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***Abstract*—We consider spectrum sensing of OFDM signals in an AWGN channel. For the case of completely unknown noise and signal powers, we derive a GLRT detector based on empirical second-order statistics of the received data. The proposed GLRT detector exploits the non-stationary correlation structure of the OFDM signal and does not require any knowledge of the noise power or the signal power. The GLRT detector is compared to state-of-the-art OFDM signal detectors, and shown to improve the detection performance with 5 dB SNR in relevant cases.**

For the case of completely known noise power and signal power, we present a brief derivation of the optimal Neyman-Pearson detector from first principles. We compare the optimal detector to the energy detector numerically, and show that the energy detector is near-optimal (within 0.2 dB SNR) when the noise variance is known. Thus, when the noise power is known, no substantial gain can be achieved by using any other detector than the energy detector.

***Index Terms*—spectrum sensing, OFDM, GLRT**

I. Introduction

The introduction of cognitive radios in a primary user network will inevitably have an impact on the primary system, for example in terms of increased interference. Cognitive radios must be able to detect very weak primary user signals, to be able to keep the interference power at an acceptable level [1]. Therefore, one of the most essential parts of cognitive radio is spectrum sensing.

One of the most basic sensing schemes is the energy detector [2]. This detector is optimal if both the signal and the noise are Gaussian, and the noise variance is known. However, all manmade signals have some structure. This structure is intentionally introduced by the channel coding, the modulation and by the insertion of pilot sequences. Many modulation schemes give rise to a structure in the form of cyclostationarity (cf. [3]), that may be used for signal detection [4].

Many of the current and future technologies for wireless communication, such as WiFi, WiMAX, LTE and DVB-T, use OFDM signalling. Therefore it is reasonable to assume that cognitive radios must be able to detect OFDM signals. The structure of OFDM signals with a cyclic prefix (CP) gives a well known and useful cyclostationarity property [5]. Detectors that utilize this property have been derived, for example in [6], [7] using the autocorrelation property, and in [8] using multiple cyclic frequencies. The detector proposed in [8] is an extension of the one in [4], to multiple cyclic frequencies. None of these detectors are derived based on statistical models for the received data that captures the nonstationarity of an OFDM signal, and they are not optimal in the Neyman-Pearson sense. We will show that it is possible to obtain much better detection performance.

In practice the detector will have imperfect or no knowledge of parameters such as the noise power, the signal power and the synchronization timing of the transmitted signal. Any parameter uncertainties lead to fundamental limits on the detection performance, if not treated carefully [9].

Like in most related literature (cf. [6], [8]) we consider an AWGN channel, in order to study the most important fundamental aspects of OFDM signal detection. The main contribution of this paper is that we derive a computationally efficient detector based on a generalized-likelihood ratio test operating on empirical second-order statistics of the received signal. The so-obtained detector does not need any knowledge of the noise power or the signal power. We compare this detector to state-of-the-art methods [6], [7]. The most relevant comparison is that with the detector of [6], which also works without knowing neither the signal variance nor the noise variance. We show that our proposed method can outperform the detector of [6] with 5 dB SNR in relevant cases.

We also present a brief summary of the optimal Neyman-Pearson detector of [10], when the signal and noise powers are known, and compare it with the energy detector and with the detectors based on second-order statistic. For example, we show numerically that when the noise power is known, the energy detector is near-optimal (within 0.2 dB SNR) for OFDM signals.

II. Model

We consider a discrete-time (sampled) complex baseband model. Assume that $\mathbf{x}$ is a received vector of length N that consists of an OFDM signal plus noise, i.e.

$$\mathbf{x} = \mathbf{s} + \mathbf{n},$$

where $\mathbf{s}$ is a sequence of K consecutively transmitted OFDM symbols, and $\mathbf{n}$ is a noise vector. The noise $\mathbf{n}$ is assumed to be i.i.d. zero-mean circularly symmetric complex Gaussian with variance σ_n^2, that is, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2\mathbf{I})$. Each OFDM symbol consists of a data sequence of length N_d, and a cyclic prefix (CP) of length N_c ($\leq N_d$).

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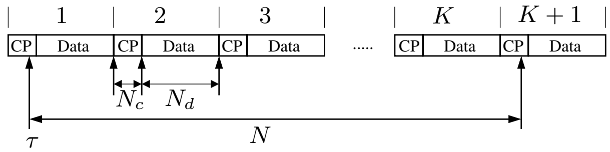

Fig. 1. Model for the N samples of the received OFDM signal.

