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Resource Allocation on the MISO Interference Channel

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"Never look down to test the ground before taking your next step;
only he who keeps his eye fixed on the far horizon will find his right road."

Dag Hammarskjöld, Markings

Abstract

The need for wireless communications has increased during the last decades. To increase the data rates of the communication links there is a need of allocating larger frequency bands. These bands are strictly regulated and the majority of the frequencies are allocated to licensed systems. The splitting of the bandwidth is orthogonal, which mean that the different systems are not interfering each other. But, orthogonal splitting is inefficient since it does not exploit all degrees of freedom in the wireless channels.

There are also unlicensed bands where different systems co-exist and operate simultaneously in a non-orthogonal manner and interfere each other. This interference degrades the performance of each system. This motivates the use of so-called spectrum sharing techniques for interference management.

The spectrum sharing can be modeled via the so-called interference channel (IFC). This consists of at least two transmitter (TX)-receiver (RX) pairs. These pairs can share resources such as frequency, time, power, code, or space. Here, the focus is on the sharing of spatial resources. By employing multiple antennas at the TXs, spatial diversity is obtained and it is possible to steer the power in any spatial direction. Assuming a single antenna at each RX we get the so-called multiple-input single-output (MISO) IFC.

There is a conflict inherent in the IFC since the TX-RX pairs optimize conflicting objectives, e.g., the data rates. To analyze this conflict we use game-theoretic concepts. In general, the situation where the TXs transmit in the directions which are optimal for their objective is inefficient. That is, it is possible increase all rates of some (or all) TX-RX pairs without decreasing the rate of any of the pairs. To do so, the TXs change their strategies such that interference is decreased.

We define several rate regions, which depend on the channel model and channel state information at the transmitters. Also, some of the most important game-theoretic operating points are described.

Acknowledgments

To be a PhD student is both inspiring and challenging. You have a lot of freedom and you can work with interesting and stimulating problems. Meanwhile, it is demanding, since you are constantly expected to deliver new scientific outputs. I have often been irritated at myself for not being more focused and efficient in my work. However, I have been supported by many persons, and I can now take the opportunity to thank you all.

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Linköping, April 2010
Johannes Lindblom

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Part I

Introduction

Chapter A

Resource Allocation for the Interference Channel

During the last decades, the need for wireless communication has increased. There is a need of transmitting more data in less time, i.e., the data rate of the communication links needs to be increased. High rates lead to broadband signaling, which means that we need large frequency bands. The use of frequency bands, or spectrum, is strictly regulated. The majority of the spectrum is allocated to licensed system such as GSM, 3GPP and DVB-T. Also, there are unlicensed bands, e.g. the Industrial, Scientific, and Medical (ISM) band and the Unlicensed National Information Infrastructure (UNII) band, which are used for systems such as 802.11 networks, Bluetooth systems, walkie-talkies. In these unlicensed bands, the different systems co-exist and operate simultaneously. Therefore, the different systems interfere with each other and this interference may substantially degrade the overall performance of each system. This motivates the use of so-called spectrum sharing techniques. For wireless systems it is possible to share or coordinate either frequency, time, power, code, or space. The focus of this thesis is on sharing of spatial resources, but we will touch on the sharing of time and power.

The spectrum sharing can be modeled via the so-called interference channel (IFC) [1]. The IFC consists of mutually interfering transmitter (TX)-receiver (RX) pairs. Sometimes, we will refer to a TX-RX pair as a user or player. For example, in wireless communications this pair can consist of a base station and an associated mobile station in a downlink cellular system. But, in wired communication it can model digital subscriber line in the same binder.

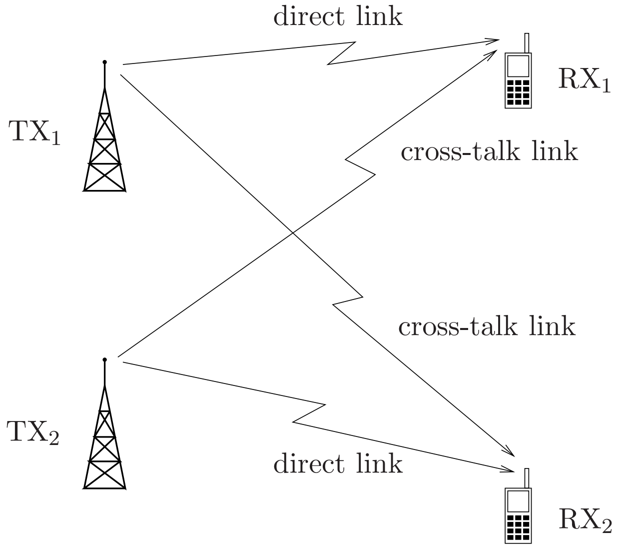


Figure 1: The 2-user IFC, illustrated for the scenario where both transmitters and receivers use a single antenna each.

The pairs share a common communication medium, i.e., they transmit simultaneously in the same frequency band. In Fig. 1 the two-user IFC is depicted. Here, TX_1 sends information to its intended receiver RX_1. But, it also interferes the unintended receiver RX_2 since there is a propagation path between them.

By employing multiple antennas at the transmitters, spatial diversity is obtained and it becomes possible to steer the power in arbitrary direction. We will show that in general it is suboptimal for the transmitters to transmit in the direction which is optimal for one system, disregarding the others. If the transmitters deviate from their optimal strategies and steer the power such that interference is decreased, then all systems might can their data rates.

Recently, the IFC has gained interest since it is a good model for conflict situations in wireless networks. A conflict occurs when two or more players who have different objectives act on the same system or share a common resource. In wireless networks, especially ad-hoc networks, the transmitters and receivers have been acting without cooperation. By cooperation we mean exchange of channel knowledge among the users and cooperative design of transmit strategies such that all users benefits, e.g., get higher rates. The lack of cooperation leads to a situation where the users act "selfishly" to maximize their own interests, disregarding the others. To analyze such conflicts, game theory has been used. This is a branch of mathematics, which servers as a tool for analyzing conflict situations. Specifically, these conflict situations correspond to optimization problems with multiple conflicting objectives. There is a rich literature on the use of game theory to study resource allocation in wireless networks. A brief survey of the literature can be found in [2]. The book [3] presents fundamental game theoretic results and their application to wireless communication and networking, while the book [4] is a standard text book in game theory. The book [5] presents a novel and comprehensive perspective for improving the performance of wireless systems by combining fundamentals of resource allocation and application examples. Sec. 6 presents a survey of the information theoretical research on the IFC.

