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Mathematical modelling in upper secondary mathematics education in Sweden

A curricula and design study

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To my family

Preface

With this thesis I end a more than 15 year long period of my life as a student at Linköping University, and many people have supported and helped me in different ways in writing and completing this work. A very special thanks goes, of course, to Christer Bergsten who has endured, guided, helped, encouraged, and supported me as my supervisor, in my pursuit of sometimes naïve and impulsive ideas. Thank you Christer! I would also like to thank Björn Textorius for his engagement and support in my research endeavours, and especially for his careful reading of parts of the manuscript. My thanks also go to Morten Blomhøj for his valuable comments and suggestions on the draft of the manuscript that was presented at my 90% seminar in October 2009, and to Magnus Österholm for suggestions of improvements and his generosity with Word templates. Thanks also go to Arne Enqvist, Göran Forsling, Ulf Janfalk, and Bengt Ove Turesson for helping me in different ways during my struggle.

Last, but definitely not least, I would like to thank my family for being available and helping out the best they could and for putting up with me, especially this last year of my Ph.D. studies. My thoughts go foremost to my loving and caring wife Pauline, whose patience, understanding, and support have been astonishing; I have many, many, many things to thank you for Pauline, and making it possible to complete this thesis is one of them.

My final words go to Elias, Tova and Sonja: Now I'm all yours!

Publications

Included in this thesis are the following five reports and papers; some which have been, or are in the process of being, scrutinized via peer review due to submission for publications in journals or conference proceedings. This applies for the Papers 1, 2, and 3 below:

- [1] Ärlebäck, J. B. (in press). Towards understanding teachers' beliefs and affects about mathematical modelling. To appear in the proceedings of CERME 6, *The Sixth Conference of European Research in Mathematics Education, Lyon, France, January 28 – February 1, 2009*.
- [2] Ärlebäck, J. B. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Montana Mathematics Enthusiast*, 6(3), 331-364.
- [3]¹ Frejd, P., & Ärlebäck, J. B. (accepted for publication after revision). First results from a study investigating Swedish upper secondary students' mathematical modelling competencies, Proceedings of ICTMA 14, *The 14th International Community of Teachers of Modelling and Applications, Hamburg, July 27-31, 2009*.
- [4] Ärlebäck, J. B. (2009). *Matematisk modellering i svenska gymnasieskolans kursplaner i matematik 1965-2000* (Rapport nr 2009:8, LiTH-MAT-R-2009-8). Linköping: Linköpings universitet, Matematiska institutionen.
- [5] Ärlebäck, J. B. (2009). *Designing, implementing and evaluating mathematical modelling modules at the upper secondary level – a design study* (Rapport nr 2009:8, LiTH-MAT-R-2009-9). Linköping: Linköpings universitet, Matematiska institutionen.

The following papers are also connected to the work here presented, especially to Paper 2 which is an extend version of Paper 6 providing more empirical data and analysis, but which not explicitly are contained in this thesis:

- [6] Ärlebäck, J. B., & Bergsten, C. (2010). On the use of realistic Fermi problems in introducing mathematical modelling in upper secondary mathematics. In R. Lesh, P. L. Galbraith, W. Blum, & A. Hurford (Eds.), *Modeling Students' Mathematical Modeling Competencies. ICTMA 13* (pp. 597-609). New York: Springer.

¹ A modified version of Paper 3 is also accepted to be presented at the ICMI/ICIAM study conference on *Educational Interfaces between Mathematics and Industry* (the EIMI-study) held in Lisbon, Portugal on April 19-23, 2010.

- [7] Frejd, P., & Ärlebäck, J. B. (2010). On Swedish upper secondary students' description of the notion of mathematical models and modelling. Paper presented at MADIF7, Stockholm 26-27 January 2010.

Finally, the first half of my graduate studies in applied mathematics focusing on research in general relativity resulted in the following two publications, which also will not be accounted for here:

- [8] Bergman, J. (2004). Conformal Einstein Spaces and Bach tensor generalizations in n dimension. Linköping Studies in Science and Technology. Theses No. 1113, Licentiate thesis. Linköping: Linköpings universitet.
- [9] Bergman, J., Edgar, S. B., & Herberthson, M. (2005). The Bach tensor and other divergence-free tensors. *International Journal of Geometrical Methods in Modern Physics*, 2(1), 13-21.

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Abstract

The aim of this thesis is to investigate and enhance our understanding of the notions of mathematical models and modelling at the Swedish upper secondary school level. Focus is on how mathematical models and modelling are viewed by the different actors in the school system, and what characterises the collaborative process of a didactician and a group of teachers engaged in designing and developing, implementing and evaluating teaching units (so called modelling modules) exposing students to mathematical modelling in line with the present mathematics curriculum. The thesis consists of five papers and reports, along with a summary introduction, addressing both theoretical and empirical aspects of mathematical modelling.

The thesis uses both qualitative and quantitative methods and draws partly on design-based research methodology and cultural-historical activity theory (CHAT). The results of the thesis are presented using the structure of the three curriculum levels of the intended, potentially implemented, and attained curriculum respectively.

The results show that since 1965 and to the present day, gradually more and more explicit emphasis has been put on mathematical models and modelling in the syllabuses at this school level. However, no explicit definitions of these notions are provided but described only implicitly, opening up for a diversity of interpretations.

From the collaborative work case study it is concluded that the participating teachers could not express a clear conception of the notions mathematical models or modelling, that the designing process often was restrained by constraints originating from the local school context, and that working with modelling highlights many systemic tensions in the established school practice. In addition, meta-results in form of suggestions of how to resolve different kinds of tensions in order to improve the study design are reported.

In a questionnaire study with 381 participating students it is concluded that only one out of four students stated that they had heard about or used mathematical models or modelling in their education before, and the expressed overall attitudes towards working with mathematical modelling as represented in the test items were negative. Students' modelling proficiency was positively affected by the students' grade, last taken mathematics course, and if they thought the problems in the tests were easy or interesting. In addition empirical findings indicate that so-called realistic Fermi problems given to students working in groups inherently evoke modelling activities

Populärvetenskaplig sammanfattning

I denna avhandling studeras olika aspekter av begreppen *matematisk modell* och *matematisk modellering* med syfte att öka förståelsen av dessa begrepp så som de förekommer i samband med matematikämnet i den svenska gymnasieskolan. Avhandlingen, som består av fem artiklar och rapporter tillsammans med en sammanfattande introduktion (kappa), belyser dessa begrepp utifrån tre olika perspektiv: ett kursplaneperspektiv, ett lärarperspektiv och ett elevperspektiv. En stor del av avhandlingen studerar ett samarbete mellan en didaktiker och två lärare som utformar och utvecklar, implementerar och utvärderar undervisning (så kallade modelleringsmoduler) med mål att introducera och exponera gymnasieelever för matematisk modellering. De två modelleringsmoduler som togs fram i detta designprojekt var avsedda för kurserna *Matematik C* respektive *Matematik D*, och bestod av ett antal lektioner där eleverna fick arbeta i grupper med små miniprojekt.

Från ett kursplaneperspektiv visar resultatet att sedan införandet av den moderna gymnasieskolan 1965 och fram till våra dagar, har gradvis allt mer tonvikt lagts på begreppen matematisk modell och modellering i kursplanerna i matematik. Däremot finns inga tydliga definitioner av begreppen i kursplanerna, utan dessa tas för givna och beskrivs bara i implicita termer, vilket öppnar för en mångfald av tolkningar. Detta gäller så väl innebörden av, såväl som funktionen av och hur man kan arbeta med matematiska modeller och modellering.

Det visade sig också att de två lärarna i designstudien inte kunde formulera och uttrycka vilken innebörd och mening begreppen matematisk modell och modellering hade för dem, men att de fann arbetet med designprojektet positivt och givande. Dock framkom vissa tveksamheter om vad eleverna faktiskt lärde sig av modulerna. Under framtagandet av modulerna i designprojektet identifierades också olika faktorer som på olika sätt påverkade och hindrade ett effektivt arbete och kommunikation. Dessa faktorer kan relateras till vanor, rutiner och praxis på den skola där projektet genomfördes, samt lärarnas attityder.

I en studie med 381 elever i gymnasieskolans årskurs 3 uppgav endast en av fyra elever att de hade hört talas om, eller använt, matematiska modellering under sin gymnasieutbildning. I studien löste eleverna sju kortare modelleringsproblem och efter genomfört test uttryckte de en allmän negativ attityd till att arbeta med matematiska modeller. Elevernas resultat på testet påverkades positivt av elevernas betyg, vilken deras senast lästa matematikkurs var, och om de tyckte att problemen i testerna var lätta eller intressanta. Å andra sidan uttryckte eleverna som arbetade med modelleringsmodulerna i designstudien generellt positiva erfarenheter av att arbeta med matematisk modellering, men upplevde en viss tidsbrist. En annan empirisk studie visar att så kallade realistiska Fermiproblem som löses av studenter i grupp har stor potential för att introducera vad matematiska modellering kan innebära.

Resultaten i denna avhandling har bidragit till att lägga en grund för att få en bättre förståelse för och olika sätt att se på begreppen matematisk modell och modellering och deras funktion, användning och potential i svenska gymnasie-matematik.

PART I

PREAMBLE – THE COAT

Chapter 1

Introduction

1.1 Setting the scene

Why should students learn mathematics at school? This is a complex question which could be addressed from many different perspectives and is hard to give a concise and fair answer to. Niss (1996) summarises the historical as well as the contemporary arguments in the literature in the following three fundamental reasons for why mathematics should be taught at school (quote):

- contributing to the *technological and socio-economic development* of society at large, either as such or in competition with other societies/countries;
- contributing to *society's political, ideological and cultural maintenance and development*, again either as such or in competition with other societies/countries;
- providing *individuals with prerequisites which may help them to cope with life* in the various spheres in which they live: education or occupation; private life; social life; life as a citizen.

(p. 13, italics in original)

In a sense, Romberg (1992) summarises all these three arguments in what he refers to as a 'functional justification' when he argues that "schools should prepare students so that they can be productive citizens in society" (p. 756). In addition, Romberg presents five other arguments invoked for the learning of mathematics in schools, also related and connected in different ways to the three more general arguments provided by Niss above. These are: mathematics enhances and improves one's ability to think logically; mathematics trains and increases the stamina, so that the students are prepared and more easily can tackle situations and problems in the future where endurance is needed for succeeding in resolving the issues at hand; the appeal to an aesthetic side of mathematics as something beautiful and enjoyable; to ensure the regrowth of coming future generations of mathematicians; and finally, the argument that mathematics is a part of our culture (Romberg, 1992, pp. 758-759).

It should be noted that there is an ongoing debate within the mathematics education community whether these arguments and reasons are valid, legitimate and justified. Ernest (2000) for example argues that “the utility of academic and school mathematics in the modern world is greatly overestimated” and specifically that “the utilitarian argument provides a poor justification for the universal teaching of the subject throughout the years of compulsory schooling”². This argument is further elaborated in Jablonka (2009). In the Swedish contexts similar ideas have been expressed by Lundin (2008); “It is necessary to clearly distinguish between the mathematics of schooling, and the actual practices of technology, science and everyday life. The mathematics of schooling establishes a link between these practices and the practices of elementary mathematics instruction. My argument is that not only is this link largely illusory – something most people would probably agree on – but also impossible.” (p. 375).

Nevertheless, the three arguments of Niss and the functional justification argument of Romberg all involve *the using of mathematics* in one way or another; to produce something; to enhance something; to achieve something; to understand something; to predict something; or generally, to do something. With the phrase *to use mathematics* I here mean to take a mathematical concept, construct, idea, or a mathematical procedure and apply it to a situation with the aim to achieve an objective that could be more or less clear and well-defined. In other words, to take a situation and to look at and examine it using mathematics. In some cases this means to reformulate the situation using mathematical entities to get a description or understanding of the situation possibly involving mathematical ideas, expression, concepts, vocabulary or reasoning. A description of a situation is a *model*, and if the description is formulated using mathematics it is a *mathematical model*. So, using mathematics often results in the formulation of a mathematical model or indeed the using of an already existing mathematical model.

In this thesis I will investigate the notions of mathematical models and the process of producing or working with mathematical models, *mathematical modelling*, with respect to how these notions are, and have been, described, understood and dealt with at the upper secondary level in Sweden.

² See also Ernest (1998b).

1.1.1 Problématique

Internationally, research in mathematics education focusing on the role, use, and the teaching and learning of mathematical modelling at different school levels has been gaining momentum since the mid 1960's (Blum, 1995). Among other things, this has manifested itself in the founding of ICTMA³ with its biannual conferences, the ICMI 14 study⁴ focusing on mathematical modelling, and two special issues of the ZDM⁵. The arguments that have been put forward for the incorporation of mathematical modelling in schools are similar to the ones presented in the previous section for the learning of mathematics generally, and have been summarised in the following five overall arguments: *the formative argument*; *the critical competence argument*; *the utility argument*; *the 'picture of mathematics' argument*; and *the 'promoting mathematics learning' argument* (Blum & Niss, 1991; Niss, 1989).

In Sweden however, no systematic research explicitly focusing on mathematical models and modelling in connection with mathematics education comparable to what has been, and is being, done in other countries has been carried out. Nevertheless, some highly focused studies exist: Lingefjärd (2000) and Lingefjärd and Holmquist (2003; 2001; 2005; 2007) investigate aspects of mathematical modelling in connection with prospective teachers and teacher education; and Palm's research (2002; 2007) with focus on authentic and realistic features and consequences of mathematical school tasks. Occasionally, mathematical models and modelling are also mentioned in the passing in connection with research focusing on problem solving in general (e.g. Håstad, 1978; Wyndhamn, 1997; Wyndhamn, Riesbeck, & Schoultz, 2000, just to mention some examples). Nevertheless, whether justified or not, Lingefjärd (2006) summarizes the recent developments and present situation in Sweden by stating that "it seems that the more mathematical modeling is pointed out as an important competence to obtain for each student in the Swedish school system, the vaguer the label becomes" (p. 96).

In the latest formulation of the written curriculum document governing the Swedish upper secondary mathematics education from 2000, *using mathematical models* is put forward as one of the four important aspects of the subject that, together with *problem solving*, *communication* and *the history of mathematical ideas*, should permeate all mathematics teaching (Skolverket, 2001). It is also

³ International Conference on the Teaching of Mathematical Modelling and Applications, which goes back to 1983.

⁴ International Commission on Mathematical Instruction, ICMI study 14: Modelling and applications in mathematics education (Blum, Galbraith, Henn, & Niss, 2007).

⁵ *Zentralblatt für Didaktik der Mathematik*, the issues in question are 38(2) and 38(3) respectively.

stressed that “[a]n important part of solving problems is designing and using mathematical models” and that one of the goals to aim for is to “develop their [the students’] ability to design, fine-tune and use mathematical models, as well as critically assess the conditions, opportunities and limitations of different models” (Skolverket, 2001, p. 61). However, no explicit definition or more elaborate description of the notions and concepts of mathematical model or modelling is given.

Although mathematical models and modelling in fact are central concepts in the governing curriculum documents, research reports on how the teaching of mathematics at almost every school level in Sweden is governed by the use and content of traditional textbooks, especially at secondary level (Johansson, 2006; Skolinspektionen, 2009; Skolverket, 2003; SOU 2004:97). As a rule, these traditional textbooks normally do not bring up, treat or systematically address mathematical models and modelling explicitly in any detail – if at all.

In other words, the state of affairs when it comes to mathematical models and modelling at the Swedish upper secondary mathematics level raises many questions: What are mathematical models and modelling? How are these notions perceived and understood by the different actors in the Swedish school system and why are they perceived and understood in this manner? In what way have, are, and could mathematical models and modelling be worked with in upper secondary mathematics education? In what ways can mathematical models and modelling be taught and learned? Why should mathematical models and modelling be taught and learnt at the upper secondary level? What approaches are there to assess students’ work in mathematical modelling? ...

It seems that there is a need to survey the whole upper secondary mathematics education to get an overall picture and understanding of how these concepts of mathematical models and modelling are being looked upon and treated by the different actors at the upper secondary educational level. Some aspects of this problématique will be addressed and discussed in the present thesis.

1.1.2 Overview of the Swedish upper secondary school system

Since this thesis focuses on the teaching and learning of mathematics at the Swedish upper secondary level, the following paragraph is devoted to provide a brief overview of the present structure of this school level. For a more detailed account with an additional historical perspective see Årlebäck (2009a)⁶.

In 1994 a reform of the secondary educational system in Sweden resulted in 16 different three year national course based programmes. The subject of mathematics, which in the previously corresponding Technical and Natural science study programmes was taught as one three year long course, was divided into five courses organized around and built up from the following areas:

⁶ Paper 4, see pages 149-244.

arithmetics; algebra; geometry; theory of probability; statistics; theory of functions; trigonometry; differential and integral calculus; and differential equations (Skolverket, 2000). In the present curriculum, due to a reform in 2000, there are 17 national programmes and seven mathematics courses in the mathematics syllabuses. The basic course structure is summarised in figure 1.1., where the different indentations indicate which courses normally are studied during the 1st, 2nd, and 3rd year respectively.