In practice one cannot know exactly when to start the detection. That is, the receiver is not synchronized to the transmitted signal that is to be detected. Let τ be the synchronization mismatch, in other words the time when the first sample is observed. That is, $\tau = 0$ corresponds to perfect synchronization. We assume that the transmitted signal consists of an infinite sequence of OFDM symbols, so that detection can equivalently start within any symbol. Then, it is only useful to consider synchronization mismatches within one OFDM symbol, that is in the interval $0 \leq \tau < N_c + N_d$. In a perfectly synchronized case ($\tau = 0$) we would observe a number (K) of *complete* OFDM symbols, in order to fully exploit the structure of the signal. Without loss of generality, we assume that the total number of samples in the vector $\mathbf{x}$ is $N = K(N_c + N_d)$. This implies that $\mathbf{x}$ will in general (when $\tau > 0$) contain samples from $K+1$ OFDM symbols, as shown in Figure 1.

III. Optimal Neyman-Pearson detector

The key observation for deducing the optimal detector is that the OFDM signal lies in a certain subspace, owing to the structure introduced by the repetition of data in the CP. In a perfectly synchronized scenario (τ known), this subspace would be perfectly known. The theory of detection of a signal in a known subspace has been extensively analyzed, both in white and colored noise [11]. In realistic scenarios, τ will be unknown. Since the signal depends on τ, the signal subspace will be only partially known in general. In what follows, we provide a very brief summary of a derivation of the optimal Neyman-Pearson detector from first principles.

We start by formulating a vector-matrix model for the received signal. Let $\mathbf{q}_i$ be the N_d-vector of data associated with the ith OFDM symbol. This data vector is the output of the IFFT operation, used to create the OFDM data. In the general unsynchronized case ($\tau \neq 0$), the received signal $\mathbf{x}$ will contain samples from symbols $1, \ldots, K+1$. Thus, let $\mathbf{q} \triangleq [\mathbf{q}_1^T \mathbf{q}_2^T \ldots \mathbf{q}_{K+1}^T]^T$ be a vector consisting of the data that correspond to $K+1$ OFDM symbols. Then, the received signal $\mathbf{s}$ can be written

$$\mathbf{s} = \mathbf{T}_\tau \mathbf{q},$$

where $\mathbf{T}_\tau$ is a sparse $K(N_c+N_d) \times (K+1)N_d$ matrix of ones and zeros, that describes the structure of the OFDM signal. The matrix $\mathbf{T}_\tau$ is known, given τ. See [10] for its explicit form.

Assuming a sufficiently large IFFT, the data vector $\mathbf{q}$ can be assumed to be Gaussian by the central limit theorem. That is, $\mathbf{q} \sim \mathcal{C}N(\mathbf{0}, \sigma_s^2 \mathbf{I})$, where σ_s^2 is the variance of the complex signal samples. Then, conditioned on τ, the distribution of the signal $\mathbf{s}$ is also Gaussian. That is, $\mathbf{s}|\tau \sim \mathcal{C}N(\mathbf{0}, \sigma_s^2 \mathbf{T}_\tau \mathbf{T}_\tau^T)$.

We wish to detect whether there is a signal present or not. That is, we want to discriminate between the following two hypotheses:

$$\begin{aligned} H_0 &: \mathbf{x} = \mathbf{n}, \\ H_1 &: \mathbf{x} = \mathbf{s} + \mathbf{n}, \end{aligned} \tag{1}$$

We start by considering detection when σ_n^2 and σ_s^2 are perfectly known.

A. Known σ_n^2 and σ_s^2

In this subsection, we derive the optimal Neyman-Pearson detector, for the unsynchronized case when τ is unknown. Under H_0, the received vector contains only noise. That is,

$$p(\mathbf{x}|H_0) = \frac{1}{\pi^N \sigma_n^{2N}} \exp\left(-\frac{\|\mathbf{x}\|^2}{\sigma_n^2}\right).$$

Under H_1, the received vector contains an OFDM signal plus noise, and the first sample is received at time τ. Since τ is unknown, we model it as a random variable, and obtain the marginal distribution:

$$p(\mathbf{x}|H_1) = \sum_{\tau=0}^{N_c+N_d-1} P(\tau|H_1) p(\mathbf{x}|H_1, \tau).$$

