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Generation of magnetic fields by the ponderomotive force of electromagnetic waves in dense plasmas

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Abstract. We show that the non-stationary ponderomotive force of a large-amplitude electromagnetic wave in a very dense quantum plasma with streaming degenerate electrons can spontaneously create d.c. magnetic fields. The present result can account for the seed magnetic fields in compact astrophysical objects and in the next-generation intense laser–solid density plasma interaction experiments.

The mechanisms for spontaneous magnetic field generation in classical plasmas include (i) non-parallel density and temperature gradients (the so-called Biermann battery [1]), (ii) the electron temperature anisotropy (known as the Weibel instability [2]), (iii) counterstreaming electron beams [3, 4], and (iv) the ponderomotive forces of laser beams [5–15]. Spontaneously generated magnetic fields are of significant interest in laser produced plasmas [16–24], in our Universe [25, 26], in many cosmic environments [27], as well as in galactic and intergalactic spaces [28–32].

However, in dense astrophysical objects [33–36] (e.g. white dwarf stars, neutron stars, magnetars, etc.) and in the next-generation intense laser–solid density plasma experiments [37,38], the electrons are degenerate. Consequently, one has to account for the quantum-statistical pressure [39, 40], quantum electron tunneling [41, 42] and electron-1/2 spin effects [43–46]. Here we thus consider the nonlinear interaction between a large-amplitude electromagnetic wave and a very dense quantum plasma with streaming electrons. It is shown that the non-stationary ponderomotive force of the electromagnetic wave creates slowly varying electric fields and currents, which generate d.c. magnetic fields.

Let us consider the propagation of an ordinary mode electromagnetic wave, with the electric field $\mathbf{E}(\mathbf{r}, t) = (1/2)\mathbf{E}_0(x, t)\exp(-i\omega t + ikx) + \text{c.c.}$, in an unmagnetized non-relativistic dense plasma with streaming electrons (with the drift velocity $u\hat{\mathbf{z}}$,

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where u is the magnitude of the electron drift speed and $\hat{\mathbf{z}}$ is the unit vector along the z-axis in a Cartesian coordinate system; typically u is much smaller than the Fermi electron thermal speed) and immobile ions. Here, $\mathbf{E}_0(x,t)$ is the envelope of the electromagnetic field at the position $\mathbf{r}$ and time t, and c.c. denotes complex conjugate. The frequency ω and the wave vector $\mathbf{k} = k\hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$ is the unit vector along the x-axis, are related by [47]

$$\frac{k^2c^2}{\omega^2} = N = 1 - \frac{\omega_{\rm pe}^2}{\omega^2} - \frac{k^2u^2\omega_{\rm pe}^2}{\omega^2(\omega^2 - k^2V_{\rm TF}^2 - \Omega_{\rm q}^2)}, \tag{1}$$

where c is the speed of light in vacuum, N is the index of refraction, $\omega_{\rm pe} = (4\pi n_0 e^2/m_{\rm e})^{1/2}$ is the electron plasma frequency, n_0 is the electron number density, e is the magnitude of the electron charge, $m_{\rm e}$ is the electron mass, $V_{\rm TF} = \hbar(3\pi^2 n_0)^{1/3}/m_{\rm e}$ is the Fermi electron thermal speed, $\Omega_{\rm q} = \hbar k^2/2m_{\rm e}$, and $\hbar$ is the Planck constant divided by 2π. Equation (1) is simply obtained from the Maxwell equations, the electron continuity equation, and the electron momentum equation including the equilibrium electron drift speed, the quantum-statistical pressure, and the quantum Bohm force [39,40]. The $\Omega_{\rm q}$ term is associated with the quantum Bohm force caused by the electron tunneling in dense quantum plasmas.

The electromagnetic wave exerts a ponderomotive force $\mathbf{F}_p = \mathbf{F}_{ps} + \mathbf{F}_{pt}$ on the plasma electrons, where the stationary and non-stationary ponderomotive forces [5,6] are, respectively,

$$\mathbf{F}_{ps} = \frac{(N-1)}{16\pi}\nabla|\mathbf{E}_0|^2, \tag{2}$$

and

$$\mathbf{F}_{pt} = \frac{1}{16\pi}\frac{\mathbf{k}}{\omega^2}\frac{\partial[\omega^2(N-1)]}{\partial\omega}\frac{\partial|\mathbf{E}_0|^2}{\partial t}. \tag{3}$$

The ponderomotive force pushes the electrons locally, and creates the slowly varying electric field

$$\mathbf{E}_s = -\nabla\phi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} = \frac{1}{n_0 e}\mathbf{F}_p, \tag{4}$$

where the scalar and vector potentials are, respectively,

$$\phi = -\frac{(N-1)}{16\pi n_0 e}|\mathbf{E}_0|^2, \tag{5}$$

and

$$\mathbf{A} = -\frac{c}{16\pi n_0 e}\frac{\mathbf{k}}{\omega^2}\frac{\partial[\omega^2(N-1)]}{\partial\omega}|\mathbf{E}_0|^2. \tag{6}$$

The induced slowly varying magnetic field $\mathbf{B}_s$ is then $\mathbf{B}_s = \nabla\times\boldsymbol{A}$. Noting that

$$\frac{\partial[\omega^2(N-1)]}{\partial\omega} = \frac{2\omega k^2u^2\omega_{\rm pe}^2}{(\omega^2 - k^2V_{\rm TF}^2 - \Omega_{\rm q}^2)^2}, \tag{7}$$

we can express the magnitude of the magnetic field as

$$|\mathbf{B}_s| = \frac{eck^3u^2|\mathbf{E}_0|^2}{2m_{\rm e}L\omega(\omega^2 - \Omega^2)^2}, \tag{8}$$

where L is scale length of the envelope $|\mathbf{E}_0|^2$ and $\Omega = (k^2V_{\rm TF}^2 + \Omega_{\rm q}^2)^{1/2} \equiv (\hbar k^2/2m_{\rm e})[1+4(3\pi^2n_0)^{2/3}/k^2]^{1/2}$. We note from (8) that the magnetic field strength is proportional to u^2, and attains a large value when $\omega \sim \Omega$. The electron gyrofrequency Ω_c is

$$\Omega_c = \frac{e|\mathbf{B}_s|}{m_e c} = \frac{k^3 V_0^2 u^2 \omega}{2L(\omega^2 - \Omega^2)^2}, \tag{9}$$

where $V_0 = e|\mathbf{E}_0|/m_e\omega$ is the electron quiver velocity in the electromagnetic fields.

To summarize, we have presented a new theory for magnetic field generation by the non-stationary ponderomotive force of a large-amplitude electromagnetic wave in a very dense plasma with degenerate electrons that are streaming. Specifically, we have shown that the non-stationary radiation pressure creates slowly varying electric fields and currents, which, in turn, can produce d.c. magnetic fields in a non-relativistic dense quantum plasma. The present results should be useful in understanding the origin of seed magnetic fields in compact astrophysical bodies containing intense X-ray sources, in quantum free electron lasers [48,49], as well as in the next generation of intense laser–solid density plasma interaction experiments and in quantum diodes [50].

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