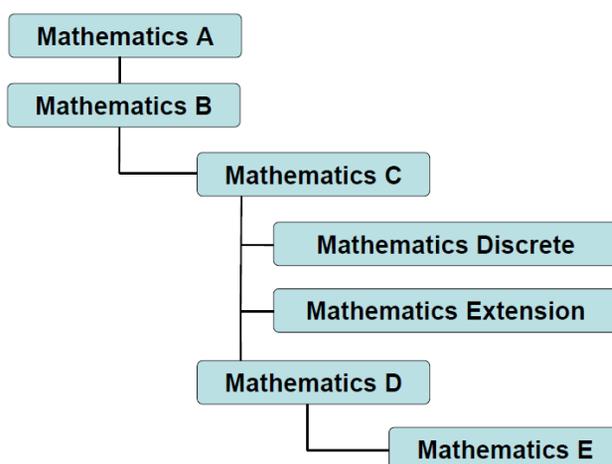


Figure 1.1. The basic structure of the mathematics courses in the Natural science and Technical programmes in Swedish upper secondary school.

Normally, in preparatory programmes for university studies such as the Natural science or Technological programme, *Mathematics A* and *B* are studied during the first year of secondary training, *Mathematics C* and *D* during the second year, and the rest during the third year. However, local variations occur, and the two optional courses, *Mathematics Discrete* and *Mathematics Extension*, are studied after the *Mathematics C* course, but not necessary before the *Mathematics D* and the *Mathematics E* course. For admission to the science and technical university programmes all universities require at least *Mathematics D*, but at some universities *Mathematics E* is the threshold course⁷.

⁷ For the academic year 2009/2010 a look [in October 2009] at the information from web pages of the universities and schools of technology offering Master of Engineering programmes, the following universities demanded *Mathematics E* for entrance: *Chalmers*, *Lund University*; and *KTH Royal Institute of Technology*. On the other hand *Blekinge Institute of Technology*; *Karlstad University*; *Linköping University*; *Luleå University of Technology*; *Mid Sweden University*; *Mälardalen University Sweden*; *Umeå University*; and *Uppsala Universitet* all had *Mathematics D* as the threshold course of admittance.

The extensiveness of the courses is indicated by the amount of so called *credits at upper secondary school*⁸ ascribed to the courses. A three year national programme is comprised of 2500 credits which approximately correspond to 25 credits per week. *Mathematics A, C, and D* are all 100 credit courses and *Mathematics B, E, Discrete and Extension* are 50 credit courses.

The upper secondary students receive a grade on each mathematics course they take ranging from, here presented in decreasing order with the used Swedish abbreviation given in parentheses, *Pass with special distinction* (MVG, Mycket Vål Godkänd), *Pass with distinction* (VG, Vål Godkänd), *Pass* (G, Godkänd), and *Fail* (IG, Icke Godkänd).

1.1.3 Personal background

How did I end up doing a PhD in mathematics education? Well, as far as I can recall, when started my upper secondary education I wanted to become a Master of Engineering focusing on electronics, computers and computing, and hence I chose to follow the Technical programme which in its third year had a ‘low voltage’ profile. However, during my upper secondary years something happened and I am not quite sure what. As most teenagers I had many things going, but at this time music was the passion in my life. Most of my spare time I spent either playing the clarinet or alto saxophone in different orchestras and bands (or in the garage, practicing) with the dream to one day become a professional musician.

In school however, mathematics was and had always been my best school subject, both in the sense that I enjoyed it, it came easy to me, and that I was rather successful (especially compared to subjects like Swedish or English which I had to struggle a lot with). In my secondary years, my mathematics and physics teacher (as well as form teacher) also inspired, encouraged, and urged me to study more mathematics, which I eventually did.

In the spring of 1994, after two years as a semi-professional musician in different bands of the Royal Swedish Army, I took a halftime 10 point course⁹ doing some basic calculus, linear algebra and statistics at Högskolan in Jönköping taught by Dan Andreasson, which was an extremely valuable experience; I failed my first mathematics exam ever which made me a more humble student and to realise that I needed to put effort into my studies. More importantly, much thanks to Dan’s entertaining, clear and inspiring lectures I also came to terms with the fact that I still had a passion for mathematics! In the autumn of 1994 I had to choose between going to Växjö to study to be a mathematics/physics upper

⁸ This is the official English translation (Utbildningsdepartementet, 2003) of the Swedish ‘*gymnsiepoäng*’.

⁹ At that time *1 point* supposed to correspond to one week of full-time study (40 hours). *Halftime* means that the course is spread out over 20 weeks instead of the normal 10 weeks.

secondary teacher, or to go Linköping University and begin the *Mathematics programme*. With friends already in Linköping and good prospects for keeping playing music in different constellation I luckily ended up in Linköping.

After five and a half years of fulltime studies of mathematics, physics and teacher training courses, including one year at the Technical University in Vienna, I finished my master in mathematics as well as my teaching diploma in mathematics and physics for the upper secondary level in 2001, and applied for a PhD position in mathematics, which I got. In 2004 I presented and defended my licentiate thesis in (applied) mathematics with the title “Conformal Einstein Spaces and Bach tensor generalizations in n dimension” and later got the opportunity to change the focus of my research to mathematics education under the supervision of Christer Bergsten. He suggested that I should look into the situation at the Swedish upper secondary school with respect to mathematical models and modelling. What I found when I unprejudiced started to pursue and investigate this suggestion closer really caught my interest and raised many questions. I was so intrigued that I readily decided that mathematical models and modelling at the Swedish upper secondary level should be the topic for my PhD studies.

When I changed my research focus from mathematics to mathematics education I in a sense went from working *within* and exploring a mathematical model (general relativity) modelling the cosmos, to working with the concepts of mathematical models and modelling more generally.

1.2 Overall aim of the thesis

In general terms the objective of this thesis can be formulated as follows. It aims to extend and deepen our knowledge about how mathematical models and modelling is, has been, and could be viewed, taught and learned in Swedish upper secondary mathematics education. However, this vast, broad and general formulated objective must naturally be delimited, transformed and made into operationalisable aims, and each of the five papers and reports contained in this thesis has its own more specific aims. Taken together however, these aims of the included papers and reports can be seen as informing and addressing the following *main aim* of the thesis:

The main aim of this thesis is to investigate how mathematical modelling as prescribed in the upper secondary mathematics curriculum can be implemented in the existing teaching practice and to identify which challenges and barriers that are connected to such an implementation process.

This main aim addresses what can be called *the implementation probématique*, which is the question of how to realise what is prescribed in the written curriculum documents with respect to mathematical models and modelling in the

existing mathematics classroom practice. The main aim can be seen as containing two interrelated components: firstly, *a design/product part*, which focus on *how*, in what way, mathematical modelling can be implemented in the existing teaching practice at the upper secondary level in line with the present governing mathematics curriculum; and secondly, *a process part* focusing on the process of doing the actual implementation at this school level.

However, before these two aspects of the main aim can be addressed, the relevant background and framing of the present situation with respect to mathematical models and modelling at the Swedish upper secondary level as indicated in some of the question presented in the problématique must be established. In other words, it is necessary to get an overview of the past and present state and status of mathematical models and modelling in Swedish upper secondary mathematics education situating and providing perspectives in which to understand the main aim of the thesis.

1.3 Research questions

The overall research question (*RQ*) studied in this thesis directly addresses the main aim described above and can be formulated as follows:

RQ. *How can mathematical modelling as prescribed in the upper secondary curriculum be implemented in the existing teaching practice and which types of barriers and challenges can be identified in relation to the implementation process?*

In the discussion of the main aim it is argued that the context in which *RQ* is addressed must be made clear and to specify this context the following sub-question (*SQ*) is addressed:

SQ. *What is the historical and present state and status of mathematical models and modelling in Swedish upper secondary mathematics education?*

However, regarded as research questions *RQ* and *SQ* are both of a quite general nature, and each of the five papers included in the thesis addresses them using more precise research questions. Nevertheless, starting with the *SQ*, this general question can be seen as constituted by following three sets of sub-questions:

- SQ 1.* *What is the historical and present state and status of mathematical modelling in the governing documents (syllabus) in mathematics for the Swedish upper secondary school? What is written in the governing documents? What could be said about the evolution over time between 1965 and 2000?*
- SQ 2.* *What happens in the mathematical classroom with respect to mathematical modelling? What are Swedish upper secondary mathematics teachers' beliefs about mathematical modelling?*

SQ 3. What do the students know and learn about mathematical modelling in Swedish upper secondary mathematics education?

These three sets of sub-questions focus on different levels of curricula, and this will be elaborated on in the methodology chapter. *SQ 1* is the focus of Paper 4; *SQ 2* is addressed in Paper 1 and partly in Paper 2 and 3; and, in Paper 3 focus is on the questions in *SQ 3*. Taken together, the answers to *SQ 1 – SQ 3* provide important aspects of the background and context for the question *RQ*.

The overall research question *RQ* can also further be specified and split up into sub-questions. However, to be able to present these sub-questions as precise as possible some notion and vocabulary needs to be introduced. The precise form of the four sub-questions emerging from the overall research question *RQ* will be given when this have been done in the methodology chapter.

1.4 Structure of the thesis

The thesis consists of five papers and reports together with this preamble (or as we call it in Sweden, ‘coat’). The results from the five papers and reports will be structured and discussed in relation to three out of four defined different curriculum levels of the Swedish upper secondary school. This curriculum framework is defined and elaborated in chapter two, where also the overall methodology of the thesis is presented. In chapter three follows a résumé of a non-exhaustive selection of the literature on mathematical models and modelling, before the five papers are briefly summarised in chapter four and finally discussed, in relation to each other and the presented background, in chapter five. The thesis ends with a few suggestions about the significance of the research results presented in this thesis and how the work here initiated could be continued and developed further.

Chapter 2

Methodology

Burton (2005) argues that researchers in mathematics education in general pay little or no attention to explaining and motivating the rationale for the actual research design they apply to be able to draw the conclusions they report on when writing up their research. In Burton's opinion, accounts of research is full of answers to *how* results were obtained, whereas answers related to *why* choices were made and decision taken to be able to arrive at the conclusions rarely are found. The *how*-question concerns the "*methods* used by the researcher to undertake their research" (p. 1, italics added), and the *why*-question focus on the rationale for the research design, the *methodology*. This distinction between *method* and *methodology* is an important one, and that more emphasis should explicitly be put on the methodology is also put forward by Wellington (2000); he describes methodology as "the activity or business of choosing, reflecting upon, evaluating and justifying the methods you use" (p. 22) and argues that it is necessary to know the methodology of a piece of research to be able to impartially judge and assess it. Ernest (1998a) argues along the same line and writes that "[e]ducational-research *methods* are specific and concrete approaches. In contrast, educational-research *methodology* is a *theory* of methods – the underlying theoretical framework and the set of epistemological (and ontological) assumptions that determine a way of viewing the world and, hence, that underpin the choice of research methods." (p. 35, italics in original). However, what to be understood by a 'theory' or a 'theoretical framework' must evidently be elaborated and specified.

I agree with Burton (2005) in that "I do not believe that there is ever a case where the researcher's beliefs, attitudes, and values have not influenced a study, nor do I believe that it is possible for a researcher ever to assume that values can be assumed as shared within a 'scientific community'" (p. 3), and hence I will try "to be clear to [myself] about the values, beliefs, and attitudes that are driving the study that [I] propose to do and to make that clarity visible to the reader" (p. 4).

2.1 Considerations of philosophical nature

As a part in trying to answer Burton's *why*-question, and in line with Ernest (1998a) argument, I will briefly account for the ontological and epistemological foundation on which this work rests. To address issues concerning ontological and epistemological matters is in my opinion very interesting, intriguing, relevant, but above all, hard. Ernest (1998a), drawing on Thomas Kuhn's book *The Structure of Scientific Revolutions*, propose to use the notion of "overall theoretical perspective or paradigm" (p. 32) to denote the underlying assumptions about knowledge, the world and what exists in the world (i.e. epistemology and ontology) together with the methodology. The discussion provided by Ernst contrasts the three paradigms *the interpretative* (which Ernest also refers to as *the qualitative*), *the scientific* (or *positivist*), and *the critical-theoretical*, which is the same division of research paradigms contrasted in Cohen, Mainon and Morrison (2000). When paradigms are discussed and contrasted, the comparison is usually made between the positivist and the interpretivist tradition (e.g. Bassey, 1999). On a very rudimentary level research carried out in an interpretive paradigm "is predicated on the view that a strategy is required that respects the differences between people and the objects of the natural sciences and therefore requires the social scientist to grasp the subjective meaning of social action." (Bryman, 2004, p. 13). Interpretive research and positivistic research are sometimes referred to as interpretative respectively normative, and according to Cohen, Manion and Morrison (2000), research carried out in a normative paradigm is typically positivistic and uses natural sciences methods to investigate rule-governed human behaviour in terms of existing or from outside forced upon theories, whereas in research in an interpretive paradigm "[i]nvestigators works directly with experience and understanding to build their theory on them" (p. 23).

During the first half of my PhD studies doing relativity I must confess myself to fit the description of 'the working mathematician' given by Davis and Hersh (1981): "the typical working mathematician is a Platonist on weekdays and a formalist on Sundays. That is, when he is doing mathematics he is convinced that he is dealing with an objective reality whose properties he attempting to determine. But then, when challenged to give a philosophical account of this reality, he finds it easiest to pretend that he does not believe in it after all." (p. 321). As a consequence, I had a 'positivistic legacy' when I change the focus of my PhD studies and started to engage in mathematics education research. However, due to extensive reading and coursework (especially the courses I had the privilege and opportunity to attend at the University of Agder, Kristiansand, Norway arranged under the auspices of NoGSME¹⁰) I rather soon found my self an interpretivist and hence the research in this thesis has been carried out in a interpretive paradigm.

¹⁰ Nordic Graduate School of Mathematics Education

2.2 Conceptual frameworks

According to experts in the community of mathematics education the use of *research frameworks*, *theories* and *philosophical foundation* are crucial aspects to consider when engaging in research activities (Lester, 2005). However, these notions are ambiguous and for instance Niss (2007a) notes that “it is not clear at all what ‘theory’ actually means in mathematics education. Nor is it clear where the entities referred to as theories invoked in mathematics education come from, how they are developed, what foundations they have, or what roles they play in the field.” (p. 97). In this section I will give my interpretation of the notion *conceptual framework* which I use to describe the tools I used in my research presented in this thesis. I do this by taking Lester’s (2005) discussion about research framework as point of departure.

Drawing on the online *Encarta World English Dictionary*, which defines a *framework* as “a set of ideas, principles, agreements, or rules that provides the basis or outline for something intended to be more fully developed at a later stage”, Lester (2005) picture a research framework as a construction *scaffold*, a basic structure, that (1) *provides a structure for conceptualizing and designing research studies*, which facilitates to determine the nature of questions asked, to formulate questions, relate involved concepts and their relations, and to make justification procedures plain; (2) *enables you to make sense of data*, data per se do not provide any information¹¹; (3) *allows us to transcend our common sense*, making it possible for us to discern and identify important problems and issues underlying the phenomena studied; and related to this last point, (4) *enables us to gain deeper understanding* by guiding and framing our research designs, research questions, methods, data interpretation and how to justify our conclusions.

Looking at figure 2.1 which presents a schematic representations of a typical research process, it is obvious that an adequate research framework imbues the whole research process, and may facilitate both the processes (represented by the arrows) and the formulation of the six ‘stages’ (represented by the rectangles) in such an endeavour.

Lester (2005) continues to discuss advantages and problems with three types of research frameworks; *theoretical*, *practical* and *conceptual frameworks*. A *theoretical framework* extensively uses and draw on what Lester calls ‘formal theories’, a notion that Lester however not defines, but provides the example of Piaget’s theory of intellectual development. *Practical frameworks* are constituted by accumulated practice knowledge, ‘what works’, and are to some extent is the antithesis compared with the theoretical frameworks. The *conceptual framework*

¹¹ The validity of this statement depends on ontological and epistemological considerations and choice. For example, from a realist perspective ‘data’ is all there is, but from a constructivist perspective data is constructed; I confess my self to the latter.

could be described as a mixture of the previous two, drawing on both theories and practical knowledge with a focus on justification, addressing Burton's *why*-question, rather than explanation.