We assume that τ is completely unknown, and model this by taking τ uniformly distributed over the interval $[0, N_c + N_d - 1]$, so that

$$P(\tau|H_1) = \frac{1}{N_c + N_d}.$$

We know that $\mathbf{s}|\tau \sim \mathcal{C}N(\mathbf{0}, \sigma_s^2 \mathbf{T}_\tau \mathbf{T}_\tau^T)$, and thus $\mathbf{x}|H_1, \tau \sim \mathcal{C}N(\mathbf{0}, \mathbf{Q}_\tau)$, where

$$\mathbf{Q}_\tau \triangleq \sigma_n^2 \mathbf{I} + \sigma_s^2 \mathbf{T}_\tau \mathbf{T}_\tau^T.$$

The optimal Neyman-Pearson test is

$$\begin{aligned} \Lambda_{\text{optimal}} &\triangleq \log\left(\frac{p(\mathbf{x}|H_1)}{p(\mathbf{x}|H_0)}\right) \\ &= \log\left(\sum_{\tau=0}^{N_c+N_d-1} \frac{1}{\det(\mathbf{Q}_\tau)} \exp(-\mathbf{x}^H \left(\mathbf{Q}_\tau^{-1} - \frac{1}{\sigma_n^2}\mathbf{I}\right) \mathbf{x})\right) \\ &\quad + \log\left(\frac{\sigma_n^{2N}}{N_c + N_d}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{optimal}}. \end{aligned} \tag{2}$$

where η_{optimal} is a detection threshold.

To compute the LLR (2), we need to compute $\det(\mathbf{Q}_\tau)$ and $\mathbf{x}^H \left(\mathbf{Q}_\tau^{-1} - \frac{1}{\sigma_n^2}\mathbf{I}\right) \mathbf{x}$. A direct computation of these quantities can be very burdensome if N is large. However, the computations can be significantly simplified by exploiting the sparse structure of $\mathbf{Q}_\tau$ [10].

B. Benchmark - Energy detection

A computationally efficient and widely used detector is the energy detector, also known as radiometer [2]. It measures the received signal energy and compares it to a predetermined threshold. That is, the test is

$$\Lambda_e = \sum_{i=0}^{N-1} |x_i|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta_e. \quad (3)$$

The energy detector does not require, and therefore does not exploit, any knowledge about the signal to be detected. Therefore it will be used as a benchmark to the optimal detector derived in Section III-A, that utilizes the knowledge of the lengths of the CP and the data. The performance of the energy detector is well known, cf. [12]. Moreover, consider the case when $N_c = 0$ (no CP), so that there is no structure in the (OFDM) signal that can be used. Then $\mathbf{T}_\tau \mathbf{T}_\tau^T = \mathbf{I}$, and $\mathbf{x}|H_1 \sim \mathcal{C}N(\mathbf{0}, (\sigma_n^2 + \sigma_s^2)\,\mathbf{I})$. In this special case, the energy detector is equivalent to the detector derived in Section III-A, and therefore optimal.

C. Unknown σ_n^2 and σ_s^2

When σ_n^2 and σ_s^2 are unknown, the optimal strategy is to eliminate them from the problem by marginalization. We need to choose proper a priori distributions for σ_n^2 and σ_s^2 , and then compute the marginalization integrals. It is not clear how these a priori distributions should be chosen. One possibility is to choose a non-informative distribution, for example the gamma distribution as we used in [13] to express lack of knowledge of the noise power. For most sensible distributions, the integrals are very hard to compute analytically. Therefore, for the case of unknown σ_n^2, σ_s^2, we proceed by instead using generalized likelihood-ratio tests.

IV. Detection based on second-order statistics

In this section, we propose a detector that exploits the structure of the OFDM signal by using empirical second-order statistics of the received data. The approach is inspired by the works of [6], [7], which also use second-order statistics although in a highly suboptimal manner, see Section IV-D for a discussion. The case of most interest is when σ_n^2 and σ_s^2 are unknown, and we start our treatment with this assumption.

