcal{G} = \mathcal{A}$. That $\mathcal{G}$ satisfies Property I follows directly from Properties 1 and 3 in Lemma 5. Set $\mathcal{G}$ also satisfies Property II by equality. For convenience, we use a set $\mathcal{M}$ of points referred to as marked (see below). Initially $\mathcal{M} = \emptyset$. If $\mathcal{G}$ does not satisfy Property III, we make some modifications to its elements, resulting in the fulfillment of the property. We start from $j_h^{d_{ih}+1}$, and consider the sequence of test points $j_h^{d_{ih}+1}, j_h^{d_{ih}+2}, \ldots, j_h^{J_{ih}}$. Let j_h^k be the first point not satisfying Property III, i.e., $j_h^k \in \mathcal{J}_{ih}^2$ and $\mathcal{G}$ does not contain any element (j, g_j) where $g_j = j_h^k$. Consider sequence $j_i^{J_{ih}}, j_i^{J_{ih}-1}, \ldots$, and let j_i^r be the first point for which $F_h'(g_{j_i^r}) > F_h'(j_h^k)$. (The inequality is strict because no point is currently associated with j_h^k.) We set $g_{j_i^r} = j_h^k$, i.e., replace the element $(j_i^r, g_{j_i^r}) \in \mathcal{G}$ with (j_i^r, j_h^k), and mark j_i^r by setting $\mathcal{M} = \mathcal{M} \cup \{j_i^r\}$. We then repeat the procedure for the next point in $j_h^{k+1}, \ldots, j_h^{J_{ih}}$ not satisfying Property III, until this property holds for $\mathcal{G}$.

We show that the above update procedure is always feasible. Assume that the update is $g_{j_i^r} = j_h^k$. Later, we need to find another point $j_i^{\tilde{r}}$ because Property III is not satisfied for $j_h^{\tilde{k}}$. As the procedure treats $j_h^{d_{ih}+1}, j_h^{d_{ih}+2}, \ldots, j_h^{J_{ih}}$ in the given order, $F_h'(j_h^{\tilde{k}}) > F_h'(j_h^k)$. Moreover, $F_h'(g_{j_i^r}) = F_h'(j_h^k)$. Therefore $j_i^{\tilde{r}} \neq j_i^r$. As a result, a point in $j_i^{J_{ih}}, \ldots, j_i^{d_{ih}+1}$ can be selected and marked at most once as j_i^r in the update procedure, i.e., j_i^r will not be used for a later update (which will break again Property III at j_h^k). So once Property III becomes satisfied at j_h^k, it will remain satisfied for j_h^k in all subsequent updates. Moreover, as long as there is a point j_h^k for which Property III does not hold, we can always find the point j_i^r. The reason is that $\mathcal{G}$ initially satisfies Property 4 in Lemma 5, and the updates will not change this property of $\mathcal{G}$.

The above procedure will obviously not cause $\mathcal{G}$ to violate Properties I-II. Next, we show that Property 2 in Lemma 5 holds after the updating procedure. This property is satisfied by $\mathcal{G}$ before the procedure starts. Consider the first update made for point j_h^k, and the update is to set $g_{j_i^r} = j_h^k$. We only need to examine the property for j_i^r and another

21

arbitrary point j_i^q in $j_i^{d_{ih}+1}, \ldots, j_i^{J_{ih}}$, because the update will not affect the property for other pairs of points. Assume $q > r$. In this case $F_h'(g_{j_i^q}) < F_h'(j_h^k)$, as otherwise j_i^r would not have been chosen. After the update $F_h'(g_{j_i^r}) = F_h'(j_h^k)$, so the property remains satisfied for j_i^r and j_i^q. Consider the case $q < r$, and thus $F_h'(g_{j_i^q}) \geq F_h'(g_{j_i^r}) > F_h'(j_h^k)$ before the update. The update changes the second inequality to equality, and hence the first inequality remains valid. Therefore Property 2 is satisfied after the update. The same argument applies to the subsequent updates, and the result follows.

Consider Property 3 in Lemma 5. We can immediately conclude that the property is satisfied by any point $j \in \mathcal{J}_{ih}^2$ if $j \notin \mathcal{M}$, because g_j is not changed. If $j \in \mathcal{J}_{ih}^2 \cap \mathcal{M}$, the property may not hold. In this case we make the observation that Property IV is satisfied for g_j. This is because any point in $j_i^{d_{ih}+1}, \ldots, j_i^{J_{ih}}$ is marked at most once, and the points in $j_h^{d_{ih}+1}, \ldots, j_h^{J_{ih}}$ causing updates and marking are all different.