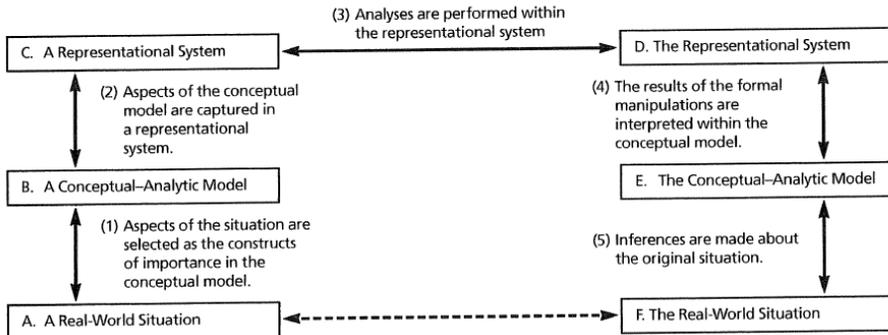


Figure 2.1 “A schematic representation of the process of conducting empirical research” (Schoenfeld, 2007, p. 73)

Niss (2007b) on the other hand talks about *investigational frameworks* which he argues in general terms at least consist of the following three components: (a) a *perspective* on what is being researched; (b) *theoretical constructs* which are in line with the perspective in (a); (c) some preferred *methods* using/involving the constructs of (b) addressing the issues in (a). Although Niss does not specify in detail what he intends with the notions ‘perspective’ or ‘theoretical constructs’, he concludes that the investigational frameworks in mathematics educational research have become more numerous and increasingly complex.

Also Cobb (2007) argues for the advantages of using conceptual/investigational frameworks and he suggests “that rather than adhering to one particular theoretical perspective, we act as bricoleurs by adapting ideas from a range of theoretical sources” (p. 29).

Both Niss’ investigational framework and Lester’s conceptual framework are ways to describe the construction and function of what tools researcher build, develop and apply for different purposes with different aims when conducting research (e.g. figure 2.1). In the construction of this ‘scaffold’ the addressing of Burton’s *why*-question is crucial for the trustworthiness of the research (Schoenfeld, 2007). In my research I consider myself to be a bricoleur, building a scaffold, a conceptual framework, that will help me to make sense and meaning to the whole research process I engage in.

I agree with Lerman (2006), and think that divergence and multiplicity in the theories used to investigate the phenomena we are researching can be very productive and is a fruitful path to extend our understanding. In addition, I also believe that taking this perspective towards doing research, is one possibility to address and better come to terms with the issue noted by Silver and Herbst (2007), that for mathematics education in general “theory, research, and practice in mathematics education should exist in a more harmonic relation that has been the case to date” (p. 40).

Regarding the use of quantitative and qualitative methods, the previous prevailing methodology perspective has been that “[q]uantitative research methods have grown out of scientific search for cause and effect expressed ultimately in a grand theory” (Stake, 1995, p. 39) and hence are ‘only’ suitable for research done in a positivistic tradition, whereas “[t]o the qualitative scholar, the understanding of human experience is a matter of chronologies more that cause and effect” (Stake, 1995p. 39) making quantitative methods ‘only’ suitable for research carried out in an interpretative paradigm. I agree with Schoenfeld (2007) who argues that this separation between the two different types of research methods is artificial. The critique presented is that qualitative and quantitative research methods are intertwined so that one really can not have one type of research without some element of the other, and that maintaining a strict distinction between the two counteract and restrain creativity and innovation in research design (Bryman, 2004; Cohen et al., 2000; Gorard, 2001; Pring, 2004). From my perspective as a bricoleur I see no problem in using qualitative and quantitative research methods along side each other; they rather complement each other, provide perspectives, and even possibly strengthen conclusion providing triangulation.

2.3 Rationale for the overall research design of the thesis

When I started to read research papers and reports on different aspects of mathematical models and modelling in mathematics education, I was struck by the fact that practically nothing was written about the past and present situation in Sweden. I found this surprising, frustrating, but also very intriguing. Mathematical models and modelling were however mentioned in the passing in research focusing on (mathematical) problem solving (e.g. Håstad, 1978; Wyndhamn, 1997; Wyndhamn et al., 2000, just to mention some examples) and in specialised studies by Lingefjärd, Lingefjärd, and Holmquist, and Palm respectively, as mentioned in the introduction chapter.

In the initial phase of my research, it was not clear and obvious to me that I should engaged in research involving aspects of actual upper secondary classroom practice. Rather, my initial research plan was more or less theoretical in the sense

that it suggested to study and analyse different types of texts; upper secondary syllabuses and curricula; mathematical textbooks; and, national tests and students' achievements on these¹². Table 2.1 gives an overview of my first research plan from 2006 which had as a central element an upcoming curriculum reform in 2007, Gy07. The idea was to study the effect and influence of this reform at three different curriculum levels¹³ with respect to the notions of mathematical models and modelling, and to look at the didactic transposition (Chevallard, 1991; Bosch & Gascón, 2006) of these notions at the upper secondary educational level. At this stage, the research questions proposed to be addressed were very similar to the questions SQ and SQ 1 – SQ 3 presented in section 1.3. Although the majority of the data I planned to analyse were different kinds of texts, there was an empirical element at the potentially implemented curricula level for the Lpf94 curriculum, which aimed to investigate teachers' beliefs about mathematical models and modelling.

Table 2.1 Overview of my research plan as presented at the NoGSME summer school in Dømmesmoen, Norway, 12-17 June, 2006.

	Time, curricular		
	Lgy70	Lpf94	Gy07
Intended Curricula	X	X	X
(Potentially) Implemented Curricula	X (textbooks)	X (textbook, new empirical data)	X (textbooks)
Attained Curricula	X (use of "central" tests)	X (use of national tests)	X (use of national test)

Nevertheless, the research plan developed, partly due to the fact that the planned curriculum reform Gy07 was revoked because of a change of government. The basic idea to look at the didactical transposition however

¹² This is perhaps not surprising considering that I was coming from a research tradition in mathematics, which, in my case anyway, made me more comfortable with the idea to do 'desk-research', than to enter the messy and complex world of schools and classrooms.

¹³ These were the *intended curriculum level* (what should be taught), the *potentially implemented curriculum level* (what is in textbooks and other teaching materials along side with teacher intentions on what to expose the students to in the classroom); and the *attained curriculum level* (what students learn). All these will be elaborated and explained in more detail in section 2.4.

remained and it was decided to include all the curricula reforms since 1965¹⁴ to get a more complete picture, but to have a main emphasis on the curriculum from 2000. Still, the research questions remained in principle the same and the empirical element was also still included, see table 2.2.

Table 2.2. Overview of my research plan as presented at the NoGSME summer school in Laugarvatn, Iceland, 4-11 June, 2007

Didactical transposition of mathematical modelling	Time, curricular					
	1965	1970	~1975	1981/82	1994	2000
	Läroplan för gymnasiet	Lgy70	Suppl. (not official)	Suppl.	Lpf94	Suppl.
Intended Curricula	X	X	X	X	X	X
(Potentially) Implemented Curricula (textbooks)	X (textbooks)	X (textbooks)	X (textbooks)	X (textbooks)	X (textbooks)	X (textbook, new empirical data)
Attained Curricula	X (use of "central" tests)	X (use of national test)	X (use of national test)			

Gradually I got more and more influenced by the courses I attended and the literature I read. This naturally gave me another understanding of the field and what doing research in the field of mathematics education could be about. Gradually I felt an increasing interest to more actively involve actual classroom practice in my work; to, as Pring (2004) puts it, make the research more *educational*. As a consequence, the emphasis on didactical transposition was in principle abandoned, and the focus of my research ended up concentrating on the first row and the last column in table 2.2 manifesting itself in the research questions *SQ* and *RQ* respectively.

2.3.1 Conceptual framework components drawing on the SQ

When I realised that the background literature on mathematical models and modelling from a Swedish perspective was sparse, it became natural to include the provision of part of such a background as part of the aims of my research. My readings soon lead me to three papers which inspired me and eventually helped me to provide the initial structure I used to conceptualise my work. The papers were *Applications and modelling in mathematics curricula – state and trends* (Niss, 1987), *Aims and Scope of Applications and Modelling in Mathematics Curricula* (Niss, 1989), and *Applied Mathematical Problem Solving, Modelling,*

¹⁴ In the reform of 1965 the upper secondary educational system was reformed more or less into its present form.

Applications, and Links to Other Subjects: State, Trends and Issues in Mathematics Instruction (Blum & Niss, 1991). All these survey papers focus on just such aspects of mathematical models and modelling which I hoped to find on the situation in Sweden. They enabled me to manoeuvre and delimit a well-defined research topic and corresponding equally well-defined and researchable questions. The three papers lead me to use a curriculum framework providing the basic structure for my whole study as part of my conceptual framework. This curriculum framework will be elaborated in section 2.4, and it is in addition this framework I am using to discuss the results presented in this thesis.

2.3.2 Conceptual framework components drawing on the RQ

The shift in focus of my research from the ‘theoretical’ to the more ‘practical’ as outlined above grew stronger as time went on. The formulation of the main aim of this thesis is a result of this process. In this section I will provide an abridged version of the methodological consideration and the argumentation of its consistence, in order to introduce a vocabulary to adequately formulate the more specified research sub-questions to the *RQ* studied. However, a full account will not be given here; this is done in section 3 in Paper 5.

My point of departure was that I wanted to take the existing classroom practice seriously, and out from these given premises investigate the notions of mathematical models and modelling. In particular, I wanted to see how it could be possible to work with these concepts in line with the present curriculum in the daily practice. During the initial phase of this part of the research I was taking courses at the University of Agder, Kristiansand, Norway arranged under the auspices of NoGSME, and through these I got my first encounter with Design-Based Research (Barab & Squire, 2004; Sandoval & Bell, 2004; The Design-Based Research Collective, 2003), cultural historical activity theory or CHAT (Engeström, 1987; Goodchild, 2007; Jaworski & Goodchild, 2006; Roth & Lee, 2007), and different forms of researcher-practitioner relationships (Jaworski, 2003; Wagner, 1997). These three ideas (together with some other influences) all came together in the so called LCM¹⁵ Project, a research project based at the University of Agder. In short, the LCM Project aimed to develop and study *communities of inquirers* consisting of groups of teachers and didacticians in a co-learner partnership. The primary objective was to “design and study classroom activity that is inquiry-based. Here, inquiry is seen as a design, implementation and reflection process in which teachers should be central.” (Goodchild & Jaworski, 2005) referring to (Jaworski, 2004). Inspired by this project and my reading of the mentioned literature above I decided to design my research study in the same spirit. The decision to use design-based research provides a possibility to address the question of *how* it could be possible to work with the notions of

¹⁵ Leaning Communities in Mathematics

mathematical models and modelling in line with the present curriculum in the daily school practice focusing on (1) *what* to use/work on in the classroom, specifying meaning and content related to the two notions; (2) the designing of the ‘what-material’ in (1), the *process* of producing teaching and teaching material mediating and realising (1); and (3) how this material works in the classroom, to see and evaluate how the *product* designed in (2) functions in a real mathematics classroom. In addition, the design-based methodology advocates and incorporates equal and close partnership of collaboration between the involved researcher and participants as a central feature. I believe such collaboration would facilitate the recruiting of teachers to participate in the research as well as make the research founded in, and relevant for, the existing teaching practice. To describe and conceptualise the research, CHAT is used to provide a language of description (Dowling, 1994) and to function as a diagnostic and analytic tool. In my opinion, CHAT is a flexible framework that can be adjusted and applied to incorporate the complexity of the research, so that it; acknowledges the importance of social interaction; includes institutional factors; offers a way to conceptualise the affects of the introduction of new concepts; and, focuses both on processes and products.

From my aim; the influences from design-based research, CHAT, and researcher-practitioner frameworks; my understanding of the situation at the upper secondary school, generally and specifically concerning mathematical models and modelling, I formulated five *guiding principles* to facilitate and support the research process. These are not to be regarded as disjoint in nature but rather overlapping and connected in an intertwined way. The guiding principles were (quote, see Paper 5, pp. 278-279):

- GP1.** The research should be as naturalistic as possible in the sense that
- it should be carried out at the participating teachers’ schools and within their practice;
 - the teachers’ ideas and initiative should be given priority;
 - my role in the implementation of the modelling modules should be kept at a minimum, preferably only involvement in connection to the collection of data.
- GP2.** The research should be of collaborative nature, where the participating teachers and I as a researcher should work together on equal footing.
- GP3.** The participating teachers should experience the research as meaningful and useful (first and foremost for their own account, secondly on behalf of their students, and thirdly with the least priority, for me and my objectives).
- GP4.** The modelling modules should be in line with the present curriculum, meaning that the mathematical content in the modules should be what is prescribed in the course syllabuses respectively.
- GP5.** The modelling modules should be small, so that they do not mess up the teachers’ ‘normal’ practice (teaching and other responsibilities).

Given these five guiding principles the *RQ* can now be formulated in terms of four sub-questions and this is done in section 2.6.

2.4 Curriculum framework

Romberg (1992) defines curriculum as an "*operational plan* for instruction that details what mathematics students need to know, how students are to achieve the identified curricular goals, what teachers are to do to help the students develop their mathematical knowledge, and the context in which learning and teaching occur" (p. 749, italics in original). There are many (conceptual) frameworks used in educational research focusing on, and capturing, different aspects of such a curriculum, its realization, and the outcomes of it in terms of students learning.

In line with the inspiring paper by Niss and Blum (1991), which among other things discusses mathematical modelling in terms of goals, implementation, and assessment, I wanted to find a curriculum framework that mirrored these 'levels' of description. I found that the most straightforward and suitable framework to be the IEA curriculum framework which in addition had the benefit to be well documented, tested, known, and widely used.

2.4.1 The curriculum framework of TIMSS

The *IEAs*¹⁶ studies, and especially that latest TIMSS¹⁷ study, are permeated by the idea that curriculum is one of the most central and important variables in trying to understand and explain differences in students' performance due to national differences. The conceptual model of curriculum originally used by IEA was a framework consisting of the three levels *intended*, *implemented* and *attained curriculum* (Robitaille et al., 1993; Travers & Westbury, 1989).

The intended curriculum is the content matter which is defined by the authorities in an educational system. Here, authority can be on a national level or on a more local level depending on the country and the level of the educational system in question. The content matter specified in the intended curriculum may be defined and described in terms of concepts, processes, skills or competencies, and attitudes which the students are expected to study and assimilate during their schooling. According to Robitaille et al. (1993) "the intended curriculum is embodied in textbooks, in curriculum guides, in the content of examinations, and in policies regulations, and other official statements generated to direct the educational system" (p. 27) and is society's principle lever to manifest and influence different aspects of the students' possibilities and opportunities to learn

¹⁶ the International Association for the Evaluation of Educational Achievement

¹⁷ the Third International Mathematics and Science Study

in an educational system (W. H. Schmidt & Houang, 2003). Notable influences on this systemic level of the curriculum are the society's goals and expectations in terms of participation rates of students and mediated values to them; the resources made available; as well as the expectations and status of the practitioners working within an educational system in relation to society as a whole (Robitaille et al., 1993).

The next level, *the implemented curriculum*, is the view and interpretation of the content that the teacher makes available to the students in the classroom. This level is affected by institutional frames such as the teaching praxis; how the classroom is organised; to what extent different kinds of recourses are used; and, the teachers' background and attitudes. Similar to the case with the intended curriculum, the implemented curriculum is influenced by the society in large, but conditions and requirement due to social, cultural and/or economic concerns and conditions on a more local level might have a notable affect.

The third level, *the attained curriculum*, is what the students de facto learn as a result from their school going. This is not affected only by the implemented curriculum, but also by how much time the students spend on studying at home, their diligence, how the students act and functions in the classroom and so on. In other words, the attained curriculum should be related to the background of the students' personal and social situation.

The mutual relationship between the intended, the implemented and the attained curriculum and the respective overall general social contexts as described by Robitaille et al. (1993) is depicted in figure 2.2 and is often referred to as the 'IEA tripartite model'.

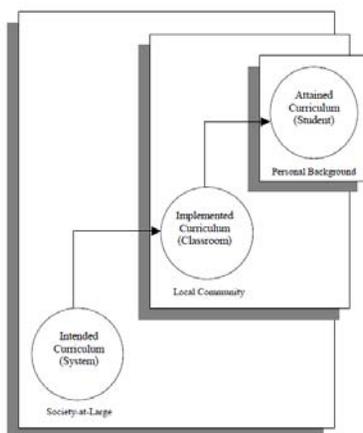


Figure 2.2. The conceptual framework for TIMSS, the IEA tripartite model (Robitaille et al., 1993, p. 26)

However, in the later IEA studies the framework presented in figure 2.2 was developed and an intermediate level between the intended and the implemented curricula was introduced, the so-called *potentially implemented curriculum* (W. H.