A. GLRT-approach for unknown σ_n^2 and σ_s^2

The repetition of data in the CP gives the OFDM signal a nonstationary correlation structure. We will propose a detector based on the generalized likelihood-ratio test (GLRT), that exploits this structure. Without loss of generality we assume throughout this section that the number of received samples is $N = K(N_c + N_d) + N_d$. Define the sample value product

$$r_i \triangleq x_i^* x_{i+N_d},\ i = 0, \ldots, K(N_c + N_d) - 1 \quad (4)$$

The expected value of r_i of an OFDM signal is non-zero, for the data that is repeated in the CP of each OFDM symbol. This property will be used for detection. The received vector $\mathbf{x}$ consists of K consecutive OFDM symbols.

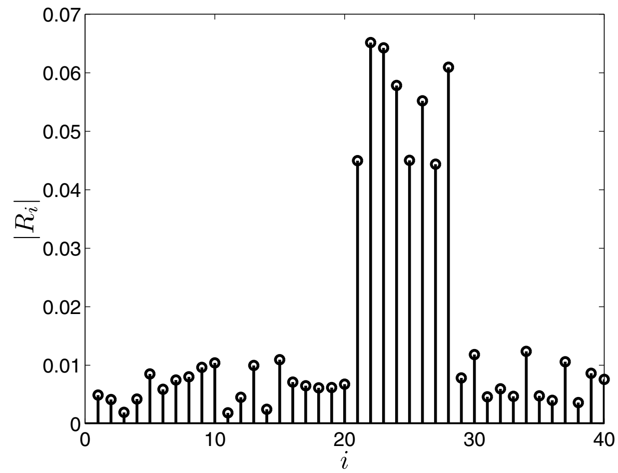

Fig. 2. Example of the correlation structure of a noise-free OFDM signal. $N_c = 8$, $N_d = 32$, $K = 50$, $\tau = 20$.

Moreover, we know that if $s_i = s_{i+N_d}$ $(= q_{i+\tau} = q_{i+N_d+\tau})$, $0 \le i < N_c + N_d$, then $s_{i+k(N_c+N_d)} = s_{i+N_d+k(N_c+N_d)}$, $k = 1, \ldots, K-1$. Analogously, if $s_i = q_{i+\tau}$ and $s_{i+N_d} = q_{i+N_d+\tau}$ are independent $(q_{i+\tau} \neq q_{i+N_d+\tau})$, then $s_{i+k(N_c+N_d)}$ and $s_{i+N_d+k(N_c+N_d)}$ are also independent. Thus, we define

$$R_i \triangleq \frac{1}{K} \sum_{k=0}^{K-1} r_{i+k(N_c+N_d)},\ i = 0, \ldots, N_c + N_d - 1. \quad (5)$$

Under H_0, all the averaged sample value products R_i are identically distributed. Under H_1, there will be N_c consecutive values of R_i (starting with R_τ) that have a different distribution than the other N_d values. Figure 2 illustrates this for a noise-free OFDM signal with $N_c = 8$, $N_d = 32$, $K = 50$ and $\tau = 20$. Since R_i is complex-valued, the figure shows $|R_i|$. It is clear that the 8 samples corresponding to the CP are significantly larger than the other.

The aim of our proposed method is to detect whether R_i are i.i.d. or whether their statistics depend on i as explained above and as illustrated in Figure 2. Essentially, our proposed method implements a form of change detection. We propose a detector based on a GLRT that deals with the difficulty of not knowing τ, σ_s, σ_n. Let $\mathbf{R} \triangleq [R_0, \ldots, R_{N_c+N_d-1}]^T$. The GLRT is then

$$\Lambda_{\text{GLRT}} \triangleq \frac{\max_{\tau, \sigma_n^2, \sigma_s^2} p\left(\mathbf{R}|H_1, \tau, \sigma_n^2, \sigma_s^2\right)}{\max_{\sigma_n^2} p\left(\mathbf{R}|H_0, \sigma_n^2\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{GLRT}}. \quad (6)$$

To simplify the derivation of the joint distribution and the maximization, we assume that the variables R_i are approximately independent. Then, the likelihood function can be approximated as

$$p(\mathbf{R}|H_k, \tau, \sigma_n^2, \sigma_s^2) \approx \prod_{i=0}^{N_c+N_d-1} p(R_i|H_k, \tau, \sigma_n^2, \sigma_s^2) \quad (7)$$