In the following, we give the system model for the so-called K-user multiple-input multiple-output (MIMO) IFC. By MIMO we mean multiple transmit and receive antennas, respectively. As special cases of the K-user MIMO IFC, we present the two-user multiple-input single-output (MISO) IFC and the two-user single-input single-output (SISO) IFC. The latter is sometimes called the power game and it is illustrated in Fig. 1. Focused on the MISO single user channel, we explain how different channel models and channel knowledge affect the rate measures. Here, we only consider the frequency flat channel. This channel can either be constant (instantaneous), slow-fading, or fast-fading. Based on this and the channel knowledge it makes sense to define instantaneous rate, outage rate, or ergodic rate. Since there is a conflict inherent in the IFC, it is meaningful to use game theory to analyze it. We have a conflict situation since the users optimize conflicting objectives, i.e., their rates, while they are sharing a common resource. To analyze this we will define the rate region and explain some of the most important game-theoretic points. Finally, we will give a historical overview of the research on IFCs.

1 Definition of Notation and Concepts

In this section, we define some frequently used notation and explain concepts such as the fast- and slow-fading channels and the instantaneous, outage, and ergodic rates.

Matrices are denoted by boldface, uppercase letters, e.g. $\boldsymbol{X}$, whereas vectors are denoted by boldface, lowercase letters, e.g. $\boldsymbol{x}$. Scalars are denoted by italic, lowercase letters, e.g. x. By $\mathbb{C}^{n \times m}$ and $\mathbb{C}^{n}$ we denote the sets of complex $n \times m$ matrices and complex n-dimensional vectors, respectively. The conjugate (Hermitian) transpose of a complex matrix or a vector is denoted by $(\cdot)^H$. Calligraphic uppercase letters are used to denote sets, e.g. $\mathcal{R}$. $\mathrm{Tr}\{\cdot\}$, $(\cdot)^{1/2}$ denote the trace and the square root of a matrix, respectively. The matrix $\mathrm{Diag}\{\boldsymbol{x}\}$ is a diagonal matrix, with the elements in the vector $\boldsymbol{x}$ as diagonal elements. $\|\cdot\|$ is the Euclidean norm of a vector, and $\mathrm{E}\{\cdot\}$ is the expectation value of a random variable. If a random vector $\boldsymbol{x}$ is circularly complex Gaussian with zero mean and has covariance matrix $\boldsymbol{Q}$, we denote this as $\mathcal{CN}(\mathbf{0}, \boldsymbol{Q})$, where $\mathbf{0}$ is the zero-vector.

Throughout this thesis, we consider the flat-fading Gaussian IFC. Flat fading means that the communication bandwidth is much less than the coherence bandwidth of the channel. That is, the channel can be represented by a single channel filter tap [6]. By Gaussian we mean that the used codebooks approach to the Gaussian ones and that the additive noise at the receivers is Gaussian.

Here, we explain the different concepts regarding the behavior of the channels. The channels might vary fast or slow, and they can be deterministic or random. When the channels are constant over a time slot corresponding to at least the entire codeword, we say that the channels are instantaneous. For this scenario it is reasonable to assume that there is time to obtain accurate channel estimates and feed them back to the transmitters. When the transmitters perfectly know the channels we will say that they have *perfect* or *instantaneous* channel state information (CSI). On the other hand, the channel may be a random variable, which is the fading scenario. The fading can be either slow or fast. By *slow-fading* we mean that the channel is drawn randomly, but remains constant for the entire transmission time. This models the scenario where the delay requirements are short compared to the coherence time of the channel, [6]. For the slow-fading scenario we will study both case of perfect CSI and the case of statistical CSI. Here, statistical CSI refers to the scenario where the transmitters only know the statistical distribution of the channel. Sometimes, we use *imperfect* or *partial* CSI instead of statistical CSI. Let us now consider a scenario where the codewords span several coherence periods. This is the scenario of *fast-fading* for which we now would like to characterize the performance limit. When the fading is fast, it is hard to obtain accurate channel estimates within a coherence period. Therefore, we will assume that the transmitters do not have perfect CSI.

2 System Model

We consider the most general form of the IFC, namely the K-user MIMO IFC. This consists of K TX-RX pairs, where both TXs and RXs are equipped with multiple antennas. Each transmission will not only be received by the intended receiver but also the remaining $K-1$ ones. We say that the transmitter causes interference. Transmitter i employs n_{T_i} transmit antennas and receiver i employs n_{R_i} receive antennas. In this thesis, we assume that no interference cancellation is performed by the receivers. This is suboptimal, especially when the interference is strong compared to the useful signal, but it is a practical assumption. The aforementioned assumptions give the baseband signal model

$$\boldsymbol{y}_i = \boldsymbol{H}_{ii}\boldsymbol{x}_i + \sum_{j\neq i}^{K} \boldsymbol{H}_{ji}\boldsymbol{x}_j + \boldsymbol{e}_i, \qquad i = 1,\ldots,K, \tag{1}$$

where $\boldsymbol{y}_i \in \mathbb{C}^{n_{R_i}}$ is the signal vector received by receiver i, while $\boldsymbol{x}_i \in \mathbb{C}^{n_{T_i}}$ is the vector transmitted by source i. The matrix $\boldsymbol{H}_{ii} \in \mathbb{C}^{n_{R_i}\times n_{T_i}}$ is the direct channel of link i while $\boldsymbol{H}_{ji} \in \mathbb{C}^{n_{R_i}\times n_{T_j}}$ is the channel matrix of the cross-talk channel between transmitter j and receiver i. The element in row k and column l of matrix $\boldsymbol{H}_{ji}$ is the gain of the propagation channel between TX_i's lth transmit antenna and RX_i's kth receive antenna. The vector $\boldsymbol{e}_i \in \mathbb{C}^{n_{R_i}}$ represents the circularly symmetric complex Gaussian noise vector with covariance matrix $\boldsymbol{R}_i$. The second term of the right-hand side of (1) represents the multi-user interference received by RX_i caused by the other $K-1$ transmitters. Due to limitations, e.g. battery and power regulations, the transmit power is bounded. Without loss of generality, we can define the power bound as

$$\mathrm{E}\left\{\|\boldsymbol{x}_i\|^2\right\} = \mathrm{Tr}\{\boldsymbol{\Psi}_i\} \leq 1, \tag{2}$$

where $\boldsymbol{\Psi}_i \triangleq \mathrm{E}\{\boldsymbol{x}_i\boldsymbol{x}_i^H\}$ is the covariance matrix of the transmitted vector $\boldsymbol{x}_i$. Since $\boldsymbol{\Psi}_i$ is a covariance matrix, it must be positive semidefinite, which we denote as $\boldsymbol{\Psi}_i \succeq 0$. Let