Schmidt et al., 2001; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). This curriculum level is in principle constituted by the textbook used in the classroom, which originally was viewed as belonging to the intended curriculum but, with reservation for variations in different national traditions and policies of course, also “can be thought of as representing the implemented curriculum since they are employed in classrooms to organize, structure, and inform student’s learning experiences” (W. H. Schmidt & Houang, 2003, p. 983). The use of the word *potential* in the label of this curriculum level refers to both the uncertainties of how well a textbook actually represents the intended curriculum on the one hand, and how the practicing teacher chooses to make use of it in the classroom, the implemented curriculum, on the other hand. The modified IEA framework is illustrated in figure 2.3, and has been used in research also outside the IEA context by for instance Johansson (2006) to study the mathematics seventh grade textbooks as part of the potentially implemented curriculum in Swedish compulsory school¹⁸.

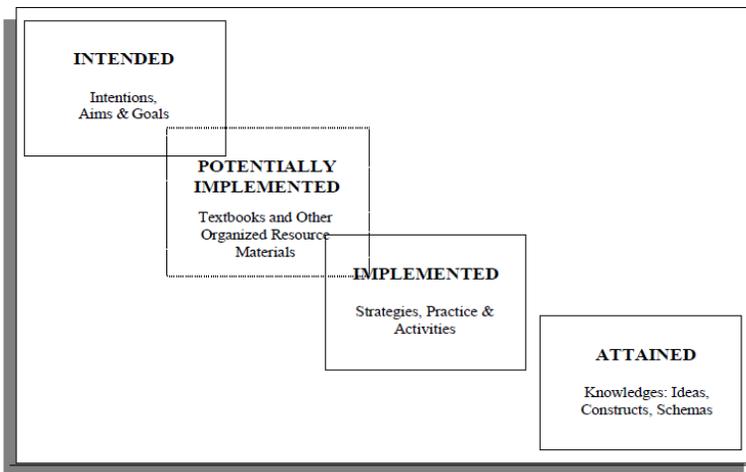


Figure 2.3. “Textbooks – The Potentially Implemented Curriculum”
(W. H. Schmidt, McKnight, Valverde, Houang, & Wiley, 1997, p. 178)

2.4.2 Other curriculum frameworks

Researchers have also developed and used other curriculum frameworks which to some extent often include aspects of the IEA framework levels *intended*, (*potentially*) *implemented* and *attained curriculum*. Bishop (2001), researching

¹⁸ See also Johansson (2003) for a nice overview of these curriculum levels and a discussion of these in relation to the Swedish context.

values in mathematics teaching¹⁹, discusses two possible extensions of the IEA framework present in the literature. The first extension introduces a framework which adds two intermediate levels between the intended and the implemented levels, and the implemented and the attained levels respectively: *the intended curriculum, interpreted by the teacher*; and, *the implemented curriculum, as interpreted by the students* (p. 239). To this framework one can see parallels to the ideas of theory of didactic transposition, although the focus of the latter is at the institutional level and goes “beyond individual characteristics of the subjects of the considered institutions” (p. 55). The second framework discussed by Bishop, which he argues could be used for constructing value revealing activities for teachers or the analysis of teachers values, pinpoints teachers’ views of *aims* (intended curriculum), *means* (implemented curriculum), and *effects* (attained curriculum) in terms of the *declared curriculum*, the *de facto curriculum*, and the *potential curriculum*. The three latter capture what the teacher states, exhibits (in class or otherwise), and the teachers (positive) developmental potential that can be discerned respectively (pp. 242-243).

Porter and Smithson (2001), developing and studying so called curriculum indicators, use a curriculum framework which distinguishes between what they call the *intended, enacted, assessed, and learned* curricula. Their *intended curriculum* coincide with the IEA framework definition, however, the *enacted curriculum* “refers to the actual curricular content that students engage in the classroom” (p. 2). Note the difference between the *enacted curriculum* and the IEA’s *implemented curriculum*, where the former focuses on what the students engage in the classroom, whereas the latter focuses on what is implemented in the classroom. Ideally, the *assessed curriculum* is in perfect alignment with the *intended curriculum*, and Porter and Smithson’s motivation for using this curriculum level is to capture potential discrepancies between what is assessed and the *intended curriculum*. Finally, the *learned curriculum* is a part of the framework to mirror what the students actually learn, which may be more or less related to the other levels of curriculum in the framework, and is analogous to IEA’s attained curriculum level.

2.4.3 Adopted and applied curriculum framework

In this thesis I will try to illuminate different aspects of mathematical models and modelling related to different Swedish upper secondary curricula levels. I will do this using slightly different conceptualisations and definitions of the intended, potentially implemented, implemented, and attained curricula respectively than the frameworks accounted for above. The reason for this is that the rationale for me to use a curriculum framework is different from the reasons invoked in the

¹⁹ In this paper Bishop defines *values in mathematics teaching* as “deep affective qualities which teachers promote and foster through the school subject of mathematics” (Bishop, 2001, p. 239)

work cited in the previous two sections; the motivation of the IEA is to find a framework which is operationalisable and provides measurable entities; Bishop focus on values; and Porter and Smithson's interest is in curriculum indicators.

For me on the other hand, the framework provides the basic structure to think about how different actors within the Swedish upper secondary school system express their views on the notions of mathematical models and modelling. The principle deviation from the IEA framework in my interpretation is in the definition of the potentially implemented curriculum.

In the present study I draw the IAE curriculum model as presented in figure 2.2 and define the *intended curriculum* level to be constituted by all the written curricula documents governing the upper secondary mathematics education. However, since the word *curriculum* could be understood in different ways, and due to the fact that there are some ambiguities regarding the translation into English of the corresponding Swedish word 'kursplan', which sometimes is translated to 'syllabus' and sometimes to 'curriculum', one has to be careful when navigating in the literature.²⁰

The *potentially implemented curriculum*, which in the IAE curriculum model is dominated by the used textbook(s) in the classroom, is in this study extended to also include what in the IAE framework is expressed as influencing the implemented curricula in terms of teachers' backgrounds, values and attitudes. The motivation for this is the way teachers' knowledge, beliefs and affects towards the learning and teaching of mathematics influence and relate to their practice is a highly active field of research (Philipp, 2007). For instance, Thompson, acknowledging the dialectic nature between beliefs and practice, argues that "[t]here is support in the literature for the claim that beliefs influence classroom practice; teachers' beliefs appear to act as filters through which teachers interpret and ascribe meanings to their experience as they interact with children and the subject matter" (Thompson, 1992, p. 138-139). In addition, the six authors of the chapters on teachers' beliefs in the book edited by Leder, Pehkonen and Törner (Leder, Pehkonen, & Törner, 2002) all infer a strong link between teachers' belief and their practice, working from a premise that could be expressed by "to understand teaching from teachers' perspectives we have to understand the beliefs with which they define their work" (Nespor, cited in Thompson, 1992, p.129). Although reviews of research on different aspects of beliefs in connection to mathematics knowing, teaching and learning often conclude that there is a great degree of variation of the involved concepts and their meaning used by different researchers (Leder et al., 2002; Pajares, 1992; Philipp, 2007; Thompson, 1992), I have chosen teachers' beliefs, values and attitudes to be part of the potentially

²⁰ For a precise specification of which material that is taken to be 'written curricula documents governing the Swedish upper secondary mathematics education' see Appendix A in Paper 4, pages 218-228.

implemented curriculum to capture the teachers' expressed opinions on what they should or want to bring to the mathematics classroom.

The *implemented curriculum* on the other hand is what the teacher brings up in the classroom and which content the students are exposed to; this might be more or less in agreement with the potentially implemented curriculum, and my definition of the implemented curriculum coincides with the original definition of the IAE framework.

Concerning the *attained curriculum* I also follow the IAE framework definition.

2.5 Reconceptualising the research questions

Recall the overall research question (*RQ*) studied in this thesis:

RQ. How can mathematical modelling as prescribed in the upper secondary curriculum be implemented in the existing teaching practice and which types of barriers and challenges can be identified in relation to the implementation process?

and the background and context providing question *SQ*:

SQ. What is the historical and present state and status of mathematical models and modelling in Swedish upper secondary mathematics education?

With the adopted curriculum framework the set of questions *SQ 1 – SQ 3*:

SQ 1. What is the historical and present state and status of mathematical modelling in the governing documents (syllabus) in mathematics for the Swedish upper secondary school? What is written in the governing documents? What could be said about the evolution over time between 1965 and 2000?

SQ 2. ~~What happens in the mathematical classroom with respect to mathematical modelling?~~ What are Swedish upper secondary mathematics teachers' beliefs about mathematical modelling?

SQ 3. What do the students know and learn about mathematical modelling in Swedish upper secondary mathematics education?

can be seen to address issues on *the intended*, *the potentially implemented*, and *the attained curriculum* respectively. Note that one of the two questions in *SQ 2* has been deleted compared to the first formulation in section 1.3. The reason for this is simply that this question is strictly speaking not addressed systematically in the sense that actual classrooms are investigated with respect to how mathematical models and modelling are taught. Just to recapitulate: *SQ 1* is the focus of Paper 4; *SQ 2* is addressed in Paper 1 and partly in Papers 2 and 3; and, in Paper 3 focus is on the questions in *SQ 3*.

It is now possible to conceptualise the overall research question *RQ* using the adapted curriculum framework. *RQ* can be seen as to take what is written about mathematical models and modelling in the *intended curriculum* as the point of departure with the objective to design an intervention for the *implemented curriculum*. In this process the *potentially implemented curriculum* is partly studied. In addition, with the introduced notion of the guiding principles *RQ* can now be further specified and split up into the following four the sub-questions:

- SQ 4. How does the collaboration respecting the five guiding principles influence the developmental/designing process and the form the modelling modules take?*
- SQ 5. How do the teachers experience the sequence of designing, implementing and evaluating modelling modules?*
- SQ 6. How are the teachers' attitudes towards mathematical models and modelling changing, if at all, as the project evolves?*
- SQ 7. How do students experience working with the designed modelling modules?*

All the questions *SQ 4 – SQ 7* are investigated in Paper 5. Assuming that the answers provided to the questions above will be communicated and disseminated to the research community of mathematics education, teacher educators, and practicing teachers, I would argue that the answers *SQ 5 – SQ 7* (potentially) affects what upper secondary mathematics teachers bring with them to the classroom. Hence, the just mentioned questions can all be seen to address issues related to the potentially implemented curriculum. *SQ 4* on the other hand addresses meta-issues related to the research design of the study from which Paper 5 reports, and does not fit in the curriculum framework.

Chapter 3

”Terminological issues are mostly tedious and boring. Nevertheless, for a scholarly or scientific discourse to be serious, in fact possible, it is essential that at least the key entities and concepts of that discourse are reasonably clear for those involved. This is particularly true with a field like mathematics education in which transparency and clarity are not easily achieved, let alone to be taken as a matter of course.” (Niss, 1996, p. 12)

Mathematical models and modelling

The Oxford Dictionary of English (2nd edition revisited)²¹ provides the following five interpretations when ‘model’ is entered as the headword entry:

- (1) a three-dimensional representation of a person or thing or of a proposed structure, typically on a smaller scale than the original...
- (2) a thing used as an example to follow or imitate...
- (3) a simplified description, especially a mathematical one, of a system or process, to assist calculations and predictions...
- (4) a person employed to display clothes by wearing them.
- (5) a particular design or version of a product...

and most people can probably relate to all these meanings and understand as well as use them in everyday speech. The word *model* originates from the Italian word *modello*, which in turn originates from the Latin word *mo’dulus*, a diminutive form of *modus*, which translates to *measure* or *size*. In scientific work and debate a *model* is often equated with some sort of representation of an object, a phenomena or and idea. As is evident in the five interpretation of the word *model*

²¹ www.oxfordreference.com, retrieved 20091123

above, it is often the case that one either discusses models as *concrete models* like replicas made in different sizes or illustrations of an idea, or *abstract models* like mental constructions or theories (NE).

A naïve, direct and intuitive meaning of the notion of a *mathematical model* is a model in any of the meanings described above, except in the sense of (4), that are expressed using mathematical nomenclature and syntax. However, to have a scientific discussion and debate it is important to have as clear and precise definitions of the involved concepts and notions as possible. In the case of *mathematical models* and *mathematical modelling* from a mathematics education perspective this is not a trivial matter, and this chapter aims to provide an (non-exhaustive) overview of some of the aspects of the past and present debate.

3.1 Mathematical modelling

Just as the words *models* and *modelling* are found with different meanings and interpretations in very day life, the same is true in mathematics education regarding the notions of *mathematical models* and *mathematical modelling*. For example, Ogborn (1994), quoted in Molyneux-Hodgson, Rojano, Sutherland and Ursini (1999, p. 176), describes modelling in general terms as “thinking about one thing in terms of simpler artificial things”. In mathematics education research these ‘simpler artificial things’ most of the time is mathematical vocabulary and syntax. Lingefjärd (2006), drawing on Swetz and Hartzler (1991) does just this: “Mathematical modeling can be defined as a mathematical process that involves observing a phenomenon, conjecturing relationships, applying mathematical analyses (equations, symbolic structures, etc.), obtaining mathematical results, and reinterpreting the model.” (p. 96). Generally, one can find many different approaches to and perspectives on mathematical modelling in the mathematics education research literature (Blum, Galbraith, Henn et al., 2007; Haines, Galbraith, Blum, & Khan, 2007). The variety of perspectives is illustrated by Sriraman and Kaiser (2006) in their report of an analysis of the papers presented in *Working Group 13: Applications and modelling* at the CERME4 conference²² written by European scholars. They conclude “that there does not exist a homogenous understanding of modelling and its epistemological backgrounds within the international discussion on applications and modelling” (p. 45) and argue and call for a more precise clarification of the concepts involved in the different approaches to make communication and discussions more simple and fruitful. Also other researchers have commented on this issue of definitional character, especially in connection to the consequences for and influences on the teaching and learning of mathematical models and modelling. Hamson (2003) for

²² The 4th conference organized by ERME, European society for Research in Mathematics Education, held in Sant Feliu de Guíxols, Spain, 17-21 February 2005.

example writes that "because the term modelling is open to misuse and misunderstanding it is necessary to sort out what should *actually* be going on in the mathematics classroom." (p. 220, italics in original).

3.1.1 The modelling cycle and modelling competency

Although the notions of mathematical models and modelling are understood and used in ambiguous ways, Kaiser et al. (2006) conclude that research on mathematical modelling in mathematics education typically uses or develops some general description of the process of mathematical modelling. One of the two prevailing general descriptions in the literature is often given or summarised in a so called *modelling cycle*, which schematically and in an idealised way illustrates how the modelling process connects the extra-mathematical world (domain) and the mathematical world (domain) (Blum, Galbraith, & Niss, 2007); for an example see figure 3.1. In this particular example Borromeo Ferri (2006) describes the modelling process in terms of 6 *phases* (real situation, mental representation of the situation, real model, mathematical model, mathematical result, and real results) and *transitions* between these phases (understanding the task, simplifying/structuring the task, mathematizing, working mathematically, interpreting, and validating). However, depending on the purpose and focus of the research these modelling cycles might look different and highlight different aspects of the modelling process (Haines & Crouch, 2010; Jablonka, 1996).

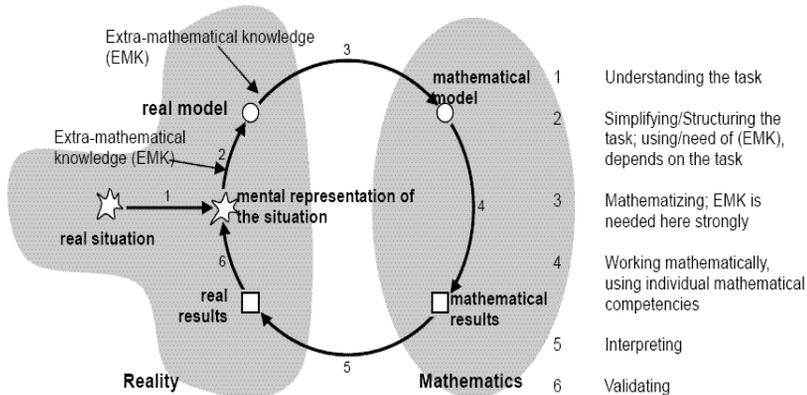


Figure 3.1: The modelling cycle by Blum and Leiß (2007) as adapted and presented by Borromeo Ferri (2006, p. 92)

It should be noted that the conceptualisation of mathematical modelling as exemplified in figure 3.1 only provides a schematic, idealised and simplified picture of the modelling process, and it has been criticised for a number of reasons. Firstly, the division of the two domains illustrated in figure 3.1 can be

considered to be artificial and raises questions about how, why and on what foundation such a division is possible to make, as well as what the consequences of such an assumption are. Secondly, empirical findings strongly suggest that the modelling process is not cyclic and that in a modelling situation the modeller normally jumps between the different stages/activities in a more non-cyclic and rather unsystematic manner (Ärlebäck & Bergsten, 2010); see also Borromeo Ferri (2007a; 2007b). To capture this ‘stochastic’ feature of a real modelling process, Voskoglou (2007) introduces a (finite) Markov chain to model and describe the processes involved in mathematical modelling as these are carried out in a mathematics classroom. In addition the ‘transitions’ between the steps/phases in the modelling process, indicated with directed arrows in figure 1, are not going just one-way, but rather are more dialectic in nature. One way to conceptualise this property of the modelling process and to make it explicit are provided by Skov Hansen, Holm and Troels-Smith (1999), see figure 3.2, which connects the phases of modelling using a net, not indicating any directions of the transitions, as well as accounting for the just mentioned ‘stochastic’ feature of modelling. Thirdly, the view of the transition between the stages/phases of *the real model* and *the mathematical model* in figure 3.1 as a move from ‘reality’ to ‘mathematics’, which generally is referred to as *mathematisation* and normally involves to express reality, or a model of reality, in mathematical terms and syntax, has been questioned by Gellert and Jablonka (2007). They point to the distinction between the real world and mathematics as depicted in figure 3.1 as being untenable, especially within the context of the mathematics classroom since what in the figure is called a ‘real model’ is strongly influenced by the mathematics available to the students, and conclude that “Mathematisation within the circular process of mathematical modelling is – epistemologically regarded – a potentially misleading construct and it is – pedagogically – of debatable value. On the one hand, the circular model of mathematical modelling adequately acknowledges the contingencies of problem definition and formalization; on the other hand it tends to obscure the informative power of mathematics. Mathematics is not only the sphere where formalized problems find their solutions; mathematics is from the outset the vantage point from which problems are construed.” (pp. 5-6).