To summarize, the new $\mathcal{G}$ has Properties I-III, and Properties 1, 2, and 4. Moreover, if a point j in $\mathcal{J}_{ih}^2$ does not satisfy Property 3, then g_j will satisfy Property IV. If Property IV does not hold for $\mathcal{G}$, we make some further modifications, without changing the fulfillment of Properties I-III. Note that violation of Property IV means that there are at least two points $j_i^k, j_i^r \in \mathcal{J}_{ih}^2$, such that $g_{j_i^k} = g_{j_i^r}$. We process points $j_i^{J_{ih}}, j_i^{J_{ih}-1}, \ldots, j_i^{d_{ih}+1}$ in the given order, and stop when we encounter a point $j_i^k \in \mathcal{J}_{ih}^2$, for which there exists another point $j_i^r \in \mathcal{J}_{ih}^2$, $r < k$, and $g_{j_i^k} = g_{j_i^r} = j_h^t$. We find a point j_h^q in $j_h^{d_{ih}+1}, j_h^{d_{ih}+2}, \ldots, j_h^{t-1}$, with the condition that there does not exist any element $(j, g_j) \in \mathcal{G}$ where $j \in \mathcal{J}_{ih}^2$ and $g_j = j_h^q$. We update $\mathcal{G}$ by setting $g_{j_i^k} = j_h^q$. This is repeated until Property IV becomes valid for $\mathcal{G}$.

We show the existence of point j_h^q as follows. Because Property IV is not satisfied at $g_{j_i^k} = g_{j_i^r} = j_h^t$, none of j_i^k and j_i^r is in set $\mathcal{M}$, and consequently Property 3 is satisfied at both j_i^k and j_i^r. Let $m - 1$ be the number of test points in $\mathcal{J}_{ih}^2$ requiring power strictly greater than P_{i, j_i^k} in cell i, then sequence $j_h^{d_{ih}+1}, j_h^{d_{ih}+2}, \ldots, j_h^{t-1}$ contains at least m points. Among them, at most $m - 1$ are associated with the $m - 1$ points in $\mathcal{J}_{ih}^2$ requiring power strictly greater than P_{i, j_i^k} in cell i. Moreover, because of Property 2, none of the points in $j_h^{d_{ih}+1}, j_h^{d_{ih}+2}, \ldots, j_h^{t-1}$ is associated with any $j_i^{\tilde{k}}$ where $d_{ih} + 1 \leq \tilde{k} < k$. Thus there is at least one point that can be chosen as j_h^q. After the update for j_i^k, Property 3 is clearly still valid for all points in $(\{j_i^{k-1}, j_i^{k-2}, \ldots, j_i^{d_{ih}+1}\} \cap \mathcal{J}_{ih}^2) \setminus \mathcal{M}$. Moreover, Property 2 remains satisfied for any pair of these points. Therefore we can repeat the aforementioned update as long as Property IV does not hold at some point. Once an update is made for j_i^k, subsequent updates will not alter this property at $g_{j_i^k}$ because of the criterion used for selecting j_h^q. Furthermore, the updates will obviously not affect Properties I-III. Hence eventually $\mathcal{G}$ will satisfy Properties I-IV. □

22

We are ready to prove Proposition 4. Solution $(\bar{x}, \bar{s})$, as defined at the beginning of the proof, clearly satisfies (1), (2), (3), and (4). For (5), the left-hand side is as follows.

$$\sum_{j \in \mathcal{J}_{ih}} \bar{s}_j^{ih} = \sum_{j \in \mathcal{J}_{ih}^2} \left(\sum_{k=F_i'(j)}^{J_i} \bar{x}_{i(k)} + \sum_{k=F_h'(j)}^{J_h} \bar{x}_{h(k)} \right) + \frac{1}{2} \sum_{j \in \mathcal{J}_{ih} \setminus \mathcal{J}_{ih}^2} \left(\sum_{k=F_i'(j)}^{J_i} \bar{x}_{i(k)} + \sum_{k=F_h'(j)}^{J_h} \bar{x}_{h(k)} \right)$$
$$- |\mathcal{J}_{ih}^2| \tag{A.2}$$