Moment	$\mathbf{H_0}$	$\mathbf{H_1}$	
		$i \notin S_\tau$	$i \in S_\tau$
$E\left[\overline{R}_i \mid \cdot\right]$	0	0	σ_s^2
Var $\left[\overline{R}_i \mid \cdot\right]$	$\frac{\sigma_n^4}{2K}$	$\frac{(\sigma_s^2+\sigma_n^2)^2}{2K}$	$\frac{\sigma_s^4+\frac{\sigma_n^4}{2}+\sigma_s^2\sigma_n^2}{K}$
$E\left[\widetilde{R}_i \mid \cdot\right]$	0	0	0
Var $\left[\overline{R}_i \mid \cdot\right]$	$\frac{\sigma_n^4}{2K}$	$\frac{(\sigma_s^2+\sigma_n^2)^2}{2K}$	$\frac{\frac{\sigma_n^4}{2}+\sigma_s^2\sigma_n^2}{K}$
Cov $\left[\overline{R}_i, \widetilde{R}_i \mid \cdot\right]$	0	0	0

TABLE I
FIRST AND SECOND ORDER MOMENTS OF R_i.

and we only need to derive the marginal distributions of R_i. Since R_i is a complex-valued random variable, its real and imaginary parts must be dealt with separately. Let $\overline{a}$ and $\widetilde{a}$ denote the real and imaginary parts of a respectively. Then, $R_i = \overline{R}_i + j\widetilde{R}_i$, where

$$\overline{R}_i = \frac{1}{K}\sum_{k=0}^{K-1} \overline{r}_{i+k(N_c+N_d)},\ i = 0, \ldots, N_c + N_d - 1,$$

$$\widetilde{R}_i = \frac{1}{K}\sum_{k=0}^{K-1} \widetilde{r}_{i+k(N_c+N_d)},\ i = 0, \ldots, N_c + N_d - 1.$$

The terms $r_{i+k(N_c+N_d)}$ and $r_{i+l(N_c+N_d)}$ of the sum (5) are i.i.d. for $k \neq l$ by construction. Hence, R_i is a sum of i.i.d. random variables. Let $\mathbf{R}_i \triangleq \left[\overline{R}_i\ \widetilde{R}_i\right]^T$. Then, for large K, by the central limit theorem (cf. [14, pp. 108–109]), $\mathbf{R}_i$ has the two-dimensional Gaussian distribution

$$\mathbf{R}_i \sim \mathcal{N}\left(\begin{bmatrix} E\left[\overline{R}_i\right] \\ E\left[\widetilde{R}_i\right] \end{bmatrix}, \begin{bmatrix} \text{Var}\left[\overline{R}_i\right] & \text{Cov}\left[\overline{R}_i, \widetilde{R}_i\right] \\ \text{Cov}\left[\overline{R}_i, \widetilde{R}_i\right] & \text{Var}\left[\widetilde{R}_i\right] \end{bmatrix}\right). \tag{8}$$

The structure of the OFDM signal incurs that the equality $s_i = s_{i+N_d}$ holds for N_c consecutive variables R_i, and that s_i and s_{i+N_d} are independent for all the other N_d variables. Let S_τ denote the set of consecutive indices for which $s_i = s_{i+N_d}$, given the synchronization mismatch τ. The expectations, variances, and covariances of $\overline{R}_i$ and $\widetilde{R}_i$ respectively are computed in [10], and are summarized in Table I.

Detection is most crucial at low SNR ($\sigma_n^2 \gg \sigma_s^2$). We use this low-SNR approximation in the remainder of this section to simplify the computations of the ML estimates of the unknown parameters. A similar approximation was used in [6]. Define $\sigma_1^2 \triangleq \frac{\sigma_n^4}{2K}$. Then, at low SNR, the variances of $\overline{R}_i$ and $\widetilde{R}_i$ are approximately equal to σ_1^2 in all cases. Using the low SNR approximation and the statistics of R_i from Table I in (8) yields

$$\begin{cases} \mathbf{R}_i \mid \{H_0\} \sim \mathcal{N}\left(\mathbf{0}, \sigma_1^2\mathbf{I}\right), i = 0, \ldots, N_c + N_d - 1, \\ \mathbf{R}_i \mid \{H_1, i \notin S_\tau\} \sim \mathcal{N}\left(\mathbf{0}, \sigma_1^2\mathbf{I}\right), \\ \mathbf{R}_i \mid \{H_1, i \in S_\tau\} \sim \mathcal{N}\left(\begin{bmatrix} \sigma_s^2 \\ 0 \end{bmatrix}, \sigma_1^2\mathbf{I}\right). \end{cases} \tag{9}$$

Under the approximations made, if we insert (7) and (9) into (6), we obtain the test

$$\max_\tau \frac{\sum_{i=0}^{N_c+N_d-1} |R_i|^2}{\sum_{k\in S_\tau}\left|R_k - \frac{1}{N_c}\sum_{i\in S_\tau}\overline{R}_i\right|^2 + \sum_{j\notin S_\tau}|R_j|^2} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{GLRT}}. \tag{10}$$

This test is computationally efficient. We only need to compute the empirical averages R_i from (4) and (5), then compute the likelihood ratio (10) for each τ, $0 \leq \tau < N_c + N_d$, and take the maximum.