$$\mathcal{D}_{n_{T_i}} \triangleq \{\boldsymbol{\Psi} \in \mathbb{C}^{n_{T_i}\times n_{T_i}} | \boldsymbol{\Psi} \succeq 0, \mathrm{Tr}\{\boldsymbol{\Psi}\} \leq 1\} \tag{3}$$

denote the set of feasible transmit covariance matrices of dimension $n_{T_i}\times n_{T_i}$. A low complexity, but not necessarily optimal, way to construct the vector $\boldsymbol{x}_i$ is to superimpose multiple streams as

$$\boldsymbol{x}_i = \boldsymbol{W}_i\,\mathrm{Diag}\{\boldsymbol{p}_i\}^{1/2}\,\boldsymbol{s}_i, \tag{4}$$

which, is the so-called linear precoding. In (4), $\boldsymbol{s}_i \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{I})$ is an n_i-dimensional vector containing the data symbols. That is, TX_i transmits n_i data symbols or streams. Element k in the n_i dimensional vector $\boldsymbol{p}_i$ is the power of data symbol k. The matrix $\boldsymbol{W}_i$ has dimension $n_{T_i} \times n_{T_i}$ and its columns are assumed to have unit norm and to be linearly independent. Then the power constraint (2) becomes

$$\mathrm{E}\Big\{ \|\boldsymbol{x}_i\|^2 \Big\} = \sum_{k=1}^{n_i} p_i \leq 1.$$

When the transmitted vector (4) contains several data symbols, i.e., $n_i > 1$, we will say that we have multi-stream transmission. If $n_i = 1$ we have single-stream transmission.

In this thesis, the focus is on the two-user MISO IFC, which is obtained from the MIMO by letting $K = 2$ and $n_{R_i} = 1$ for $i = 1, 2$. These values give channels represented as row vectors. In purpose of having a notation coherent with the included papers we let the channel vector $\boldsymbol{h}_{ij}$ from TX_i to RX_j to be a column vector. To simplify the math, the element of the vectors are the complex conjugates of the channel coefficients. The resulting baseband model of the received data at RX_i then becomes[1]

$$y_i = \boldsymbol{h}_{ii}^H \boldsymbol{x}_i + \boldsymbol{h}_{ji}^H \boldsymbol{x}_j + e_i, \quad i, j \in \{1, 2\}, \;\; j \neq i, \tag{5}$$

where $\boldsymbol{x}_i$ is defined as in (4). For the scenario of single antenna receivers, it is practical to use single-stream transmission. That is, $\boldsymbol{x}_i = \boldsymbol{w}_i s_i$, where $\boldsymbol{w}_i \in \mathbb{C}^{n_{T_i}}$ is the beamforming vector used by TX_i. Here, $\boldsymbol{w}_i = \sqrt{p_i}\boldsymbol{u}_i$, where $\boldsymbol{u}_i$ is a unit-norm vector. The channel vector $\boldsymbol{h}_{ji} \in \mathbb{C}^{n_{T_i}}$ is a column vector, which can be interpreted as a matrix of size $n_{T_i} \times 1$. The noise term e_i is $\mathcal{CN}(0, \sigma_i^2)$. Now, the power constraint is included in the beamforming vector and we can write (2) as

$$\mathrm{E}\Big\{ \|\boldsymbol{x}_i\|^2 \Big\} = \|\boldsymbol{w}_i\|^2 \leq 1. \tag{6}$$

Based on the power constraint (6), we can define the set of allowed beamforming vectors for transmitter i to be

$$\mathcal{W}_{n_{T_i}} = \{\boldsymbol{w} \in \mathbb{C}^{n_{T_i}} \,|\, \|\boldsymbol{w}\|^2 \leq 1\}. \tag{7}$$

[1]Whenever an expression is valid for both system, it is given for system i, whereas $j \neq i$ is interpreted as the interfering system.

Now, we assume that both transmitters and receivers employ a single antenna each, and we get the SISO IFC. Sometimes this setup is referred to as a power game [2]. The baseband signal received by RX_i then becomes

$$y_i = h_{ii} p_i s_i + h_{ji} p_j s_j + n_i, \tag{8}$$

where $h_{ji} \in \mathbb{C}$ is the scalar channel coefficient between TX_j and RX_i and $p_i \in \mathbb{R}$ is the power TX_i uses to transmit symbol s_i. The power constraint is now $p_i \leq 1$ and the set of feasible powers is

$$\mathcal{P}_i \triangleq \{p_i | 0 \leq p_i \leq 1\}. \tag{9}$$

3 Instantaneous, Ergodic, and Outage Rates and Rate Regions

In this section we consider the two-user MISO IFC to define different rate regions. Depending on whether the channel is instantaneous or fading (slow or fast fading) and the channel knowledge at transmitters and receivers, it makes sense to define different kind of rates. Each assumption on channel model and amount of CSI at the transmitters leads to different rate regions. We assume that no interference cancellation will be made at the receivers. Instead, the receivers treat interference as additive Gaussian noise. This is a suboptimal, but highly practical assumption, and for low and moderate interference it is close to optimal. Given the channel vectors $\boldsymbol{h}_{ii}$ and $\boldsymbol{h}_{ji}$, the beamforming vectors $\boldsymbol{w}_i$ and $\boldsymbol{w}_j$, and the noise variance σ_i^2 the achievable rate (in bits/channel use (bpcu)) at RX_i becomes [7]

$$R_i(\boldsymbol{w}_i, \boldsymbol{w}_j) = \log_2\left(1 + \frac{|\boldsymbol{h}_{ii}^H \boldsymbol{w}_i|^2}{\sigma_i^2 + |\boldsymbol{h}_{ji}^H \boldsymbol{w}_i|^2}\right). \tag{10}$$

Rate expressions for MIMO and SISO IFC's can for example be found in [8] and [9], respectively. We will refer to the rate (10) as the *instantaneous rate.* The best TX_i can do to maximize the rate R_i, given the power constraint $\|\boldsymbol{w}_i\|^2 \leq 1$ and disregarding what TX_j does, is to use the maximum ratio (MR) beamforming vector $\boldsymbol{w}_i^{\mathrm{MR}} = \boldsymbol{h}_{ii}/\|\boldsymbol{h}_{ii}\|$. The rate region given the rates in (10) will be defined as

$$\mathcal{R} = \bigcup_{\boldsymbol{w}_1, \boldsymbol{w}_2 \in \mathcal{W}_{n_{T_i}}} (R_1(\boldsymbol{w}_1, \boldsymbol{w}_2), R_2(\boldsymbol{w}_2, \boldsymbol{w}_1)). \tag{11}$$