We construct a subset $\mathcal{G}_s \subseteq \mathcal{G}$. First, we add, for each $j_i^k \in \mathcal{J}_{ih}^2$ where $d_{ih} \le k \le J_{ih}$, the unique element $(j_i^k, g_{j_i^k}) \in \mathcal{G}$ to $\mathcal{G}_s$. The existence of the element and its uniqueness follow from Properties I and IV. Next, for each $j_h^k \in \mathcal{J}_{ih}^2$, where $d_{ih} + 1 \le k \le J_{ih}$ and no association $(j, g_j) \in \mathcal{G}$ with $g_j = j_h^k$ has been added to $\mathcal{G}_s$ in the first step, we choose one association $(j, g_j) \in \mathcal{G}$ where $g_j = j_h^k$, and add it to $\mathcal{G}_s$. The existence of such an element follows from Property III. If there are several associations with $g_j = j_h^k$, any of them can be chosen. Several observations follow from the construction of $\mathcal{G}_s$. First, each test point $j_i^k \in \mathcal{J}_{ih}^2$ with $d_{ih} + 1 \le k \le J_{ih}$ appears exactly once as j in one element $(j, g_j) \in \mathcal{G}_s$. Second, each test point $j_h^k \in \mathcal{J}_{ih}^2$ with $d_{ih} + 1 \le k \le J_{ih}$ appears exactly once as g_j in one element $(j, g_j) \in \mathcal{G}_s$. Third, for any two elements $(j_1, g_{j_1}) \in \mathcal{G}_s$ and $(j_2, g_{j_2}) \in \mathcal{G}_s$, $j_1 \ne j_2$ and $g_{j_1} \ne g_{j_2}$. Fourth, for all $(j, g_j) \in \mathcal{G}_s$, either $j \in \mathcal{J}_{ih}^2$, or $g_j \in \mathcal{J}_{ih}^2$, or both. For any $(j, g_j) \in \mathcal{G}_s$, (A.2) contains, within its parentheses, each of $\sum_{k=F_i'(j)}^{J_i} \bar{x}_{i(k)}$ and $\sum_{k=F_h'(g_j)}^{J_h} \bar{x}_{h(k)}$ exactly once. By Property II, constraint (8) and Proposition 2, we obtain the following inequality for any $(j, g_j) \in \mathcal{G}_s$.

$$\sum_{k=F_i'(j)}^{J_i} \bar{x}_{i(k)} + \sum_{k=F_h'(g_j)}^{J_h} \bar{x}_{h(k)} \ge \sum_{k=F_i'(j)}^{J_i} \bar{x}_{i(k)} + \sum_{k=F_h'(a_j)}^{J_h} \bar{x}_{h(k)} \tag{A.3}$$

$$= \sum_{k=F_i'(j)}^{J_i} \bar{x}_{i(k)} + \sum_{k=L(j,h)}^{J_h} \bar{x}_{h(k)} \tag{A.4}$$

$$\ge \sum_{k=F_i'(j)}^{J_i} \bar{x}_{i(k)} + \sum_{k=1}^{F_i'(j)-1} \bar{x}_{i(k)} \tag{A.5}$$

$$= 1$$

The next observation is $F_i'(j) \ge d_{ih} + 1$ and $F_h'(g_j) \ge d_{ih} + 1$ for any $(j, g_j) \in \mathcal{G}_s$. And it follows from the construction of constraint (8) that $\sum_{k=F_i'(j_i^m)}^{J_i} \bar{x}_{i(k)} = 1$ and $\sum_{k=F_h'(j_h^m)}^{J_h} \bar{x}_{h(k)} = 1$ for all $m, 1 \le m \le d_{ih}$. Clearly, each of these sums appears exactly

once within the parentheses of (A.2). For $\mathcal{G}_s$, let n, n_i, and n_h denote the numbers of elements for which $j \in \mathcal{J}_{ih}^2$ and $g_j \in \mathcal{J}_{ih}^2$, $j \in \mathcal{J}_{ih}^2$ and $g_j \notin \mathcal{J}_{ih}^2$, and $j \notin \mathcal{J}_{ih}^2$ and $g_j \in \mathcal{J}_{ih}^2$, respectively. From the observations, we obtain the following inequality that completes the proof.

$$(A.2) \geq n + \frac{1}{2}n_i + \frac{1}{2}n_h + (|\mathcal{J}_{ih}^2| - n - n_i) + (|\mathcal{J}_{ih}^2| - n - n_h)$$
$$+ \frac{1}{2}(d_{ih} - (|\mathcal{J}_{ih}^2| - n - n_i)) + \frac{1}{2}(d_{ih} - (|\mathcal{J}_{ih}^2| - n - n_h)) - |\mathcal{J}_{ih}^2| = d_{ih}$$
$$\text{(A.6)}$$

References

[1] Aardal K, van Hoesel CPM, Koster AMCA, Mannino C, Sassano A. Models and solution techniques for the frequency assignment problem. 4OR 2006;1;261–317.