Any knowledge of the parameters σ_n^2, σ_s^2 or τ can easily be incorporated in the proposed detector by inserting the corresponding true parameter value into (6). See the following two subsections for a brief discussion. If the synchronization mismatch τ is known, then the maximization in (10) can be omitted.

B. Special case: Known σ_n^2 and σ_s^2

If σ_n^2, σ_s^2 are known, they can be directly inserted into (6). In this case we do not need to use the low-SNR approximation, since both σ_n^2 and σ_s^2 are known. Using the statistics from Table I in (8) and some algebra, the LLR is given by

$$\begin{aligned} \max_\tau \Bigg(& \frac{1}{\sigma_1^2}\sum_{k=0}^{N_c+N_d-1}|R_k|^2 - \frac{1}{\gamma_1^2}\sum_{i\notin S_\tau}|R_i|^2 \\ & - \sum_{j\in S_\tau}\left(\frac{\left(\overline{R}_j - \sigma_s^2\right)^2}{\overline{\gamma_1}^2} + \frac{\widetilde{R}_j^2}{\widetilde{\gamma_1}^2}\right)\Bigg), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \gamma_1^2 &\triangleq \text{Var}\left[\overline{R}_i \mid H_1, i \notin S_\tau\right] = \text{Var}\left[\widetilde{R}_i \mid H_1, i \notin S_\tau\right] \\ &= \frac{1}{2K}\left(\sigma_s^2 + \sigma_n^2\right)^2, \\ \overline{\gamma_1}^2 &\triangleq \text{Var}\left[\overline{R}_i \mid H_1, i \in S_\tau\right] = \frac{1}{K}\left(\sigma_s^4 + \frac{\sigma_n^4}{2} + \sigma_s^2\sigma_n^2\right), \\ \widetilde{\gamma_1}^2 &\triangleq \text{Var}\left[\widetilde{R}_i \mid H_1, i \in S_\tau\right] = \frac{1}{K}\left(\frac{\sigma_n^4}{2} + \sigma_s^2\sigma_n^2\right). \end{aligned}$$

Note that complete knowledge of the parameters for the proposed GLRT detector is *not* equivalent to the optimal genie detector (2). Therefore, the detector in (11) is suboptimal. However, it is interesting to use for comparison purposes, since a comparison between (11) and (2) provides a feeling for how much performance that is lost by basing the detection on the second-order statistics R_i instead of on the received raw data $\mathbf{x}$.

C. Special case: Known σ_n^2 and unknown σ_s^2

If σ_n^2 is known but σ_s^2 is unknown, we may use the ML estimate $\widehat{\sigma_s^2}$ in lieu of σ_s^2. In this case, the low-SNR approximation is necessary. After some algebra, and removing contants, the test statistic becomes

$$\max_\tau \left(\sum_{i\in S_\tau}\overline{R}_i\right)^2. \tag{12}$$

The detector (12) may be compared with the energy detector, since both only need to know σ_n^2 in order to set the decision threshold.

D. Benchmarks

In the following, we present two competing detectors [6], [7] that are also based on second-order statistics of the received signal. To our knowledge, [6], [7] represent the current state-of-the-art for the problem that we consider.

1) Autocorrelation-based detector of [6]: The method of [6] was called an autocorrelation-based detector and it uses the empirical mean of the sample value products r_i, normalized by the received power, as test statistic. More precisely, the test proposed in [6] is

$$\Lambda_{\text{AC}} = \frac{\sum_{i=0}^{(N_c+N_d)-1} \overline{R}_i}{\frac{N_c+N_d}{N}\sum_{i=0}^{N-1}|x_i|^2} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{AC}}. \tag{13}$$

The detector proposed in [6] does not require any knowledge about the noise variance σ_n^2.