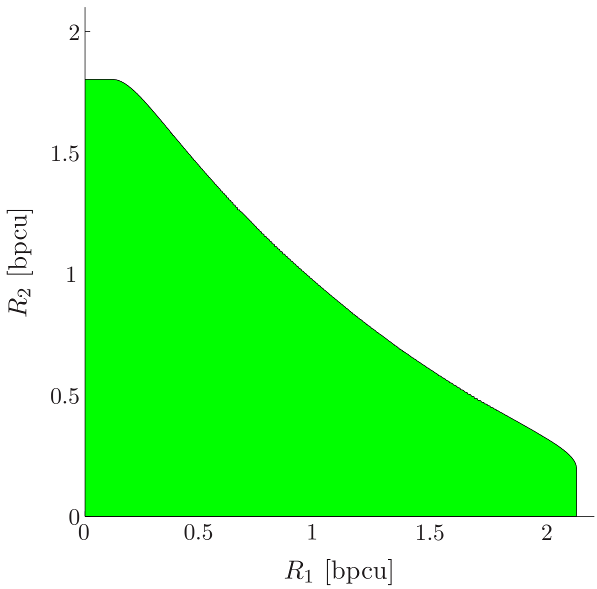


Figure 2: The shaded area is an example of a rate region.

The rate region consists of all rate pairs (R_1, R_2), which can be simultaneously achieved. In Fig. 2 we illustrate a rate region for the MISO IFC where the transmitters use three antennas each.

Now, we consider the fading scenario, i.e. the channels are random variables. Here, we model the channels as $\boldsymbol{h}_{ji} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{Q}_{ji})$. First, we consider the scenario of slow fading. Since the channel is a random variable, the rate (10) will become a random variable. Suppose that the TX_i encodes data at rate r_i. RX_i will only be able to decode the data if r_i is chosen such that $r_i < R_i(\boldsymbol{w}_i, \boldsymbol{w}_j)$. It does not matter which code the transmitter uses, the probability of decoding error cannot be arbitrarily small when the inequality is not satisfied. If the receiver fails to decode the data correctly, we say that the channel is in *outage*. The *outage probability* will become a measure of how reliable the communication links are. Assuming that the transmitters have statistical CSI, we define the *individual* outage probability ϵ_i of link i for a given pair of beamforming vectors $(\boldsymbol{w}_i, \boldsymbol{w}_j)$ and data rate r_i as

$$\Pr\left\{ r_i > \log_2\left(1 + \frac{|\boldsymbol{h}_{ii}^H \boldsymbol{w}_i|^2}{\sigma_i^2 + |\boldsymbol{h}_{ji}^H \boldsymbol{w}_i|^2}\right)\right\} = \epsilon_i. \tag{12}$$

The terms on the form $|\boldsymbol{h}_{ji}^H \boldsymbol{w}_j|^2$ are now exponentially distributed with mean $\mathrm{E}\{|\boldsymbol{h}_{ji}^H \boldsymbol{w}_j|^2\} = \boldsymbol{w}_j^H \boldsymbol{Q}_{ji} \boldsymbol{w}_j$, since $\boldsymbol{h}_{ji}^H \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{Q}_{ji})$. The transmitters can only adopt their beamforming vectors to the channels' second order statistics. We say that the systems are in *common* outage if at least one of the receivers fails to decode its intended message. We denote the common outage probability with ϵ. Then the probability that none of the links is in outage, given a pair of $(\boldsymbol{w}_1, \boldsymbol{w}_2)$ and data rates (r_1, r_2), becomes

$$\Pr\left\{r_1 < R_1(\boldsymbol{w}_1, \boldsymbol{w}_2),\ \ r_2 < R_2(\boldsymbol{w}_2, \boldsymbol{w}_1)\right\} = \epsilon, \tag{13}$$

where $R_i(\boldsymbol{w}_i, \boldsymbol{w}_j)$ is defined in (10). Since the fading process is slow, it makes sense to assume that the transmitters know the channels. When they do so, they can adopt their beamforming vectors to the current channel realization. We say that the channel is in outage given the rates r_1, r_2, if there is no pair of beamforming vectors such that the rate pair is in the rate region defined by (10)–(11). The common outage probability ϵ is then

$$\Pr\left\{(r_1, r_2) \notin \mathcal{R}\right\} = 1 - \epsilon. \tag{14}$$

Definitions of the corresponding outage rate regions can be found in Paper C, where we also provide closed form expressions of (12) and (13).

Let us now consider the fast-fading scenario for which we now would like to characterize the performance limit. Due to the fast fading we can interpret the scenario as transmission, including coding, over L parallel channels, for some number L. The average rate over these channels is then

$$R_i(\boldsymbol{w}_i, \boldsymbol{w}_j) = \frac{1}{L} \sum_{l=1}^{L} \log_2 \left(1 + \frac{|\boldsymbol{h}_{ii,l}^H \boldsymbol{w}_i|^2}{\sigma_i^2 + |\boldsymbol{h}_{ji,l}^H \boldsymbol{w}_j|^2}\right), \tag{15}$$

where $\boldsymbol{h}_{ji,l}$ is the channel vector for the lth channel from TX_j to RX_i and the beamforming vector $\boldsymbol{w}_i$ remains the same for all channels. If we let $L \to \infty$, the law of large numbers says that the quantity (15) becomes

$$\bar{R}_i(\boldsymbol{w}_i, \boldsymbol{w}_j) = \mathrm{E}\left\{\log_2 \left(1 + \frac{|\boldsymbol{h}_{ii}^H \boldsymbol{w}_i|^2}{\sigma_i^2 + |\boldsymbol{h}_{ji}^H \boldsymbol{w}_i|^2}\right)\right\}, \tag{16}$$

where the expectation is over the channels. The ergodic rate region will be defined in the same way as the instantaneous rate region (11). We will refer to the rate (16) as *ergodic rate*. This is maximized when $\boldsymbol{w}_i$ is parallel to the eigenvector corresponding to the largest eigenvalue of $\boldsymbol{Q}_{ii}$, see Paper B.

For the scenario of fast fading we will assume that the channel varies too fast for the transmitter to get perfect CSI, but still it knows the channels' statistical distribution. Again, the transmitter has to adopt its beamforming strategy to the channel statistics. In [10] it was shown that beamforming, i.e., single-stream transmission, under certain conditions is suboptimal when we only have statistical channel knowledge. When we transmit with a transmit covariance matrix of rank larger than the rank of the channel matrix, the detection problem will be hard to solve [11]. Therefore, it is practical to assume single-stream transmission for the MISO channel.