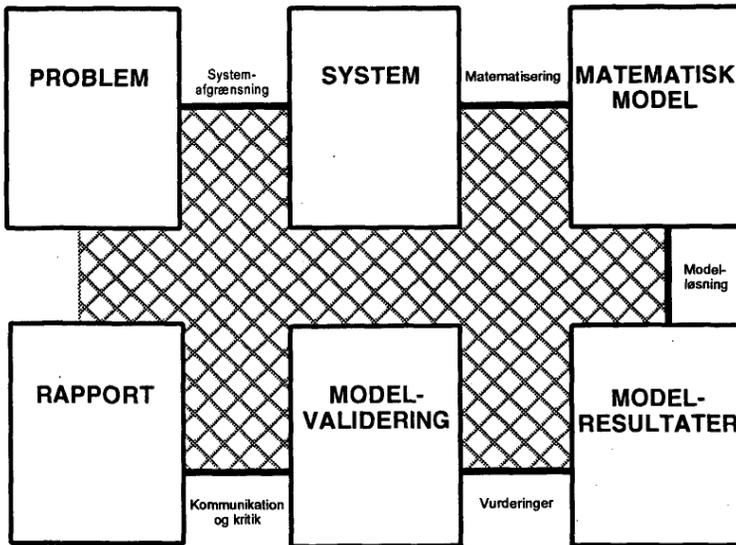


Figure 3.2. The structure of the modelling process as depicted by Skov Hansen, Holm and Troels-Smith (1999, p. 12)

The other of the two prevailing general descriptions is to use *modelling competence*, *modelling competency* or *modelling competencies*. However, as in the case with the modelling cycle there are different definitions of modelling competency and modelling competencies in the literature as well (Blomhøj & Højgaard Jensen, 2007; chapter 3.3 in Blum, Galbraith, Henn et al., 2007; Maaß, 2006). Indeed, Blomhøj and Højgaard (2007) remark that the word ‘competence’ in recent years have become a ‘buzzword’ in mathematics education with the function to “add flavour to an analysis, discussion or the planning of a teaching practice just by being mentioned.” (p. 45). Greer and Verschaffel (2007) suggest to describe and differentiate between *competencies for implicit*, *explicit*, and *critical modelling* respectively; implicit modelling competencies entail modelling activities in which students are engaged in without really being aware of that they essential are performing modelling; explicit modelling competencies are connected to explicit attention to the modelling process; and critical modelling competencies are about critical reflection about the roles, uses and consequences of modelling in mathematics, science, other disciplines and society. These three levels of descriptions aim to provide a framework that captures the ongoing discussion of modelling activities in terms of competencies incorporating a continuum of activities from basic skills to more philosophical aspects. An analogue but slightly different three-level progressive characterisation is suggested and used by Henning and Keune (2007) in terms of *Recognition and understanding of modelling (1st level)* which is “[c]haracterized by the abilities to recognize and describe the modelling process and to characterize, distinguish and

localize phases of the modelling process” (p. 227); *Independent modelling* (2nd level), basically meaning that students can solve modelling problems on their own; and *Meta-reflection on modelling* (3rd level), when the students critically can analyse and evaluate the process and outcomes from modelling as well as reflect on the purpose and applications of modelling.

Such ‘critical modelling’ or ‘Meta-reflection on modelling’ strongly relates to the level of validation of models, which goes beyond the narrow link between the situation modelled and the mathematics but calls for a wider perspective including extra-mathematical knowledge and societal values. Such analyses were made by Jablonka (1996), who studied, from a curriculum perspective, a large number of examples of mathematical models from different teaching materials with a focus on the epistemological status of the models. She identified the validation of mathematical models in the classroom situation as a problematic issue, which was often avoided or trivialised. Jablonka and Gellert (2007) describe the process of mathematisation and demathematisation as social, because “mathematics is a means for the generation of new realities not only by providing description of real world situations, but also by colonizing, permeating and transforming reality.” (p. 6).

Often modelling competency is defined drawing on or referring, either directly or indirectly, to a view of modelling expressed in terms of a modelling cycle. This is the case with Blomhøj and Højgaard Jensen (2003) who define that “[b]y mathematical modelling competence we mean being able to autonomously and insightfully carry through all aspects of a mathematical modelling process in a certain context.” (p. 126), and by ‘mathematical modelling process’ they refer to a modelling cycle similar, but more nuanced, to the one illustrated in figure 3.1. Also Maaß’s (2006) definition refers to a ‘modelling process’, but in her case she draws on the Blum and Kaiser (1997) which specifies modelling competencies by listing a number of sub-competencies: “Modelling competencies include skills and abilities to perform modelling processes appropriately and goal-oriented as well as the willingness to put these into action.” (p. 117).

The definition of modelling competency by Blomhøj and Højgaard Jensen (2003; 2007) is based on and originates from the Danish KOM-project (Niss & Jensen, 2002; Niss, 2003). Two of the advantages of adapting this definition are firstly that it offers a description of what it means to master mathematical modelling with respect to a holistic view of mathematical modelling. Secondly, using the notion of modelling competency provides a way to describe, support, and assess how the students progress and develop along the three dimensions (inherent from the KOM-project) of *degree of coverage*; *technical level*; and *radius of action* respectively. To some extent these dimensions are self-evident. However, the degree of coverage focuses on what different parts of modelling proves the student work on and their abilities to reflect on these; the technical level on the other hand reflects what mathematics the students use and how flexible they are in using it; and finally, the radius of action accounts for the contexts and domains in which the students are capable to perform mathematical modelling.

3.1.2 Perspectives on mathematical modelling

Despite the diverse definitions and used notions of mathematical models and modelling Kaiser, Blomhøj and Sriraman (2006) express an optimistic view on the chances for an understanding of the different approaches to these concepts and how they are connected and related. Indeed, they argue that there already in certain respects exists “a global theory for teaching and learning mathematical modelling, in the sense of a system of connected viewpoints covering all didactical levels” (p. 82), but that this “theory of teaching and learning mathematical modelling is far from being complete” (p. 82).

Recent efforts have been made to clarify and differentiate different approaches to mathematical models and modelling (Kaiser & Sriraman, 2006; Kaiser, Sriraman, Blomhøj, & García, 2007)²³. According to Kaiser and Sriraman (2006), different perspectives on, and approaches to, mathematical modelling can be classification as *realistic or applied modelling*; *contextual modelling*; *educational modelling* (with either a *didactical* or *conceptual* focus); *socio-critical modelling*; *epistemological or theoretical modelling*, or *cognitive modelling*. For a discussion of these perspectives see Kaiser and Sriraman (2006); Kaiser, Blomhøj and Sriraman (2006); Sriraman, Kaiser and Blomhøj (2006); and, Kaiser, Sriraman, Blomhøj and García (2007).

3.2 Problem solving, applications and modelling

Stanic and Kilpatrick (1989) begin their chapter on problem solving from a historical perspective with the words “[p]roblems have occupied a central place in the school mathematics curriculum since antiquity, but problem solving has not.” (p. 1), and it was first during the 19th century that problem solving started to get gradually more attention. Nevertheless, since then, Schoenfeld (1992) notes that “[i]ndeed, ‘problems’ and ‘problem solving’ have had multiple and often contradictory meanings through the years – a fact that makes interpretation of the literature difficult.” (p. 337).

However, following Blum and Niss (1991) a *problem* can be defined as “a situation which carries with it certain open questions that challenge somebody intellectually who is not in immediate possession of direct methods/procedures/algorithms etc. sufficient to answer the questions.” (p. 37). A consequence of this definition is that what constitutes a problem becomes something subjective. A *mathematical problem* is considered to be either a *pure problem* if the problem situation in question is embedded entirely within ‘the mathematical universe’ (the mathematical domain), or on the other hand, if the problem situation addresses some other disciplines or real world situations (the extra-mathematical domain)

²³ See also (Blomhøj, 2008).

where mathematical notation and syntax are allowed to be invoked in the process of solving the problem, the problem is called an *applied problem*. The entire process of trying to solve the problem is what is referred to as *problem solving*.

Looking back a few decades, the using of mathematics to solve a problem in an extra-mathematical domain was often referred to as *applying mathematics* and to be an example of *an application of applied mathematics* (Blum & Niss, 1991). As noted by Palm (2002), there are several different definitions of applied mathematics which complicated the discussion²⁴. Recently however, Blum et al. (2007) remark that the term *applications and modelling* taken together is being used more and more often to express all sorts of connects made between the mathematical and the extra-mathematical domain. Nevertheless, they argue that *modelling* generally focus on the transition from the extra-mathematical domain to the mathematical domain, whereas application focus on the reversed transition.

For a more elaborate discussion of the notions of problem, problem solving, applications, modelling, and the relations between these, see Blum and Niss (1991); Niss, Blum and Galbraith (2007); and, Paper 4.

3.3 Arguments for and against mathematical modelling in mathematics education, obstacles and barriers

3.3.1 Arguments for mathematical modelling

According to Blum and Niss (1991)²⁵ the following five principle arguments are invoked in the literature for the inclusion of mathematical modelling in mathematics education:

1. *The formative argument* focuses on the students' development of general capabilities and attitudes like fostering explorative and creative problem solving competencies as well as open-mindedness and self-confidence;
2. *The 'critical competence' argument* emphasises the importance to make students aware of the use and possible misuse of mathematics in society;
3. *The utility argument stresses* the use of mathematics in extra-mathematical professional and private domains;
4. *The 'picture of mathematics' argument* aims to provide the students with a rich faceted picture of mathematics as a science and an integral part of society and culture;

²⁴ Pollak (1979) for example identifies and elaborates four different definitions.

²⁵ This paper from 1991 collects and extend the ideas and analysis presented in the earlier papers (e.g. Blum & Niss, 1989; Blum, 1991; Niss, 1989). Lingefjård (2000, p. 8) refers to these as the *formative, critical, practical, cultural, and instrumental* respectively.

5. The 'promoting mathematics learning' argument emphasising instrumental aspects of modelling in the students learning of mathematical knowledge.

These arguments, and arguments falling under these arguments, are sometimes also referred to as *motives* for the inclusion of mathematical modelling in mathematics education.

Comparing the five arguments presented above with the general arguments for a mathematics education discussed in the introduction one can see clear parallels. Although one can argue that the arguments for the incorporation of mathematical modelling in mathematics education can be seen as more specific than the ones for mathematics education generally, these arguments can be criticised for being too generally formulated and applicable to almost any field of study (Jablonka, 2009), explaining their 'popularity' among mathematics educators but also making them obsolete and unproductive²⁶.

3.3.2 Arguments against mathematical modelling – obstacles and barriers

As one can find arguments for the inclusion of mathematical modelling in mathematics education, one can also find arguments against this inclusion. These latter arguments are often referred to as *obstacles* (Blum & Niss, 1991; Blum, 1996) or *barriers* (Burkhardt, 2006). However, Schmidt (in press) comments that "these [constructs] are based almost exclusively on experience and have not been subjected to empirical analysis."

Blum and Niss (1991), drawing on Pollak (1979); Blum (1985); and Blum and Niss (1987), discuss obstacles *from the point of view of instruction, from the learners' point of view, and from the teacher's point of view* respectively. The examples they provide of obstacles from the point of view of instruction are that teachers often are of the opinion that there is no time or space to include applications and modelling in an already overstuffed curriculum. Another obstacle is that some teachers are not convinced that modelling, applications and connections to other subjects should belong to mathematics instruction at all. From the students' point of view, Blum and Niss argue that to work with modelling and applications to other disciplines make the mathematical classroom less predictable and much more demanding, which is manifested as a student inherent obstacle. Finally, from the teacher's perspective, the introduction of modelling requires more from the teachers than just pure mathematical knowledge, i.e. "additional 'non-mathematical' qualifications are necessary" (p. 54), which make many teachers feel uneasy and unable to deal with applied problems and examples originating from subjects and disciplines they not studied themselves and lies outside their field of expertise. Also, as a consequence, it

²⁶ The critique from Ernest (2000) and Lundin (2008) also applies on these arguments.

becomes more complex and difficult to assess the students' progress and achievements.

These three kinds of obstacles presented by Blum and Niss are not mutually independent. Burkhardt (2006, pp. 190-193) makes another (non-disjoint) classification and instead talks about four types of *systemic barriers* which counteract larger-scale implementation of mathematical modelling in mathematics education. These systemic barriers are:

- **The systemic inertia barriers;** barriers related to teachers' *habits* and *beliefs*, as well as *teaching skills* of teachers and teacher educators, but also the *power balance* within the subject regarding for example basic skills v. problem solving, or pure v. applied mathematics.
- **The real world barrier;** introducing the real world in the mathematics classroom makes the already demanding task of teaching mathematics (mathematics in the sense as a pure abstraction) even more demanding and complicated. In addition, is modelling 'proper mathematics'?
- **The limited professional development barrier;** a changing curriculum calls for professional development of practicing teachers through for instance adequate in-service courses. In addition teacher education programmes must be up-to-date and include aspects of the teaching and learning of mathematical modelling. Generally such courses and programmes are rare.
- **The role and nature of research and development in education;** the argument is that educational research "is not well organised for turning research insights into improved practice" (p. 192).

Obstacles and barriers like the ones mentioned above are also highlighted and reported in other research papers. For instance, Maaß (2005) as well as Kaiser and Maaß (2007) focus on teachers' and students' beliefs as obstacles, and Artaud (2007) concludes that the issue of making more time available is one of the central requirements if modelling is going to become a reality in the mathematics classroom on a large-scale.

3.4 Mathematical modelling in Swedish upper secondary school

Since the reformation of the Swedish school system in 1965 the governing curricula documents for the upper secondary mathematics education or syllabus have been reformed or revised in 1970, 1972, 1982/83, 1994 and 2000, resulting in a gradually more explicit emphasis put on mathematical modelling (Ärlebäck,

2009a). Focusing on the present governing curriculum²⁷ from 2000, one of the four important aspects of the subject which should permeate all mathematics teaching, along side *problem solving*, *communication* and *the history of mathematical ideas*, is *using mathematical models* (Skolverket, 2000). Further, the document stresses that “[a]n important part of solving problems is designing and using mathematical models” and that one of the goals to aim for is to “develop their [the students’] ability to design, fine-tune and use mathematical models, as well as critically assess the conditions, opportunities and limitations of different models” (Skolverket, 2000). Nevertheless, a definition or more detailed description than given above of what a mathematical model is or what it means to model mathematically, is not provided.