Referring to Figure 2, the detector of [6] essentially uses the average of the 40 samples, and does not exploit the fact that only 8 of the samples have non-zero mean and the other 32 have zero mean. Thus, the detector of [6] ignores the fact that the received signal under H_1 is not stationary. Taking the average of the sample value products as in (13) does not exploit all of the structure in the problem.

2) Sliding-window detector of [7]: The detector of [7] uses a sliding window that sums over N_c consecutive samples, and takes the maximum. The test statistic is

$$\max_\tau \left| \sum_{i=\tau}^{\tau+N_c-1} r_i \right|.$$

The statistic (14) only takes one OFDM symbol at a time into account. A straightforward extension of this detector for K symbols, is to use the test

$$\Lambda_{\text{SW}} \triangleq \max_\tau \left| \sum_{i=\tau}^{\tau+N_c-1} R_i \right| \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{SW}}. \tag{14}$$

We will use the extended statistic (14) in our comparisons. The main drawback of the detector proposed in [7] is that it requires knowledge about σ_n^2 to set the decision threshold.

V. Numerical Results

We show some numerical results for the proposed detection schemes, obtained by Monte-Carlo simulation. All simulations are run until at least 100 detections (and missed detections) are observed. Performance is given as the probability of missed detection, P_{MD}, as a function of SNR. The SNR in dB is defined as $10\log_{10}(\sigma_s^2/\sigma_n^2)$. The noise variance was set to $\sigma_n^2 = 1$, and the SNR was varied from -20 dB to 5 dB. The data vector $\mathbf{q}$ was drawn randomly with the distribution $\mathbf{q} \sim \mathcal{C}N(\mathbf{0}, \sigma_s^2\mathbf{I})$. In the simulations, the probability of false alarm P_{FA} was fixed to find the detection threshold, η, and the probability of missed detection, P_{MD}. The IFFT size was set to $N_d = 32$ and the CP was chosen as $N_c = N_d/4 = 8$. The probability of false alarm was set to $P_{\text{FA}} = 0.05$. All detectors and their parameter knowledge requirements are summarized in Table II.

ID	Ref.	Detector	Test	σ_n^2	σ_s^2
i	[6]	Autocorrelation	(13)	$-$	$-$
ii	Proposed	2nd order, GLRT	(10)	$-$	$-$
iii	[7]	Sliding Window	(14)	$\times$	$-$
iv	Proposed	2nd order, GLRT	(12)	$\times$	$-$
v	[2], [12]	Energy	(3)	$\times$	$-$
vi	Proposed	2nd order, GLRT	(11)	$\times$	$\times$
vii	Proposed	Optimal	(2)	$\times$	$\times$

TABLE II
Summary of detectors, where $-$ means unknown and $\times$ means known parameter respectively.

Example 1: Detector comparison (Figure 3).
We start by comparing the performance of the proposed detectors. The detectors (vi)-(vii) require knowledge of both σ_n^2 and σ_s^2, the detectors (iii)-(v) require knowledge of σ_n^2, and the detectors (i)-(ii) do not require any knowledge of these parameters. In this example, the number of received symbols is set to $K = 10$. Figure 3 shows the results.

If both σ_n^2 and σ_s^2 are perfectly known, it is clear that the detectors based on second-order statistics are suboptimal. In this scenario there is a $2-3$ dB gain in using the optimal detector (vii) based on the received data compared to the detector based on second order statistics (vi). Parts of this performance loss can be attributed to the approximations made when deriving the second-order statistics detector. It is worth noting that the energy detector is near-optimal (within 0.2 dB SNR) when σ_n^2 is known, even though the signal has a substantial correlation structure. This is also in line with [1], where the optimal detector for a BPSK modulated signal was derived, and it was shown that knowing the modulation format does not appreciably improve the detector performance over the energy detector. Moreover, the performance gain of the optimal detector (vii) (and the energy detector) over the GLRT detector (ii) is approximately 5 dB SNR. Thus, perfect knowledge of σ_n^2, can substantially improve the detection performance. Notable is also that knowledge of σ_s^2 does not significantly improve the detection performance, since the energy detector only requires knowledge of σ_n^2 to set the decision threshold. To conclude, if σ_n^2 is known, no significant improvement over the energy detector can be achieved.