4 Convex Hull and Important Operating Points

In this section we define a convex hull of the rate region, and give a physical interpretation for it. Also, we introduce some important operating points and game theoretic concepts, which can be used to analyze the resource conflict. We will use the two-user MISO IFC with instantaneous CSI to explain the concepts. Also, we assume that both transmitters employ n transmit antennas.

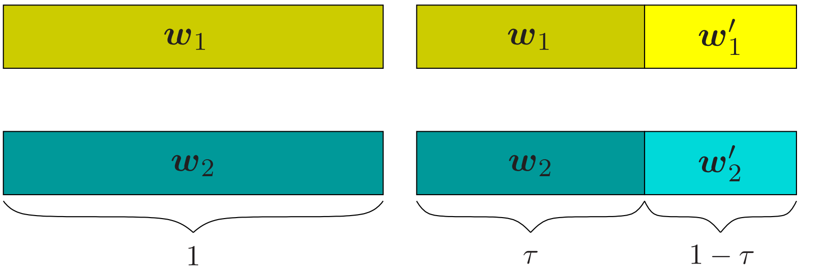
As shown in Fig. 2, the rate region defined in (11) is in general nonconvex. To find the convex hull of a region, one can take the convex combinations between all points in the region. First let (R_1, R_2) and (R'_1, R'_2) be two points in the rate region $\mathcal{R}$, defined in (11). These points are achieved by the beamformer pairs $(\boldsymbol{w}_1, \boldsymbol{w}_2)$ and $(\boldsymbol{w}'_1, \boldsymbol{w}'_2)$, respectively. The convex hull is then defined as

$$\bar{\mathcal{R}} \triangleq \bigcup_{(R_1,R_2),(R'_1,R'_2)\in\mathcal{R},\tau\in[0,1]} (\tau R_1 + (1-\tau)R'_1, \tau R_2 + (1-\tau)R'_2). \tag{17}$$

This can also be interpreted as time sharing between two operating points. During a fraction τ of the transmission period the beamforming vector pair $(\boldsymbol{w}_1, \boldsymbol{w}_2)$ is used and under the remaining time $(1-\tau)$ the pair $(\boldsymbol{w}'_1, \boldsymbol{w}'_2)$ is used. Time-sharing is illustrated in Fig. 3. In Fig. 4 is the outer boundary of the convex hull $\bar{\mathcal{R}}$ of rate region $\mathcal{R}$ illustrated.

4.1 Pareto Optimality and Pareto Boundary

The outer boundary of the rate region (or any utility region) is called the Pareto boundary of the region. This boundary consists of Pareto optimal operating points, where Pareto optimality is defined as follows:



(a) No time-sharing. (b) Time-sharing.

Figure 3: Illustration of time sharing - (a) no sharing, (b) time sharing where $\boldsymbol{w}_i' \neq \boldsymbol{w}_i$ for at least one transmitter.

Definition 1 A rate tuple $(R_1, R_2) \in \mathcal{R}$ is Pareto optimal if there is no other tuple $(Q_1, Q_2) \in \mathcal{R}$ with $(Q_1, Q_2) \geq (R_1, R_2)$ and $(Q_1, Q_2) \neq (R_1, R_2)$. (The inequality is component-wise.)

This definition will be later used to characterize the transmit strategies which achieve points on the Pareto boundary. In simple words, we say that a rate pair is Pareto optimal if there is no other rate pair such that we can increase the rate of one user without decreasing the rate of the other. Graphically, the Pareto boundary is the north east boundary of the rate region. The Pareto boundary of our example region is shown in Fig. 4.

Note that the horizontal and vertical parts of the boundary of rate region in Fig. 4 do not satisfy Def. 1. For these parts we talk about weak Pareto optimality, whereas for Def. 1 we talk about strong Pareto optimality.

4.2 Important Operating Points

Here, we present a number of important operating points in the rate region. Some points are valid for general games, others are more communication specific. The points, and in some cases how they can graphically be obtained, are illustrated in Fig. 5.

- The *utilitarian* point is the point on the Pareto boundary where the sum of the users' utilities is maximized. When we use the rate as utility,

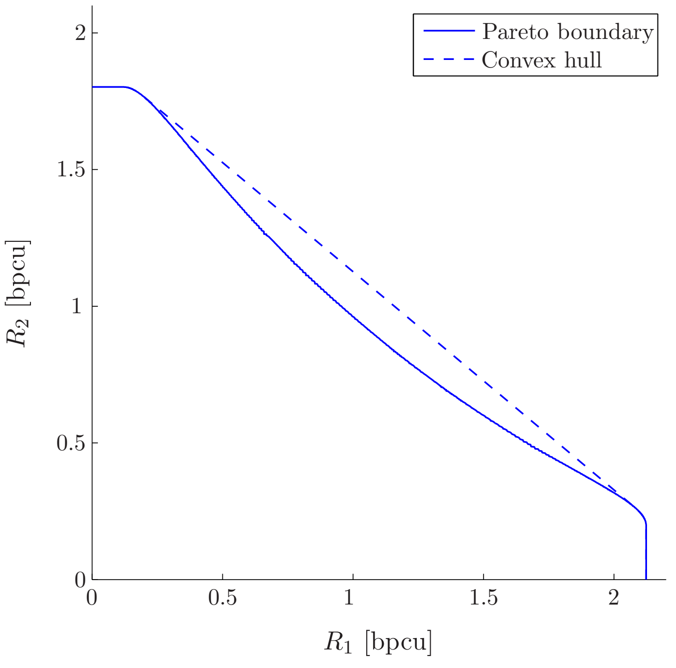


Figure 4: Illustration of the Pareto boundary of a rate region and the outer boundary of the convex hull of the region.

the point is also called the *maximum sum-rate point (SR)*. Graphically, this point can be found at the point where a line with slope -1 touches the boundary.

- The *egalitarian point* is the point where the minimum utility is maximized. It can graphically be found as the point where the Pareto boundary intersects with the line $R_1 = R_2$. This point is fair in the sense that all players get the same utility.

- The *single-user (SU)* points are the points on the boundary where one user gets maximum utility, whereas the others get no utility. For the IFC it is the case where only one transmitter is active and uses its MR beamforming vector. Then, we can say that the IFC is reduced to a single TX-RX pair.