Looking at the explicit goals for the different mathematics courses some complementary information or ideas about what the notion of mathematical models and modelling might involve is provided. In the *Mathematics A* course two of the goals are that “[p]upils should... be able to set up, interpret and illustrate linear functions and simple exponential functions and models for real events in private finances and in society” and to “...be familiar with how mathematics affects our culture in terms of, for example, architecture, design, music or the arts, as well as how mathematical models can describe processes and forms in nature.” (p. 65). In *Mathematics B*, “[p]upils should... be able to explain the properties of a function, as well as be able to set up, interpret and use some non-linear functions as models for real processes, and in connection with this be able to work both with and without computers and graphic drawing aids.” (p. 67). Also in the goals of *Mathematics C* and *Mathematics D* models using a specific mathematical function or construction is prescribed: “Pupils should... be able to use mathematical models of different kinds, including those which build on the sum of a geometric progression, ... be familiar with how computers and graphic calculators can be used as aids, when studying mathematical models in different application areas” (p. 70) and “Pupils should... be able to draw graphs of trigonometric functions, as well as use these functions as models for real periodic processes” (p. 73) respectively. Finally, in *Mathematics E*, “[p]upils should... be able to interpret, explain and set up differential equations as models for real situations” (p. 76)

Looking at the literature one can however find *interpretations* of these two concepts. Wyndhamn (1997), writing about mathematics and the school subject of mathematics in the curriculum documents for the compulsory Swedish schools, provides both a graphical interpretation in terms of “phases and steps in a mathematical model” (p. 44, my translation) and a description of “three characteristic processes of working with mathematical models” (p. 45, my translation). However, no rationale is provided indicating where this interpretation

²⁷ For a historic discussion of the development from 1965 to 2000 and a more general overview and treatment see Ärlebäck (2009a)

is grounded, where it originates from, or other sources of inspiration. Also Håstad (1978) accounts for “more precisely how I [Håstad] would like to interpret the concept [mathematical model]” (p. 64, my translation) and describe “the use of mathematical models” (p. 64, my translation) in an analogue manner to the three processes of Wyndhamn. A third example is provided by Palm Bergqvist, Eriksson Hellström and Haggström (2004) when they interpret the written upper secondary mathematics governing curricula documents in discerning the consequences for the construction of national test items. Their interpretation, as well as the ones made by Wyndhamn and Håstad, seems influenced by the international discussion of mathematical models and modelling in mathematics education (e.g. figure 1), but no references are provided and in my opinion the anchorage for their interpretation lacks foundation.

3.5 My view on mathematical modelling; a brief reflection

When I entered this field of research I was very appealed by the view of mathematical modelling as presented in figure 3.1; it seemed structured, logical, functional and reasonable. During my initial reading of the literature as a freshmen researcher in mathematics education, modelling as perceived in figure 3.1, or similar descriptions, were almost exclusively what I encountered. I was also very fond of the hi-level description of modelling in terms of the triple (S, M, R) , where S represent some real problem situation; M are some collection of mathematical entities; and R a collections of relations between objects in S and M (Blum & Niss, 1991). The latter a beautiful abstraction! Considering from where I came when I entered this new research field, it is not surprising that I found these descriptions appealing with their emphasis on structure. Hence, the first studies I conducted used and draw on an understanding of the mathematical modelling process in line with the one exemplified in figure 3.1.

However, alongside with the empirical experiences I made and more reading, I started to acknowledge the critique of the cyclic view of modelling and started to search for alternative perspectives and descriptions. For a while I was into the notion of modelling competency, but although there are advantages and benefits of using this notion, I consider the connection with the modelling cycle too strong, and I am not sure exactly what new will come out of adapting it. This is not to say that I have rejected the notion of modelling competency, but rather that I have to engage in further reading and research before deciding on how to design future studies.

For me, thinking about and working with mathematical models and modelling have drawn my attention, not just to what these notions actually (could) mean and imply in different contexts, but to more general, very hard and intriguing questions about the teaching and learning of mathematics at a more philosophical and basic level.

Chapter 4

Summary of papers

In this chapter I make a brief summary of each of the five papers enclosed in this thesis and discuss how they inform each other. In particular, I will elaborate on the relevance and relationship of the Papers 1 – 4 with respect to Paper 5.

4.1 Paper 1

- [1] Ärleback, J. B. (in press). Towards understanding teachers' beliefs and affects about mathematical modelling. Proceedings of CERME 6, *The Sixth Conference of European Research in Mathematics Education, Lyon, France, January 28 – February 1, 2009*.

The aim of Paper 1 is partly theoretical in that it seeks to develop a framework trying to capture and conceptualize expressed beliefs about mathematical models and modelling and relate these to other types of beliefs studied in the literature. The need for such a framework is grounded in the problématique presented in the introduction chapter, which suggests that mathematical models and modelling are notions that are not treated in the upper secondary mathematics classrooms in Sweden. Hence practicing teachers do not necessarily have an elaborated understanding and conceptualization of these notions. Nevertheless, it also aims to provide background about the two teachers participating in the study reported on in Paper 5.

Using the terminology of Törner (2002), the *belief object* of Paper 1 is *mathematical models and modelling as perceived by upper secondary mathematics teachers*, not to be confused with the teachers' beliefs of the teaching and learning of mathematical models and modelling. Following Goldin (2002), who defines beliefs as one out of four “subdomains of affective representation[s]” (p. 61) and distinguishes between *emotions, attitudes, beliefs, and values, ethics and morals*, beliefs are here taken to be “multiply-encoded cognitive/affective configurations, usually including (but not limited to) prepositional encoding, to which the holder attributes some kind of *truth value*” (p. 64, emphasis in original). For an individual, a collection of mutually reinforcing or supporting non-contradictory beliefs taken together with the individual's justifications for their

‘coherence’ constitutes a *belief structure*²⁸, and it is the teachers’ belief structure of mathematical models and modelling that is the construct investigated in Paper 1.

Drawing on the literature on beliefs related to mathematics; mathematics teaching; and mathematics learning, in combination with the analysis of the view of modelling represented in figure 3.1, Paper 1 suggests that the teachers’ belief structure of mathematical models and modelling fruitfully can be seen and explored as constituted by the following five (sub-)belief objects: beliefs about *the nature of mathematics*; *the real world*; *problem solving*; *school mathematics*; and *applying, and the applications of, mathematics*.

This suggested framework is exploited using data from two interviews with the two teachers participating in the study reported on in Paper 5. Although this empirical data was not primarily collected with the testing of the above framework in mind, it is relevant for discussing the viability and usefulness of the framework since the interviews focus on teachers’ views on mathematical modelling as well as on mathematics and mathematics learning and teaching generally.

The interviews were partly structured around five mathematical problems to serve as a basis for the discussion and reflection²⁹. In order to structure how the teachers talked about the five (sub-)beliefs objects listed above, the interviews were recorded; transcribed and analysed using a categorization scheme based on these sub-beliefs objects. Due to the nature of the data, beliefs about *the real world* and *applications and applying mathematics* surfaced only sporadically and could therefore not be fully accounted for, and hence their corresponding beliefs in the framework were not validated or explored.

²⁸ The justification could be made by the individual using what Green (1971) refers to as *quasi-logic*, which captures the fact that some beliefs only are in consensus with other beliefs within a belief structure provided that a non-standard and personal logical explanation is made use of.

²⁹ See the appendix D of Paper 5 for details.

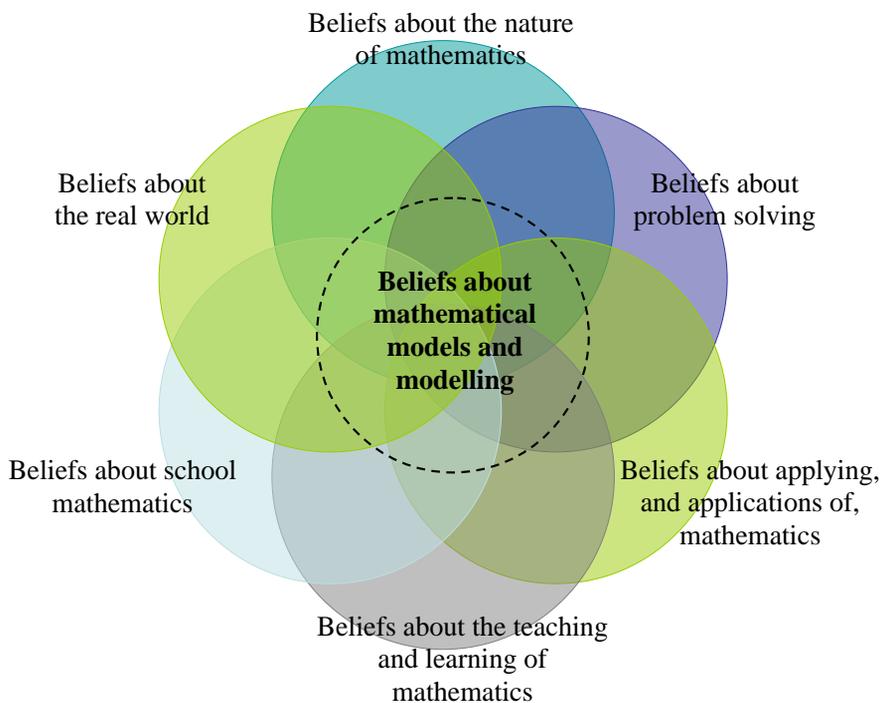


Figure 4.1. The suggested conceptualization of the belief structure of mathematical models and modelling

The result of the analysis in Paper 1 is that neither of the two teachers could clearly express their conceptions of the notions of mathematical models and modelling. They rather had to formulate their views as the interviews went on. However, no flaws in the quasi-logic holding together the different sub-beliefs structure were detected in neither teacher's sub-beliefs structures.

In addition, during the analysis of the teachers' expressed views it became clear that they answered many of the questions referring and relating to aspects of the learning and teaching of mathematics. This suggests that it would be reasonable to extend the suggested belief structure of mathematical models and modelling of the teachers to additionally include beliefs of the teaching and learning of mathematics; see figure 4.1 for an illustration of the modified suggested belief structure of mathematical models and modelling.

4.2 Paper 2

- [2] Ärleböck, J. B. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Montana Mathematics Enthusiast*, 6(3), 331-364

Paper 2 addresses the issue of how to introduce mathematical modelling to upper secondary students using an indirect approach relying on, what I call, *realistic Fermi problems*. Generally, Fermi problems are open, non-standard problems requiring the students to make assumptions about the problem situation and estimate relevant quantities before engaging in, often, simple calculations. More specifically, realistic Fermi problems are defined in terms of the following five characteristics:

- their *accessibility*; that they can be approached by all individual students or groups of students, and solved both on different educational levels and on different levels of complexity and hence not necessarily demand any specific pre-mathematical knowledge;
- their clear real-world connection, that they are *realistic*, just not intellectual exercises;
- the *specifying and structuring of the relevant information and relationships* needed to tackle the problem. This characteristic prescribes the problem formulation to be open, not immediately associated with a known strategy or procedure to solve the problem. Hence it urges the problem solvers to invoke prior constructs, conceptions, experiences, strategies and other cognitive skills in approaching the problem;
- the absence of numerical data, that is *the need to make reasonable estimates* of relevant quantities. An implication of this characteristic is that the context of the problem must be somewhat familiar, relevant and interesting for the subject(s) working in it;
- (in connection with the last two points above) their inner momentum to *promote discussion*, that as a group activity they invite to discussion on different matters such as what is relevant for the problem and how to estimate physical entities.

The aim of the study reported on in Paper 2 is to investigate if realistic Fermi problems could be used to introduce mathematical modelling at the Swedish upper secondary level by addressing the question ‘*What mathematical problem solving behaviour do groups of students display when engaged in solving realistic Fermi problems?*’. The basic idea and motivation being that *if* groups of students engaged in solving realistic Fermi problems display problem solving behaviour resembling sub-activities of the modelling process, then their problem solving experiences of such an encounter in the classroom could be used as a basis and

point of departure for a classroom discussion on what mathematical modelling is and what it might mean to engage in a modelling activity. Such a discussion might end up in a first heuristic picture of modelling of the type represented by figure 3.1.

To answer this question an analytical tool called *Modelling Activity Diagram* (the *MAD* framework) was developed in which Schoenfeld's 'graphs of problem solving' (Schoenfeld, 1985) was adapted. The tool incorporates the features of realistic Fermi problems and the view of modelling as illustrated in figure 3.1 to get a schematic picture of the problem solving process of students working on the realistic Fermi problem 'Empire State Building':

There is an information desk on the street level in the Empire State Building. The two most frequently asked questions to the staff are:

- *How long does the tourist elevator take to the top floor observatory?*
- *If one instead decides to walk the stairs, how long does this take?*

Your task is to write a short letter answering these questions, including the assumptions on which you base your reasoning, to the staff at the information desk.

The work on the problem³⁰, done by three groups consisting of totally seven volunteered students enrolled in a university preparatory year taking the upper secondary courses in mathematics taught by myself, was filmed and transcribed using a modified and simplified version of the TalkBank conversational analysis codes³¹ as a guide for the transcription. The students' written short answers were also collected.

The transcriptions were coded using the categories of the six modelling sub-activities of the developed MAD framework:

Reading: this involves the reading of the task and getting an initial understanding of the task

Making model: simplifying and structuring the task and mathematizing

Estimating: making estimates of a quantitative nature

Calculating: doing maths, for example performing calculations and rewriting equations, drawing pictures or diagrams

Validating: interpreting, verifying and validating results, calculations and the model itself

³⁰ The study from which Paper 2 reports involved the groups work on two realistic Fermi problems and the Empire State Building problem was the first of them the students encountered.

³¹ www.talkbank.org.

Writing: summarizing the findings and results in a report, writing up the solution

The analysis was made both on the level of *utterances*, here taken to be “stretch of continuous talk by one person, regardless of length and structure” (Linell, 1998, p. 160), and on the level of *dialogues* constituted by a sequence of utterances made by the group members taking turns making utterances. The question guiding the categorization was ‘*What sub-activity is the utterance/dialogue indicating that the student/group is engaged in?*’. This coding process was done repeatedly to refine the coding and to test the reliability of the process. The procedure was validated by looking at the video-recordings as well as the written short answers from the three groups. The result of this analysis (on the group level) was graphed in a *modelling activity diagram* for each group, showing the time spent on and the moves between the different modelling sub-activities during their work on the problem. The diagram was made partitioning time into 15 seconds intervals to make the description as clear and readable as possible. An example of a modelling activity diagram is displayed in figure 4.2, based on a 30 minutes problem solving session.

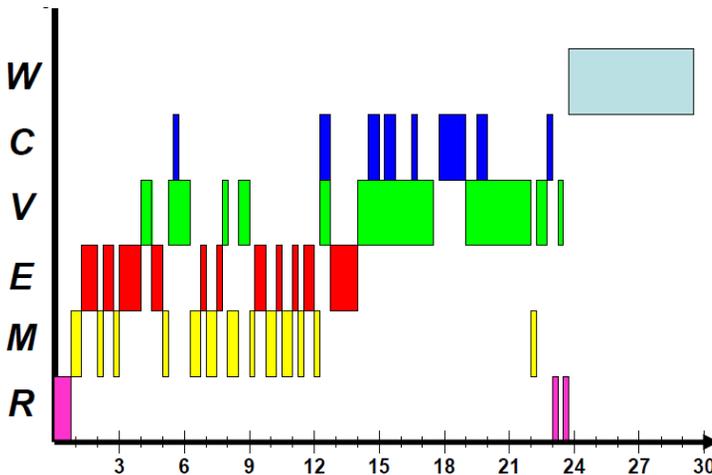


Figure 4.2. The modelling activity diagram for the Empire State Building Problem, group A (Ärlebäck, 2009b, p. 345).

Paper 2 concludes that all the modelling sub-activities proposed by the MAD framework (*Reading, Making model, Estimating, Validating, Calculating, and Writing*) were richly and dynamically represented and contributed in a dialectic progression towards a solution when the students engaged in solving the Empire State Building realistic Fermi problem. Hence, small group work on realistic Fermi problems may provide a good and potentially fruitful opportunity to

introduce mathematical modelling at upper secondary school level if this activity is followed up appropriately

In addition, one of the most evident results produced by the MAD framework is the non-cyclic nature of the modelling process, pointing out that the presented view on modelling (see Figure 3.1) as a cyclic process is highly idealised, artificial and simplified. Although it was useful to conceive mathematical modelling in this manner for the developing of the MAD framework, however a real authentic modelling processes is better described as haphazard jumps between different stages and activities.

Paper 2 also concludes that the data provides numerous examples where personal extra-mathematical knowledge is used by the students in the validation of both models and estimates as well as in the validation of calculations. In addition, the group dynamics are essential for the activation of and the evolution of the different sub-activities during the problem solving process. It is also noted that the displayed group behaviour is strongly influenced by individual preferences and group composition, making it one of the most important task variables to consider.

From the collected three letters produced by the groups, it is noted that although they spent quite a large amount of time on composing these, they contain almost no evidence about the groups' activities during the 30-minutes long problem solving session.

4.3 Paper 3

[3] Ärlebäck, J. B. & Frejd, P. (accepted for publication after revision). First results from a study investigating Swedish upper secondary students' mathematical modelling competencies, Proceedings of ICTMA 14, *The 14th International Community of Teachers of Modelling and Applications, Hamburg, July 27-31, 2009*.

Paper 3 reports on the quantitative part from a study using both qualitative and qualitative methods aiming to investigate what Swedish upper secondary students know about mathematical modelling and how capable they are of solving modelling problems. In other words the object of inquiry is the attained curriculum or to be more precise what students know about mathematical models and modelling as a result from the existing teaching practice.