However, if σ_n^2 is unknown, there can be a significant gain. We note that the GLRT detector (ii), proposed in this paper, outperforms the autocorrelation-based detector (i) in the low P_{MD} region, using the same prior knowledge. Moreover, the improvement increases with decreasing P_{MD} (increasing SNR). At low P_{MD} (below 10^{-3}), the performance improvement is in the order of 5 dB SNR. However, at high P_{MD} the autocorrelation-based detector (i) slightly outperforms the GLRT detector (ii). In the scenario considered here this occurs approximately for $P_{\text{MD}} > 0.8$. We believe this effect appears owing to the suboptimality of GLRT, especially with respect to the synchronization error. The introduction of cognitive radios in a primary network will require a smaller probability of

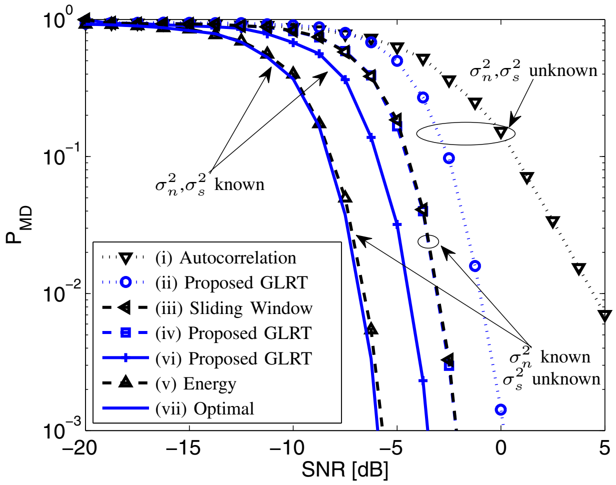

Fig. 3. Comparison of all detectors for $P_{FA} = 0.05$, $N_d = 32$, $N_c = 8$, $K = 10$.

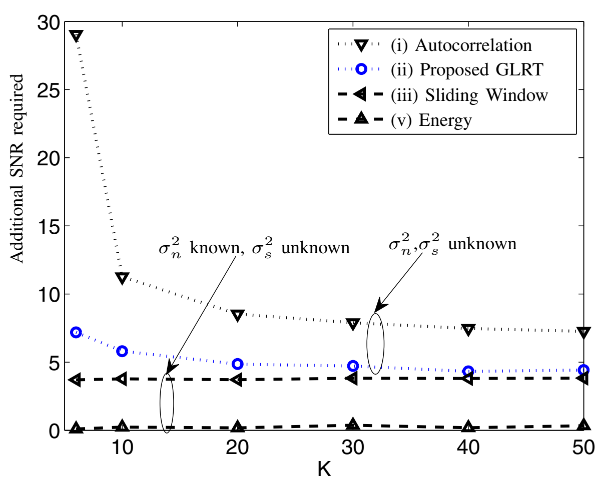

Fig. 4. Additional SNR relative to the optimal detector required to obtain $P_{MD} = 10^{-2}$, for different number of received OFDM symbols K.

missed detection to avoid causing too much interference. Then, in most relevant cases, the GLRT detector (ii) is preferable over the autocorrelation-based detector (i).

Example 2: Dependence on K (Figure 4).
In this example, we show the effect of increasing the number of received OFDM symbols K. We compare the additional SNR relative to the optimal detector for different values of K, required to obtain $P_{MD} = 10^{-2}$. Figure 4 shows the results. For the schemes (iii) and (v), that have complete knowledge of σ_n^2, the distance is constant independent of K. However for the schemes (i)-(ii), that do not have any knowledge of σ_n^2 (or σ_s^2), the distance decreases with increasing K. That is, the impact of not knowing σ_n^2 can be decreased by increasing the number of received samples.

VI. Concluding Remarks

In this work, we only considered an AWGN channel, which is a somewhat ideal assumption. In practice the channel is time-dispersive and parts of the correlation will be destroyed. However, the received signal will still be correlated, because the length of the cyclic prefix is designed with some margin to deal with the problem of intersymbol interference. Thus, the proposed detectors still works, although with degraded performance.

For simplicity, we made a few approximations in the derivation of the proposed GLRT detector. We used a Gaussian approximation via the central limit theorem, assumed approximately independent averaged sample value products, and assumed low SNR. The detector performance might be further improved by not making these approximations. In this work we used a GLRT-approach, which is not optimal. There are other ways of dealing with the unknown parameters, for example by estimation from the received data or by marginalization. These are topics for future research.

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