- The *Nash equilibrium (NE)* is the point where none of the users can improve its utility by unilaterally changing its own strategy. For the

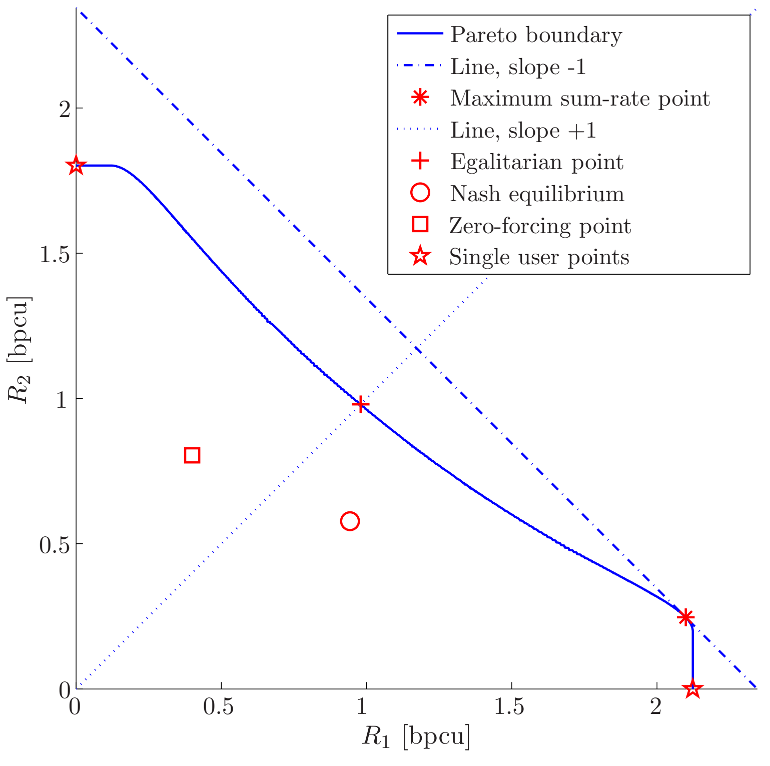


Figure 5: Illustration of the maximum sum-rate, egalitarian, NE, ZF, and single user points.

SISO IFC it is the point where all the transmitters use maximum power and for the MISO IFC all TX's use their maximum-ratio strategies. Finding the NE and corresponding transmit covariance matrices for the MIMO IFC is more involved and depends on the transmit covariance matrix of the other transmitter. For SISO and MISO IFC's the NE is unique, but for the MIMO IFC it is not necessarily unique, [8]. The NE is the only reasonable outcome if the users do not cooperate. In general, this point lies far inside the boundary of the utility region.

- The *zero-forcing (ZF)* point is achieved when all transmitters use strategies such that they maximize the useful power under a null-shaping constraint. By null-shaping we mean that the transmitters cause zero interference to the unintended receivers. We say that they use their ZF beamforming vectors. A trivial zero-forcing strategy is to not transmit at all, that is, $\boldsymbol{w} = \mathbf{0}$. To find a non-trivial ZF beamformer, we need to transmit in the null-space of the cross-talk channel. The null-space is non-empty when the cross-talk channel is known or has low-rank covariance matrix. Two examples of when the ZF strategy is to not transmit are the SISO IFC (channels are 1×1 matrices) and the case of MISO IFC with statistical CSI and full-rank channel covariance matrices. Also, the ZF point lies in general strictly inside the boundary of the region. We say that the ZF strategy is an altruistic strategy, since it helps the other user to maximize its rate, by causing no interference. But the user using its ZF strategy cannot expect to get something back from the other user.

4.3 Formulation of the Interference Channel as a Game

A game $\mathcal{G}$ can be represented by three elements as

$$\mathcal{G} = (\{1, \ldots, K\}, S, \{u_1, \ldots, u_K\}), \tag{18}$$

where in our case the players (or users) $\{1, \ldots, K\}$ are the TX-RX pairs which share wireless resources. The set S is the strategy space. In wireless systems the strategy space may consist of power, beamforming vectors, time-sharing parameters and the used spectral band. The third element, $\{u_1, \ldots, u_K\}$, is the set of utilities or payoffs. The utility in wireless communication can for example be rate at receivers, which we use here, bit-error rate, or signal-to-interference-plus-noise ratio (SINR).

The two-user covariance (MIMO) game can be represented as

$$\mathcal{G}_{\text{MIMO}} = (\{1, 2\}, \mathcal{D}_{n_{T_1}} \times \mathcal{D}_{n_{T_2}}, \{R_1, R_2\}), \tag{19}$$

where $\mathcal{D}$ is defined in (3). For the beamforming (MISO) game we get

$$\mathcal{G}_{\text{MISO}} = (\{1, 2\}, \mathcal{W}_{n_{T_1}} \times \mathcal{W}_{n_{T_2}}, \{R_1, R_2\}). \tag{20}$$

In [7] it was shown for the MISO IFC with instantaneous CSI that points on the Pareto boundary of the rate region are obtained when both transmitters use convex combinations of their MR and ZF beamformers. By using this characterization, the strategy space is reduced to $[0, 1]^2$. While we for the

MIMO and MISO games assume no time-sharing, we will use time-sharing for the power (SISO) game. We then get

$$\mathcal{G}_{\text{SISO}} = (\{1,2\}, \mathcal{P}_1 \times \mathcal{P}_2 \times [0,1], \{R_1, R_2\}), \tag{21}$$

where $\mathcal{P}_1$ and $\mathcal{P}_2$ are defined according to (9) and $[0,1]$ is the interval of the time-sharing parameter τ.

5 Cooperation on the Interference Channel

In cooperative games, the players, or transmitters, are allowed to strike deals with each other. Here, we are interested in the so-called nontransferable utility games [4]. In nontransferable utility games the players are not allowed to make side payments of utility in order to reach agreement. In the bargaining, a player can be both rational and cooperative. That is, the players can accept a found bargaining solution if it is good enough for both. This behavior can be mathematically modeled by using the bargaining theory of John Nash [12]. Nash formulated a number of axioms, (see, [4, 12] for details) and proved that the unique solution is given by

$$(\tilde{R}_1, \tilde{R}_2) = \arg \max_{(R_1^*, R_2^*) \leq (R_1, R_2) \in \mathcal{R}} (R_1 - R_1^*)(R_2 - R_2^*), \tag{22}$$

where (R_1^*, R_2^*) is the threat point, which will be the result if one player does not decide to bargain. Often it is natural to choose the NE as threat point. The solution of (22) is the so-called Nash bargaining solution (NBS). This point can graphically be found as the point where the hyperbola $(R_1 - R_1^*)(R_2 - R_2^*) = c$, where c is a constant that has a unique intersection with the Pareto boundary. It is natural to use the NE as the threat point, since it is the only reasonable outcome if the systems are not able to agree on a solution. The NBS is only defined on convex utility regions. That is, in general we have to consider the convex hull of the rate region. For simplicity, we will sometimes call the solution to the corresponding optimization problem the NBS. The NBS and how it is obtained graphically is illustrated in Fig. 6.