[2] Amaldi E, Capone A, Malucelli F. Planning UMTS base station location: Optimization models with power control and algorithms. IEEE Transactions on Wireless Communications 2003;2;939–952.

[3] Amaldi E, Belotti P, Capone A, Malucelli F. Optimizing base station location and configuration in UMTS networks. Annals of Operations Research 2006;146;135–151.

[4] Ceria S, Nobili P, Sassano A. Set covering problem. In: Dell'Amico M, Maffioli F, Martello S (Eds), Annotated Bibliographies in Combinatorial Optimization. John Wiley; 1997.

[5] Eisenblätter A, Geerdes HF, Koch T, Martin A, Wessäly R. UMTS radio network evaluation and optimization beyond snapshots. Mathematical Methods of Operations Research 2005;63;1–29.

[6] Eisenblätter A, Koch T, Martin A, Achterberg T, Fügenschuh A, Koster A, Wegel O, Wessäly R. Modelling feasible network configurations for UMTS. In: Anandalingam G, Raghavan S (Eds), Telecommunications Network Design and Management. Kluwer Academic Publishers; 2002.

[7] Eisenblätter A, Geerdes HF, Grötschel, M. Planning UMTS radio networks. OR/MS Today 2008;35;41–46.

[8] Fischetti M, Romanin Jacur G, Salazar Gonzalez JJ. Optimisation of the interconnecting network of a UMTS radio mobile telephone system. European Journal of Operational Research 2003;144;56–67.

[9] Garcia-Lozano M, Ruiz S, Olmos JJ. UMTS optimum cell load balancing for inhomogeneous traffic patterns. In: Proceedings of IEEE VTC fall 2004; 2004. p. 909–913.

[10] Geerdes HF. Dynamic aspects in W-CDMA: HSPA performance. European COST action 293 Short Term Scientific Mission (STSM) report, 2007.

[11] Jüttner A, Orbán A, Fiala Z. Two new algorithms for UMTS access network topology design. European Journal of Operational Research 2005;164;456–474.

[12] Nawrocki M, Aghvami H, Dohler M (Eds). Understanding UMTS radio network modelling, planning and automated optimisation: Theory and practice. Wiley; 2006.

[13] Glover F, Laguna M. Tabu Search. Kluwer Academic Publishers; 1997.

[14] ILOG CPLEX 10.0 User Manual. ILOG; 2006.

[15] Kalvenes J, Kennington J, Olinick E. Base station location and service assignments in W-CDMA networks. INFORMS Journal on Computing 2006;18;366–376.

[16] Mathar R, Schmeink M. Optimal base station positioning and channel assignment for 3G mobile networks by integer programming. Annals of Operations Research 2001;107;225–236.

[17] MOMENTUM Project; 2003 (updated in 2005). `http://momentum.zib.de`

[18] Olinick EV, Rosenberger JM. Optimizing revenue in CDMA networks under demand uncertainty. European Journal of Operational Research 2008;186;812–825.

[19] Siomina I, Värbrand P, Yuan D. Automated optimization of service coverage and base station antenna configuration in UMTS networks. IEEE Wireless Communications Magazine 2006;13;16–25.

[20] Siomina I, Yuan D. Minimum pilot power for service coverage in WCDMA networks. Wireless Networks 2007;14;393–402.

[21] Sun Y, Gunnarsson F, Hiltunen K. CPICH power settings in irregular WCDMA macro cellular networks. In: Proceedings of IEEE PIMRC 2003; 2003. p. 1176–1180.

[22] Touhami S, Bourjolly JM, Laporte G. Partial integration of frequency allocation within antenna positioning in GSM mobile networks. European Journal of Operational Research 2009;193;541–551.

[23] Zhang HY, Xi YG, Gu HY. A rolling window optimization method for large-scale WCDMA base stations planning problem. European Journal of Operational Research 2007;183;370–383.

[24] Zhang J, Yang J, Aydin ME, Wu JY. UMTS base station location planning: A mathematical model and heuristic optimisation algorithms. IET Communications 2007;1;1007–1014.