Paper 3 makes use of the notion of *modelling competency* and adapts the definition of Blomhøj and Højgaard Jensen (2003) who “[b]y mathematical modelling competence [we] mean being able to autonomously and insightfully carry through all aspects of a mathematical modelling process in a certain context.” (p. 126). Here the ‘mathematical modelling process’ is described analogously to figure 3.1.

Using this notation, the explicit aim of Paper 3 is to get an initial indication of the level of the mathematical modelling competency of Swedish upper secondary

students and in addition to investigate if factors such as grade, gender, last taken mathematics course, and different attitudes might affect the level of success of students solving modelling problems. The two research questions posed to address these issues were:

1. What modelling competency do Swedish upper secondary students in 12th grade display?
2. Are there any connections between the students' modelling competency relation to mathematical achievement in general (grade), gender, students' interest, last taken mathematical courses, or to previous experiences?

Drawing on multiply choice test items with a partial assessment model originally designed and developed by Haines, Crouch and Davis (2000) a research instrument was constructed consisting of seven test items. Each of the test items captures one of the following *modelling sub-competencies*: (sC1) *to make simplifying assumptions concerning the real world problem*; (sC2) *to clarify the goal of the real model*; (sC3) *to formulate a precise problem*; (sC4) *to assign variables, parameters, and constants in a model on the basis of sound understanding of model and situation*; (sC5) *to formulate relevant mathematical statements describing the problem addressed*; (sC6) *to select a model*; and (sC7) *to interpret and relate the mathematical solution to the real world context* (cf. Kaiser (2007, p. 115-116)). Additional background data such as gender, latest taken upper secondary mathematics course, and their latest received grade in mathematics, as well as answers to seven attitude Likert questions was also collected.

The data analysed to answer the posed questions comes from 400 upper secondary students (12th grade) distributed in 21 classes from all over Sweden, and due to the non-normality nature the data displayed non-parametrical statistical methods were used in the statistical analysis (e.g. the Mann-Whitney test, the Kruskal-Wallis test, and the Kendall's tau to be more specific).

The result in Paper 3 shows that only 23 % of the students stated that they had encountered the notions of mathematical models and modelling before in their secondary mathematics education. However, this did not significantly ($p > 0.05$) affect the students' total score on the tests. The analysis also shows that Swedish upper secondary students were most proficient in the questions relating to sC3 and sC4, but exhibited more difficulties in the questions relating to sC1, sC2 and sC6. The students' grade and their last taken mathematics course turned out to significantly affect the students' modelling competency in a positive way. However, no significant difference between the two highest grades (VG and MVG) was manifested, nor if the students last taken mathematics course was *Mathematics C* or *Mathematics D*. In addition, no gender effect could be discerned.

Concerning the students' attitudes towards working with mathematical modelling as represented in the test items, an overall negative tendency in all answers to the attitude questions was found. In general, the students found the

problems very hard and did not express any excitement or joy in tackling them. Neither did the students express that they found the problems especially interesting nor that they wanted to (have) work(ed) more on similar problems in their mathematics classes. However, the students to some extent seemed to recognize the value to use mathematics to solve the problems on the tests, and in addition regarded the types of questions asked as relevant and good to use in mathematics classrooms. Students finding the problems *interesting* or *easy* got a statistically ($p < 0.05$) significantly higher score on the used test.

In addition, Paper 3 reports on results related to the method used, especially on the inconsistencies in compatibility in four of the pair of test items used to test each modelling sub-competency, previously claimed by other researchers to be consistent.

One conclusion made in Paper 3 is that students' attitudes towards mathematical models and modelling may present an obstacle in the implementing of mathematical models and modelling in the mathematics classrooms at this school level.

4.4 Paper 4

[4] Ärlebäck, J. B. (2009). *Matematisk modellering i svenska gymnasieskolans kursplaner i matematik 1965-2000* [Mathematical modelling in the Swedish upper secondary mathematics education between the years 1965-2000] (Rapport nr 2009:8, LiTH-MAT-R-2009-8). Linköping: Linköpings universitet, Matematiska institutionen.

Paper 4 focuses is on the intended curriculum represented by the six syllabuses that have governed the Swedish upper secondary mathematics education for the Science and Technical programmes from 1965 and to date.

The overall question studied is *How are mathematical models and mathematical modelling described and discussed in the mathematics curriculum for the Swedish upper secondary school between the years 1965 - 2000?* More specifically however, the following questions were addressed:

- How and to what extent is mathematical models and modelling described in the different curriculum?
- Is it possible to discern some change of how mathematical models and modelling is treated over time, and if that is the case, how can this change be described?
- What are the arguments and reasons can be seen to lie behind the descriptions of mathematical models and mathematical modelling?
- Is it valid to say that the concept of mathematical modelling has gradually been undermined and impaired?

- Specifically, what can be said about mathematical models and modelling in relation to the present curriculum Gy2000? Is modelling a goal or an instrument of teaching for fulfilling other goals? Are there any specific teaching methods connected to the teaching and learning of mathematical modelling in Gy2000?

In Paper 4 it is argued that in order to get a holistic understanding of the meaning and content of the notions of mathematical models and modelling as given in the intended curriculum, it is necessary to analyse the notions of *mathematical models and modelling*, *applications (of mathematics)*, and *problem solving* jointly. In addition, to better understand the linguistic usage of these words and concepts, and to capture the interrelationships of these notions and their evolution, a historical dimension is included in the analysis. The evolution of these notions are manifested in the inclusion of the six curricula or syllabuses that have governed the Swedish upper secondary mathematics education for the Science and Technical programmes from 1965 and to date in the study.

The six syllabuses are scrutinised using a three level analysis: (1) *an initial content analysis*, an ordinary content analysis focusing on different occurrences of the words (or inflected forms of) (*mathematical*) *model* and (*mathematical*) *modelling*; (2) *a qualitative analysis* where the occurrences found in (1) are looked at in the context which they were written; (3) *an adjusted and deepened quantitative content analysis* using a developed analytic tool based on emerging sub-categories to the categories *modelling*, *application*, and *problem solving*.

The results in Paper 4 show that different ways of using and working with mathematical modelling is something that successively have gained more and more explicit emphasis in the Swedish upper secondary mathematics curriculum. The initial analysis shows that the notions of mathematical models and modelling were nearly non-existent in the early syllabuses until the reform in 1994 (Lpf94). The analysis also makes it palpable that the emphasis on mathematical models and modelling in the present curriculum (Gy2000) is twice as many in absolute numbers as in the preceding curriculum (Lpf94). In addition, it is also plain from the qualitative analysis that the formulations have become stronger and more elaborated in Gy2000. Nevertheless, the adjusted and deepened quantitative content analysis reveals that mathematical models and modelling *implicitly* have been part even of the earlier syllabuses. This is manifested in (implicit) formulations involving the students' ability to formulate and interpret mathematical connections or problem situations and relate these to the real world; to create, formulate or work with already existing models; and, to apply mathematics as a tool in other subjects and disciplines.

The results also indicate a trend of displacement in how the authors of the curriculum use the different notions modelling, applications, and problem solving; from *applications and applied problem solving* to *modelling and problem solving in more general terms*.

Using the terminology by Blum and Niss (Blum & Niss, 1991) it appears that the argument for the inclusion of working with mathematical models and modelling in the Swedish upper secondary mathematics classrooms put forward in Lpf94 primarily is *the utility argument*, but with some element of *the formative* and *the 'critical competence' arguments* as well. However, in Gy2000 on the other hand, more arguments can be discerned with a more equal emphasis; namely *the utility argument*, *the formative argument*, *the 'critical competence' argument*, as well as *the 'picture of mathematics argument'*. Nevertheless, depending on how some of the formulations are interpreted it is possible to argue that some of the arguments that at first hand seem to be *utility arguments* are more in line with *the 'promoting mathematics learning' argument*.

Relating to whether if it is valid to say that the concept of mathematical modelling has gradually been undermined and impaired, this is not supported by the results in Paper 4. On the contrary, one can interpret the results as there has been an attempt to specify and describe the concept. This is, however, done in an implicit and indirect manner in the two last curricula.

It is also concluded that mathematical modelling as described in Gy2000 can be interpreted both as a goal in itself and as didactical tool, as an instrument for fulfilling other curriculum goals. However, this dual function of mathematical modelling is manifested in different part of the curriculum. Modelling as a goal in itself is found in the part where the *goals to aims for* and *structure and nature of the subjects* are presented, and is described very explicit and clear. Formulations prescribing modelling as a didactical tool on the other hand can be found in the more concrete *goals that pupils should have attained on completion of the course*, foremost in the *Mathematics A, B, C, and E* courses respectively, and are not that immediate. The formulations in these latter goals permits interpretations, which make it possible to conclude that modelling should be used as a didactical tool. However, in the same time it is also possible to legitimately interpret the formulations about possible application of a given mathematical concept or field such as geometrical sums or differential equations.

Concerning the question if there are any specific teaching methods connected to the teaching and learning of mathematical modelling in Gy2000, no definite conclusions can be made. Mathematical modelling should permeate all mathematics teaching and in the same time function to provide opportunities for the students to apply specific mathematical content knowledge in extra-mathematical contexts

4.5 Paper 5

- [5] Ärlebäck, J. B. (2009). *Designing, implementing and evaluating mathematical modelling modules at the upper secondary level – a design study* (Rapport nr 2009:8, LiTH-MAT-R-2009-9). Linköping: Linköpings universitet, Matematiska institutionen.

Paper 5 reports on a study aiming to investigate how mathematical modelling can be incorporated at the Swedish upper secondary level in line with the present mathematics curriculum. Its focus is on the collaborative design and developmental process of two so called *modelling modules* carried out by a researcher and two teachers. A modelling module consists of a number of not necessarily sequential lessons focused on the introduction and exposing of mathematical modelling to Swedish upper secondary students. For future reference, the study from which Paper 5 reports is referred to as *the design study*.

The conceptual framework used to address this aim is drawing and based on the principles of *design-based research* (Barab & Squire, 2004; The Design-Based Research Collective, 2003); *cultural-historical activity theory* (CHAT) in Engeström's interpretation (Engeström, 1987); a *co-learning agreement* between researcher and practitioners (Wagner, 1997); and, the view of mathematical modelling illustrated in figure 3.1. The principles of design-based research is used as the overall methodology; CHAT is used as the language of description and to analyse the design process; the co-learning agreement is used to establish the relation between the researcher and the participating teachers; and mathematical modelling as described in figure 3.1 represents the conception of mathematical modelling adapted in the study.

The research reported on in Paper 5 use the following five *guiding principles* to facilitate and support the research process:

- GP1.** The research should be as naturalistic as possible in the sense that
- it should be carried out at the participating teachers' schools and within their practice;
 - the teachers' ideas and initiative should be given priority;
 - my role in the implementation of the modelling modules should be kept at a minimum, preferably only involvement in connection to the collection of data.
- GP2.** The research should be of collaborative nature, where the participating teachers and I as a researcher should work together on equal footing.
- GP3.** The participating teachers should experience the research as meaningful and useful (first and foremost for their own account,

secondly on behalf of their students, and thirdly with the least priority, for me and my objectives).

- GP4.** The modelling modules should be in line with the present curriculum, meaning that the mathematical content in the modules should be what is prescribed in the course syllabuses respectively.
- GP5.** The modelling modules should be small, so that they do not mess up the teachers' 'normal' practice (teaching and other responsibilities).

In addition, these guiding principles are also partly used in the formulation of the research questions studied, which are:

- RQ 1. How does the collaboration respecting the five guiding principles influence the developmental/designing process and the form the modelling modules take?
- RQ 2. How do the teachers experience the sequences of designing, implementing and evaluating the modelling modules?
- RQ 3. How are the teachers' views and attitudes towards mathematical models and modelling changing, if at all, as the project evolves?
- RQ 4. How do students experience working with the designed modelling modules?

Part of the result of Paper 5 are two modelling modules, one for the *Mathematics C* course and one for the *Mathematics D* course, both consisting of an introductory lesson followed by a number of lesson were groups of students engaged in project work. The *Mathematics C* course module consists of totally five lessons and the seven projects developed therein focus the whole modelling cycle (e.g. figure 3.1) in connection to the area of geometrical sums. The *C* module includes a three-tiered assessment model consisting of written group reports; the producing of a poster and oral presentation; and, in writing to comment on another groups report, poster and oral presentation. The *Mathematics D* course module consists of totally six lessons and the six projects developed therein used mathematical modelling as theme, not focusing on a particular mathematical topic, and do not emphasis the cyclic and iterative nature of modelling. The assessment included in the *D* modules consists of written groups reports. Both the differences and similarities in how the modules turned out and how they are constituted are consequences of the guiding principles in actions.

Paper 5 also concludes that the five guiding principles are incompatible. If they are taken together equally weighted they give raise to 'conflicts' when it comes to consider options and making decisions in the design and developing process of the modules. In addition, the importance of having an explicit and clear, negotiated and agreed upon, way of communication among the participants in a collaborative research based on a co-learning agreement is elucidated.

The teachers report that they think the project was successful in every aspect. Both of them are of the opinion that the designing the modules were carried out on equal footing were all participants contributed regarding form and content of the

modules. Additionally, they both express the wish to continue the collaboration and do something similar in the future. The teacher teaching the *D* course module is overall positive to the whole project; the collaborative developing and designing of the *Mathematics D* course module; the implementation of the module; and, what the students achieved and presented. The teacher teaching the *C* course module on the other hand is disappointed about what the students achieved and produced in forms of written reports, which he considered defective and insufficient.

Concerning whether the teachers' attitudes toward mathematical models and modelling changed during the project, more analysis is needed. However, there are indications that the attitudes of at least one of the teachers were not affected by her participation in the study.

Turning to how the students express how they experienced working with the modules, 58% and 70% of the students in the *Mathematics C* module respectively in the *Mathematics D* module stated that on the whole it was a positive experience. The common argument they presented was the variation in classroom activity the modules implied, and additionally in the *Mathematics D* module, the *D* module's feature to use and apply mathematics to real life contexts. Nevertheless, the students expressed that they would have liked more scheduled time to work on the tasks and to prepare the different assessment moments in the modules (58% and 38% in the *C* respectively the *D* module). Beside this issue of time, the students generally expressed to be content with the format of the modules, specially the introduction lesson using the Empire State Building problem.

4.6 Coherence of the papers, limitations and possible extensions

In the following the coherence of the included papers in the thesis is discussed with special focus on how Paper 1 – 4 inform the main aim of this thesis, which is addressed in Paper 5. In addition, limitations, possible extensions and other studies that can be carried out to enlighten important aspects related to the aim are discussed.

4.6.1 Paper 1

The relevance of Paper 1 is connected to the main aim of the thesis in that it explores and provides a framework to characterise the part of the potentially implemented curriculum which is constituted by teachers' beliefs about mathematical models and modelling. Knowledge, or at least indications, of the preconceptions about mathematical models and modelling of the participating teachers in the design study is important to consider and take into account when engaging in the collaborative design activity. It might well be the case that

different configurations of beliefs might affect both the designing process as well as the ‘designed product’ in either positive or negative respectively in productive or unproductive ways. Nevertheless, the suggested framework in Paper 1 is not robustly tested due to the nature of the data used, but the first applications of it seem promising in that it provides a structured way to think about what the teachers ‘bring with them’ in terms of prior understanding and beliefs about the object of implementation. In addition, it can be used to identify beliefs structures that either might facilitate or be a barrier for the implementation processes or different stages in the implementation process. The application of the framework on the data from the individual interviews from the design study (e.g. Paper 5) provides an answer to the following question: *Are the beliefs systems of mathematical models and modelling of the two teachers participating in the design study individually consistent?*, which is ‘no’.

The approach adapted in Paper 1 is qualitatively based on interviews and captures what sometimes is referred to as *professed beliefs*, which in this case is what the teachers say. *Attributed beliefs* on the other hand is what is reflected in the teachers’ practice, and although Speer (2005) argues that this distinction is artificial and indeed that “[t]he distinction between professed and attributed beliefs is a false dichotomy” (p. 370) due to inappropriate research design, information about the participating teachers’ attributed beliefs would have provided yet another dimension of the potentially implemented curriculum. Indeed, visiting classrooms during normal classroom activity with the objective to see what beliefs the teachers manifest in the daily practice of his/her mathematics teaching would in addition provide information about the implemented curriculum. Although such investigations would be important and relevant for the elucidation of the problématique, no such studies were conducted within the scope of this thesis.

It would also have been useful to have had results from large quantitative studies of the different sub-beliefs objects collected using surveys, which could give a more general and more representative indication of these aspects of the potentially implemented curriculum. Such an approach would make it possible to situate the two participating teachers in the design study relatively to the larger population of Swedish upper secondary mathematics teachers.