How fair are then the NBS and the points discussed in Sec. 4.2? The NBS yields a Pareto optimal point where both users gain compared to the noncooperative NE. The maximum sum-rate point is also Pareto optimal, but it can be unfair, as evidenced in Fig. 5. Here, user 1 gets a rate close to its single user rate, while user 2 gets a rate which is below its NE rate. The

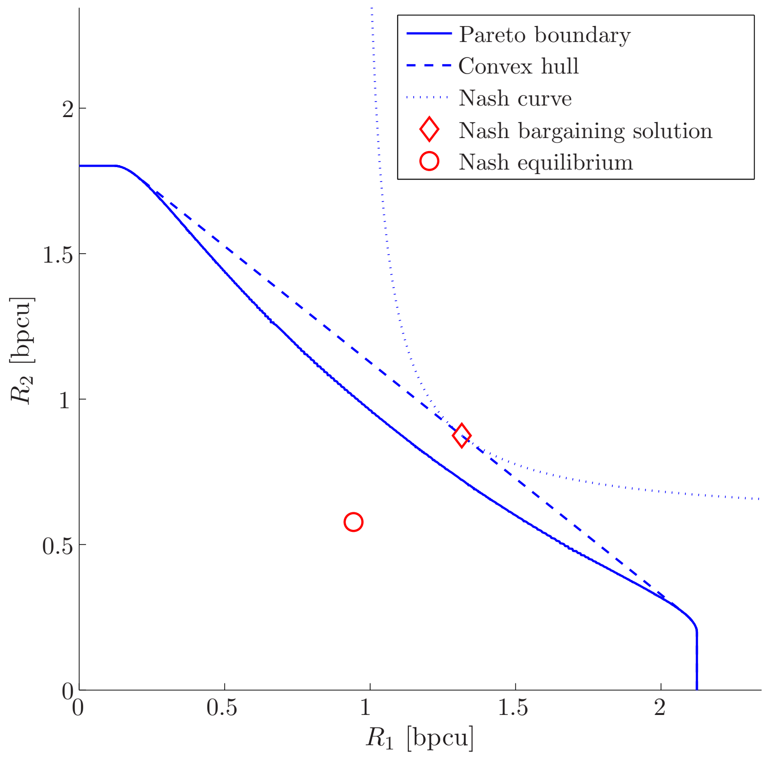


Figure 6: Illustration of the Nash bargaining solution, and the Nash curve.

egalitarian point is fair in the sense that both users get the same rate, but it might happen that one of the users gets a rate less than its NE rate.

Generally, it is hard to find Pareto optimal operating points such as the Nash bargaining solution and the maximum sum-rate point. Even if we know the rate pairs constituting one of these points, we do not directly have the corresponding transmit strategies. One way of finding optimal resource allocations is to let a central network manager do the allocation. The resource manager will then collect channel state information about all links, determine allocations for all transmitters, and tell the transmitters to use those allocations. These solutions require an exchange of information, the optimization problem might be non-convex (i.e., the solution cannot be obtained within polynomial time), and the complexity of the problem increases exponentially will the number of users and degrees of freedom. In addition to these concerns, there may be scenarios where it is impractical to have a centralized manager. For example, the two TX-RX pairs can be owned by different operators, which are not willing to share a common network unit. Another problem with a centralized method is the fact that the users might not report the true channel knowledge, but an adjusted version in order to benefit by increased rates. To address the concerns above, we would like to find algorithms which are distributed and require minimum of information exchange and channel information. It is also good if the algorithm is self-enforcing, in the sense that a transmitter which misbehaves will be punished. Of course, such algorithms may not achieve a Pareto optimal point, but the outcome may still be close to optimal, and far better than the non-cooperative outcome, i.e., the NE, for all involved players.

Different types of game approaches have been applied to several types of wireless resource conflicts. Price of anarchy has been one of the most important approaches. Here, a price for causing interference is used to force the users to act "more altruistically" [5]. In [13] the spectrum sharing for the SISO IFC with time sharing was studied. First the one-shot game was analyzed. Since the systems often coexist for a longer period, a repeated game is more appropriate to study. By building reputation and applying punishment, a larger set of self-enforcing outcomes is allowed. A distributed power control scheme for wireless ad hoc networks was studied in [14]. There, an asynchronous distributed algorithm for updating power levels and prices was presented. For the frequency-selective SISO IFC it was shown in [15] that the Nash bargaining solution could be computed using convex optimization. Also, the result was extended to the scenario of statistical CSI.

For the two-user MISO IFC there are a number of proposed algorithms for finding operating points on the Pareto boundary or close to the boundary, [16, 17, 18]. In the iterative algorithm proposed in [16] the transmitters start by using their MR strategies. In each iteration they add a portion of the altruistic ZF beamformer to the previous vector in each step. This algorithm finds a point which is better than the NE for both users. In [17] virtual SINR [19] was introduced and a specific Pareto optimal point was found. In [18], the previous works were extended to multicell MIMO channels.

There is not much work on cooperative transmissions schemes for the MIMO IFC. So far, the focus has been on algorithms for finding the non-cooperative NE, [8]. A distributed algorithm for finding a point close to the true maximum sum-rate of the MIMO IFC was presented in [20]. This iterative algorithm used prices to update the transmit covariance matrices.

6 Review of Prior Art

The problem of computing the capacity region of the general IFC is still an open problem in information theory. But, during the last 50 years some progress has been made. The study of IFCs was initiated by Shannon, [21] in the early 1960s. Ahlswede, [1], continued the work by Shannon by giving simple but fundamental inner and outer boundaries of the capacity region. By applying a superposition coding technique, originally devised for the broadcast channel, Carleial [22] was able to improve the achievable rate region of the memoryless IFC. Sato [23] was able to obtain several inner and outer bounds of the capacity region by transforming the original problem to either multiple-access or broadcast channels. So far, the best known achievable rate region for the general IFC was defined by Han and Kobayashi [24]. The computation of the Han and Kobayashi rate region is, in general, too complex to be practical attractive. Therefore, significant work has been spent on obtaining outer bounds on the rate region. Especially, the scenario of weak interference has been studied. Cheng and Verdu [25] proved that the restriction to Gaussian codebooks provides only an inner bound of the capacity region. Despite the fact that it is subobtimal, Gaussian codes are often assumed since they give us attractive math. Recently, Etkin et al. [26] were able to give a boundary, which they showed is within one bit of the capacity of the IFC.

During the last ten years, the so-called vector IFC has been studied. By vector IFC we mean the scenario were either the transmitter or the receivers have multiple antennas. The vector Gaussian IFC was introduced by Vishwanath and Jafar in 2004 [27]. They shown that the capacity region of the MISO IFC in strong interference is much harder to characterize than the corresponding capacity region for the SISO IFC. Another important contribution of [27] was that the rank of the precoding matrix is bounded by the number of users in the IFC. In [28], outer boundaries of the MIMO IFC were derived. Many of these boundaries are extensions of the results in [26] for the SISO IFC.

The later years research has been focused on finding various achievable rate regions of the IFC. Especially, the outer boundary, the so-called Pareto boundary, of the achievable rate region has been characterized for different setups. Some important contributions on the characterization of the rate region of the SISO IFC were made by Etkin et al. [13] and Charafeddine et al. [9]. Shang and Chen [29] showed that it is optimal to use single-stream transmission for the two-user MISO IFC with Gaussian signaling. The outer boundary of the rate region for the MISO IFC was characterized by Jorswieck et al. [7]. For the two-user MISO IFC they showed that boundary can be parameterized with two real valued scalar parameters.

7 Contributions of the Thesis

Paper A: Parameterization of the MISO IFC Rate Region: The Case of Partial Channel State Information

Authored by J. Lindblom, E. G. Larsson, and E. A. Jorswieck.

Published in the IEEE Transactions on Wireless Communications, Feb. 2010.

We study the achievable rate region of the multiple-input single-output (MISO) interference channel (IFC), under the assumption that all receivers treat the interference as additive Gaussian noise. We assume the case of two users, and that the channel state information (CSI) is only partially known at the transmitters. Our main result is a characterization of Pareto-optimal transmit strategies, for channel matrices that satisfy a certain technical condition. Numerical examples are provided to illustrate the theoretical results.

Paper B: Selfishness and Altruism on the MISO Interference Channel: The Case of Partial Transmitter CSI

Authored by J. Lindblom, E. Karipidis, and E. G. Larsson.

Published in the IEEE Communication Letters, Sep. 2009.

We study the achievable ergodic rate region of the two-user multiple-input single-output interference channel, under the assumptions that the receivers treat interference as additive Gaussian noise and the transmitters only have statistical channel knowledge. Initially, we provide a closed-form expression for the ergodic rates and derive the Nash-equilibrium and zero-forcing transmit beamforming strategies. Then, we show that combinations of the aforementioned selfish and altruistic, respectively, strategies achieve Pareto-optimal rate pairs.

Paper C: Outage Rate Regions for the MISO IFC

Authored by J. Lindblom, E. Karipidis, and E. G. Larsson.

Published at the 43rd Asilomar Conference on Signals, Systems, and Computers (ACSSC), Nov. 2009.

We consider the two-user multiple-input single-output (MISO) interference channel (IFC) and assume that the receivers treat the interference as additive Gaussian noise. We study the rates that can be achieved in a slow-fading scenario, allowing an outage probability. We introduce three definitions for the outage region of the IFC. The definitions differ on whether the rates are declared in outage jointly or individually and whether there is perfect or statistical information about the channels. Even for the broadcast and the multiple-access channels, which are special cases of the IFC, there exist several definitions of the outage rate regions. We provide interpretations of the definitions and compare the corresponding regions via numerical simulations. Also, we discuss methods for finding the regions. This includes a characterization of the beamforming strategies, which are optimal in the sense that achieve rate pairs on the Pareto boundary of the outage rate region.

Paper D: Pareto-Optimal Beamforming for the MISO Interference Channel With Partial CSI

Authored by E. Karipidis, A. Gründinger, J. Lindblom, and E. G. Larsson.

Published at the IEEE Third International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP), Dec. 2009.

We consider the problem of finding Pareto-optimal (PO) operating points for the multiple-input single-output (MISO) interference channel when the transmitters have statistical (covariance) channel knowledge. We devise a computationally efficient algorithm, based on semidefinite relaxation, to compute the PO rates and the enabling beamforming vectors. We illustrate the effectiveness of our algorithm by a numerical example.

Paper E: Cooperative Beamforming for the MISO Interference Channel

Published at the European Wireless Conference (EW), Apr. 2010.

Authored by J. Lindblom and E. Karipidis.

A distributed beamforming algorithm is proposed for the two-user multiple-input single-output (MISO) interference channel (IFC). The algorithm is iterative and uses as bargaining value the interference that each transmitter generates towards the receiver of the other user. It enables cooperation among the transmitters in order to increase both users' rates by lowering the overall interference. In every iteration, as long as both rates keep on increasing, the transmitters mutually decrease the generated interference. They choose their beamforming vectors distributively, solving the constrained optimization problem of maximizing the useful signal power for a given level of generated interference. The algorithm is equally applicable when the transmitters have either instantaneous or statistical channel state information (CSI). The difference is that the core optimization problem is solved in closed-form for instantaneous CSI, whereas for statistical CSI an efficient solution is found numerically via semidefinite programming. The outcome of the proposed algorithm is approximately Pareto-optimal. Extensive numerical illustrations are provided, comparing the proposed solution to the Nash equilibrium, zero-forcing, Nash bargaining, and maximum sum-rate operating points.

7.1 Publications not Included in the Thesis

The following publications contain work done by the author, but they are not included in this this.

- J. Lindblom, E. G. Larsson, and E. A. Jorswieck, "Parameterization of the MISO Interference Channel with Transmit Beamforming and Partial Channel State Information," in *Proc. Forty-Second Asilomar Conf. Signals, Systems Computers*, Pacific Grove, CA, USA, Oct. 2008, pp. 1103–1107.

We study the achievable rate region of the MISO IFC, under the assumption that all receivers treat the interference as additive Gaussian noise. The main result is a parameterization of the Pareto boundary for the case where there are two users, the transmitters use beamforming, and the channel state CSI is only partially known at the transmitters. The result is illustrated by two numerical results.

This paper is a conference version of Paper A, and treats a special case of the contribution in that paper.

- E. G. Larsson, E. Jorswieck, J. Lindblom, and R. Mochaourab, "Game theory and the flat fading Gaussian interference channel," *IEEE Signal Processing Mag.*, vol. 26, no. 5, pp. 18–27, Sep. 2009.

In this tutorial paper we describe some basic concepts from non-cooperative and cooperative game theory and illustrated them by three examples using the IFC model, namely, the power allocation game for SISO IFC, the beamforming game for MISO IFC and transmit covariance game for MIMO IFC.

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