A thorough analysis of the ‘traditional’ constituent of the potentially implemented curriculum according to the IEA framework, the written mathematical textbooks, is another line of investigation that would present insight in how mathematical models and modelling are presented to the Swedish upper secondary students. Such a study might give a more nuanced picture of the actual situation in the Swedish upper secondary school system, reinforcing or modifying the argument in the introduction that mathematical models and modelling are not given the attention and priority in the everyday classroom practice as prescribed by the curriculum. Related to the investigation of the textbooks it would also be relevant to include the often accompanied teacher manuals provided by the publishing houses.

Another study that would shed light on the potentially implemented curriculum and provide background to the present situation in Sweden could be to take a critical look at the teacher educational programs and in-service courses to see how mathematical model and modelling is brought up and promoted in these contexts. Nevertheless, such research is beyond the scope of this thesis, and is to be engaged in the future.

4.6.2 Paper 2

Paper 2 is relevant with respect to the thesis's aim as it provides evidence that one can 'induce' modelling behaviour in students' group work using realistic Fermi problems. This result suggests, that including realistic Fermi problems in the actual *introduction* of mathematical modelling might inform the answer to the *how*-question related to the aim.

Although the characteristics of realistic Fermi problems do not provide a sharp definition, problems with these features are easy to use in the existing teaching practice; they are direct in the sense that they both are easy to understand and that students can start working on them immediately. In addition, the preparation time needed on behalf of the teacher is minimal and the activity does not require more than 20 to 30 minutes. Varying the context of the used Fermi problem also facilitate involvement of critical thinking on different topics and reflection (about social or environmental issues for example). Arguably, including 'the realistic Fermi problem of the week' in the existing teaching practice could be a part of fulfilling at least some aspects of the intended curriculum with respect to mathematical modelling. However, this latter suggestion must be researched before any general recommendations could be made.

It is important however to stress that mathematical modelling can not be equated with realistic Fermi problems and that the mastering of Fermi problems is not equivalent to the mastering of mathematical modelling. Although it is easy to adapt the view that realistic Fermi problems are modelling problems in miniature, mathematical modelling cannot be reduced to solving realistic Fermi problems. There are many differences. Firstly, a Fermi problem, like any other word problem, might at best display authenticity, but could never capture the complexity of reality. Secondly, it is a big difference for a teacher to handle Fermi problems in class compared with coping with solving modelling problems. Thirdly, Fermi problems could be inadequate to use to fulfil some of the curriculum goals explicit connected to modelling, such as for example to use modelling in connection with geometrical sums as described in the syllabus of the *Mathematics C* course.

Paper 2 has a macroscopic and quantitative focus in the sense that it looks at the general behaviour of students engaged in solving Fermi problems. An additional analysis on a microscopic and qualitative level of what students actually do in more detail when they engage in solving Fermi problems could provide

further insight into the question of what Fermi can and can not do in relation to mathematical modelling.

4.6.3 Paper 3

Paper 3 focuses on the attained curriculum, the result of the present existing teaching practice, and is relevant in connection to the main aim of the thesis in that it seeks to highlight what Swedish secondary students know and are capable of with respect to mathematical modelling. In order to be able to adapt the introduction of mathematical modelling into the teaching practice, it is important to have some initial picture and understanding of the level on which students in general are capable to handle modelling problems, and also of their attitudes towards them.

To get an overview on a broader national scale this paper reports on a quantitative study, but other studies aiming to investigate the attained curriculum would also be relevant to conduct. Another source of information could be the national exams, on which problems related to modelling could be identified and students' solutions and results analysed, as well as teacher constructed exams and the students' solutions and results on those³².

The results from Paper 3 could also be put into an international perspective using the existing research which uses the same test items³³. However, there could never be a simple comparison between nations since previous research does not focus on upper secondary level and uses tests that are composed of other test items.

An important limitation of the test items used in this paper is that they only test fragments of the modelling process, and not the whole cycle.

One can note that the statistical methods used were the simplest possible and that the data is being reanalysed using more advanced statistical methods, accounting also for the covariance between the variables. This analysis will thus be able to provide a more nuanced description of the data.

4.6.4 Paper 4

The relevance and implications of Paper 4 with respect to the main aim of this thesis is evident since the first part of the main aim explicitly is to investigate how mathematical modelling as *prescribed in the upper secondary mathematics curriculum* can be implemented in the existing teaching practice. Hence, knowing what is written in the governing curriculum documents with respect to the notions

³² Research reports that in general national assessment test and teacher constructed assessment test focus on very different aspects of mathematics (Boesen, 2006) implying that the analysis of the two different sorts of test might provide different information of the attained curricula.

³³ See the references in Paper 3

of mathematical models and modelling is a prerequisite to be able to address the main aim and Paper 5. This is also manifested in the guiding principle number 4 of Paper 5 which partly states that the modelling modules should be *in line with the present curriculum*.

Although Paper 4 focuses on the governing written curriculum documents, the intended curriculum, it would have been relevant and interesting for the aim of the thesis to investigate the different discourses where these texts are produced, read, respectively used. Especially relevant would be to study the interpretations made by teachers as a part of the potentially implemented curriculum. The other mentioned discourse, the production of the curriculum, is also something that would be interesting to look into and follow up the work by Skolverket (2004b) and IKUM (2008).

4.6.5 Paper 5

Paper 5 is the paper that holds this thesis together and draws on all the other papers. It directly addresses the main aim of the thesis in that it tries to answer both the *how-* and address the *process part*.

In the study from which Paper 5 reports there are a lot of data that due to time limitations not could be analysed but which would have provided valuable contributions. The focus of Paper 5 is on the design and developmental processes of the two modelling modules which 'as products' can be considered to belong to the potentially implemented curriculum. Much of the data collected are about the implementation of these modules and the outcomes in terms of what the students produced. The natural extensions of Paper 5 are firstly to analyse the implementation of the modules, which would provide a case study account of aspects of the implemented curriculum focusing on mathematical modelling. Secondly, the other extension is to analyse the attained curriculum in terms of what the students produced. In addition, the focus groups interviews of one group in each module could provide further information on how the students experienced the modules and how to improve them.

Chapter 5

Conclusions and discussion

In this final chapter of the thesis I will briefly recapitulate the main results and some of the experiences made from the five papers included in this thesis. I will also discuss these findings in relation to each other as well in the light of earlier research done in the field. The chapter, as well as the thesis, closes with some remarks about possible implications and how this research might be continued, deepened and extended.

5.1 Conclusions

In this section I very briefly collect the results from Paper 1 – Paper 5 in relation to the adopted and applied curriculum framework: the intended curriculum, the potentially implemented curriculum; and, the attained curriculum respectively.

5.1.1 The intended curriculum

- Formulating and working with mathematical modelling have been implicit components in the Swedish upper secondary mathematics curricula between 1965 and 1994.
- From the 1994 curriculum there has been gradually more and more explicit emphasis put on mathematical modelling. However, the notions of mathematical models and modelling are not defined explicitly but only described in implicit terms.
- There is a trend of displacement in how the authors of the curricula use the notions; from *applications and applied problem solving* to *modelling and problem solving in more general terms*.
- The argument for the inclusion of working with mathematical models and modelling in the Swedish upper secondary mathematics classrooms put forward in Gy2000 are *the utility argument/the 'promoting mathematics learning' argument, the formative argument, the 'critical competence' argument, as well as the 'picture of mathematics argument'*.

- No support for the claim that the concept of mathematical modelling has gradually been undermined and impaired as a consequence of the recent curricula reforms has been found. On the contrary, attempts are made to clarify things.
- The Gy2000 mathematics curriculum can be interpreted as to see mathematical modelling both as a goal in itself and as a didactical tool for fulfilling other curriculum goals.
- No specific teaching methods connected to the teaching and learning of mathematical modelling can be discerned in Gy2000.

5.1.2 The potentially implemented curriculum

- The two participating teachers in the design study could not clearly express their conceptions of the notions of mathematical models and modelling.
- Small group work on realistic Fermi problems may provide a good and potentially fruitful opportunity to introduce mathematical modelling at upper secondary school level if this activity is followed up appropriately.
- Two modelling modules introducing and working with mathematical modelling in the Swedish upper secondary *Mathematics C* and *D* courses respectively have been designed and developed, (implemented,) and evaluated:
 - The *Mathematics C* module focuses on the whole modelling cycle and explicitly addresses the specific mathematical content of geometrical sums. The students were assessed by a written project report; a produced poster; an oral presentation; and, by the written comments they made on another group's report, presentation and poster.
 - The *Mathematics D* module uses mathematical modelling as an overarching theme and was integrated with the comprehensive task in the *Mathematics D* syllabus. The students in the *D* course module were assessed via written reports only, but the module included an oral presentation as well.
 - The participating teachers expressed that they on the whole experienced the sequence of designing, implementing and evaluating the modules positive and rewarding. In their opinion the designing of the modules was carried out on equal footing and all participants contributed in the shaping of the modules with regards to both form and content. However, in the *C* module, the teacher questions what the students actually learnt.
 - No evidence that the participating teachers' attitudes towards mathematical models and modelling changed as a consequence of participating in the design study could be established.

- A majority of the students found working with the modelling modules a positive experience, but requested more scheduled time for completing the tasks.

5.1.3 The attained curriculum

- 77 % of the students, see Paper 3, had never encountered the notions of mathematical models and modelling before in their upper secondary education.
- Students working on short test items
 - expressed an overall negative attitude towards working with the mathematical modelling;
 - were most proficient with respect to the sub-competencies *to formulate a precise problem* and *to assign variables, parameters, and constants in a model on the basis of sound understanding of model and situation*, and least proficient in *clarify the goal of the real model* and *to select a model*;
 - found the modelling problems hard.
- The students' grade, last taken mathematics course, and if they thought the problems in the tests were easy or interesting, were factors positively affecting the students' modelling competency.

5.1.4 Meta-level result

- The guiding principles behind the research design of Paper 5 (the design study) are partly incompatible and give rise to many tensions and contradictions in the designing and developing process of the two modelling modules.
- The importance to have clearly negotiated and explicit rules, norms and routines for communication between the involved parties in research based or drawing on a design-based methodology is manifested during the work on designing and developing the modules.

5.2 Discussion and implications

This thesis contributes to the understanding of different aspects of the notions of mathematical models and mathematical modelling at the Swedish upper secondary level. It highlights some instances related to different curriculum levels and provides an overview of the past and present state and trends of mathematical modelling at this school level. The picture is far from complete, but through this thesis a general picture has begun to emerge.

One of the most palpable conclusions that can be drawn from this thesis is that there is a big discrepancy between the intended and attained curriculum. In the

intended curriculum mathematical models and modelling are prescribed to *permeate all teaching*, and yet 77 % of the 400 students in Paper 3 state that they never encountered these notions in their upper secondary education before. Although mathematical modelling and different ways to work with mathematical models, in the Swedish upper secondary mathematics education, is something that successively have been more emphasised in the curriculum since the reformation and introduction of the modern upper secondary school system in Sweden 1965, there seems to be systemic barriers (Burkhardt, 2006) hindering the actual implementation and realising of the intended curriculum. These barriers manifest themselves in different ways exemplified in the research presented in the thesis.

The fact that the notions of mathematical models and modelling are described in implicit and general terms in the intended curriculum can create an insecurity about what it is that should be implemented in the classroom; the implemented curriculum. This in turn may affect what the students assimilate. The risk of having an undefined and unclear element in the intended curriculum is that it opens up to a wide variety of interpretations and pure arbitrariness. As an effect, teachers leave out and skip the element in question rather than putting the energy and effort in to first make up their own minds about the element, and then secondly, to develop, plan and realise their interpretation in their classroom practice. This insecurity is reflected in the result of Paper 1, and it was also manifested in Paper 5. Gjone (2001) explains part of this phenomena as a dilemma that the curriculum authors are confronted with: “On the one hand you want it [the intended curriculum] to be as precise and unambiguous as possible, but on the other hand, it might be desirable to have a certain amount of flexibility that allows teachers the opportunity to implement for example a non-traditional approach to teaching.” (p. 98, my translation). In my opinion, the intended curriculum must provide better and more precise definitions and descriptions about what it is the students are to learn. This is in line with Burkhardt’s conviction, that “[a]mong the **key levers** for tackling resistance to change [the existing teaching practice] are *curriculum descriptions*, supported by *well-engineered materials* to support *assessment, teaching, professional development* and *public relations* (in the literal sense) that are well-aligned with the each other – and have been shown to work well in realistic circumstances of personnel and support.” (p. 190, italics in original).

However, one can argue that the remark about the vague formulations in the written curriculum not only applies to the notion of mathematical modelling, but also, for instance, to the notion of *problem solving*, and is a general issue in the intended curriculum. Some of the questions, both of a general and a more specific nature, which comes to mind when reflecting over these results, are:

- What is the function of the written curriculum documents and how do these influence the daily classroom practice? (Both in general and more specifically regarding the mathematics curriculum.)

- What are the processes and procedures that form the basis for producing the written curriculum? Who are the authors? On what criteria have these been selected and how? How does research in mathematics education influence the work of producing written curriculum documents? To what extent are old traditions and former written curricula influencing the work of producing a new written curriculum?
- What are the intentions, reasons and aims from the curriculum authors when they bring mathematical models and modelling to the fore as much as they do in the two latest syllabuses and at the same time formulate the concepts in vague manners?
- Since the written curriculum documents allow considerable space for interpretations, at least regarding the notions of mathematical models and modelling, it is relevant to ask: How are these notions interpreted by the different actors in the educational system on the different curriculum levels, and how much does ideology come into play here? Is there any instance or authority which is ascribed priority when it comes to this interpretation, and if that is the case, which one and why?

Some of these issues are addressed by Skolverket (2004a; 2004b) and IKUM (2008). However, a theoretical framework that could be useful to investigate some of these questions further might be the theory of didactical transposition (see Bosch & Gascón, 2006).

Although one can not generalise the results with the teachers' un-conceptions of mathematical models and modelling from Paper 1, it draws attention to a potential and probable weak link between the intended and the potentially implemented curriculum. To strengthen this link one can try to take different measures. One way is to focus on teacher education and teacher in-service courses (e.g. Holmquist & Lingefjärd, 2003; Lingefjärd, 2007), perhaps by drawing and using the work carried out within the LEMA project³⁴. In addition, due to the situation in Sweden where the teaching and learning of mathematics is relying and based on the use of traditional textbooks, an effective way to influence the potentially implemented curriculum could be to put pressure on the textbook authors and distributors to increase the focus on mathematical modelling. However, both these suggestions presume that the intended curriculum specifies, defines and clearly states what it means by mathematical models and modelling, and its role at the upper secondary level.

However, the experiences made in the design study as well as the result from the study on Fermi problems (Paper 2) provide some encouraging results pointing to the possibility to introduce and let students work with mathematical models and

³⁴ LEMA (Learning and Education in and through Modelling and Applications), an EU funded project involving six countries (Sweden did not participate), see www.lemma-project.org.

modelling in line with the upper secondary curriculum, using only small means. Using realistic Fermi problems on a regular basis might contribute to fulfilling some of the curriculum goals with respect to mathematical modelling, especially if this approach to teaching is complemented and followed up with occasional project work in the form of modelling modules.

If other approaches to incorporate mathematical modelling at this school level are tried, the results from Paper 3 and 5, respectively, infer that what form in which this is done is crucial from the students' perspectives. The students' attitude towards working with modelling varied drastically between the students working on 'traditional word problems with modelling features' contra the student engaged in more complex problems which they worked on for four to five lessons. The former expressed an overall negative attitude and did not want more of mathematical modelling in their education. The students in the design study however expressed overall positive attitudes and stated that they wanted to work more with mathematical modelling in their mathematics classes.

The thesis also contributes to the research field of mathematics education in that it exemplifies the construction and use of a conceptual framework for design-based research with CHAT as an important cornerstone. In addition, it points to methodological issues about how the premises of a design-based research project affect the courses of events in the research and what consequences these initial premises might have on the whole research process.

The result also to some extent points to limitations in the prevailing most common perspective taken on mathematical modelling in this field of research, represented by figure 3.1, and calls for a critical and more reflecting attitude towards the conception of mathematical modelling and its role in society and mathematics education (see Jablonka, 1996; Jablonka & Gellert, 2007). That would also have the potential to give mathematical modelling an attention in mathematics teaching which is more in line with the general goals of mathematics education, as discussed in the introduction of this thesis, as well as the Swedish curriculum goals for mathematics, and for mathematical modelling in particular.

5.3 Future research

The research presented in this thesis can be extended and continued in numerous ways, as has already been elaborated in section 4.6 and in the discussion. Rather than just repeating some of these here, I would like to stress what I from my personal view think would be the most interesting and rewarding issues to address in future research. In short, I would like to continue the design-based research approach initiated in Paper 5 and try to improve and further develop the modelling modules. However, I have come to realise that doing research of this type requires a lot of resources and a project involving a research group to cope with all the data and analysis. In my opinion, such a research project would both enhance our

understanding about the teaching and learning of mathematical modelling, but foremost, it would bring about a change in the existing teaching practice.

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