Some aspects of iterative identification and control design schemes

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Abstract

In this report we study some different aspects of schemes for iterative identification and control design. By formulating the control objective as a criterion minimization task the problem of finding a model well suited for control appears to be closely related to a prediction error minimization problem in system identification. We discuss two ways of matching the control and identification criteria and evaluate their properties in simulations.

1 Introduction

The model is a fundamental concept in many engineering disciplines. Whenever a model is used one has to keep in mind that it only gives a simplified description of the real system. This is particularly important in control design, since the stability of the actual closed loop control system depends on the quality of the model used for control design. It is therefore natural to have the future use in mind during the modeling procedure. This fact has led to a big interest in the interplay between models obtained by system identification and control design, and the creation of a number of schemes for system identification aiming for control design. See [1] for a survey.

One important issue in the iterative schemes is how the model quality can be affected in system identification such that the control system performance is optimized. Since the schemes typically involves system identification carried out using closed loop data there is consequently also a strong relationship between the actual controller and the data that are generated in the control loop. The prefilters proposed in e.g. [2] and [3] have big similarities, however derived for different model structure. The aim of the paper is to evaluate the different prefiltering strategies in some simulation experiments.

The paper is organized as follows. Section 2 presents an introductory discussion on model based control design, while Section 3 contains some background facts in system identification. In Sections 4 - 5 we then review some different approaches to the problem of matching identification and control criteria and Section 6 contains a simulation example. Finally Section 7 contains some conclusions.

2 Model Based Control Design

Consider a discrete-time system

$$y(t) = G_0(q)u(t) + v(t) \tag{1}$$

where $u(t), y(t)$ and $v(t)$ denote input, output and disturbance respectively. The input-output dynamics is given by the transfer operator $G_0(q)$ for which we have a nominal model $G(q)$. Since the model $G(q)$ is an incomplete description of the actual system we introduce the relative model error

$$\Delta G(q) = \frac{G(q) - G_0(q)}{G(q)} \tag{2}$$

i.e.

$$G_0(q) = G(q)(1 + \Delta G(q)) \tag{3}$$

Assume now that we design a regulator

$$u(t) = C_r(q)r(t) - C_y(q)y(t) \tag{4}$$

using the model $G(q)$ such that the closed loop system is given by

$$y_d(t) = T_d(q)r(t) \tag{5}$$

where

$$T_d(q) = \frac{C_r(q)G(q)}{1 + C_y(q)G(q)} \tag{6}$$

is the designed closed loop transfer operator and $r(t)$ is the reference signal. When the regulator is applied to the actual system the output becomes

$$y(t) = T_0(q)r(t) + S_0(q)v(t) \tag{7}$$

where

$$T_0(q) = \frac{C_r(q)G_0(q)}{1 + C_y(q)G_0(q)} \tag{8}$$

is the actual closed loop transfer operator and where

$$S_0(q) = \frac{1}{1 + C_y(q)G_0(q)} \tag{9}$$

is the actual sensitivity function. The main question is now how well the designed regulator performs when it is applied to the real system. We will answer this question by considering the signal

$$\tilde{y}(t) = y(t) - y_d(t) = (T_0(q) - T_d(q))r(t) + S_0(q)v(t) \tag{10}$$

which, via some straightforward calculations, gives

$$\tilde{y}(t) = S_0(q)\Delta G(q)T_d(q)r(t) + S_0(q)v(t) \tag{11}$$

By then neglecting the disturbance v we obtain

$$\tilde{y} = S_0 \Delta G T_d r \tag{12}$$

or equivalently

$$\tilde{y} = \frac{S_d \Delta G}{1 + Q_d \Delta G} T_d r \tag{13}$$

where

$$S_d = \frac{1}{1 + C_y G} \tag{14}$$

is the designed sensitivity function and where

$$Q_d = \frac{C_y G}{1 + C_y G} \tag{15}$$

is the designed complementary sensitivity function. We have here also for convenience omitted the arguments.

Equation (12) can now be viewed in, at least, two different ways. First, when G and ΔG are fixed the problem of minimizing $\tilde{y}$ is a pure control design problem. On the other hand, when G is free the minimization of $\tilde{y}$ becomes a combined modeling and control problem. To emphasize this we will let the model and the regulator be parameterized by the parameter vector θ. The regulator transfer operators C_r and C_y depend on θ indirectly since the regulator coefficients typically are calculated from θ via some design step. We can then rewrite $\tilde{y}$ as

$$\tilde{y}(\theta) = \frac{1}{(1 + C_y(\theta)G_0)}(G(\theta) - G_0)C_r(\theta)\frac{1}{(1 + C_y(\theta)G(\theta))} \cdot r \tag{16}$$

which shows that $\tilde{y}(\theta)$ depends on the model θ in a fairly complicated way. The minimization of $\tilde{y}$ can then be expressed as

$$\min_{\theta} J(\theta) \tag{17}$$

where

$$J(\theta) = E[\tilde{y}^2(\theta)] \tag{18}$$

Using Parsevals relationship the criterion is expressed in the frequency domain as

$$J(\theta) = \int \mid G_0 - G(\theta) \mid^2 \mid \frac{1}{1 + C_y(\theta)G_0} \mid^2 \mid C_r(\theta) \mid^2 \mid \frac{1}{1 + C_y(\theta)G(\theta)} \mid^2 \Phi_r d\omega \tag{19}$$

where Φ_r is the spectrum of the reference signal. Minimizing $J(\theta)$ represents the ultimate goal in modeling for control design, and below we will discuss how the concept of optimal input design discussed in [4] and [5] and some of the proposed iterative schemes can be seen as approximate ways to minimize $J(\theta)$. Before that we shall however recall some background facts concerning system identification.

3 System Identification

Adopting the framework of [4] the starting point is a parametrized model structure

$$y(t) = G(\theta)u(t) + H(\theta)e(t) \tag{20}$$

where $e(t)$ is a zero mean white noise. The corresponding one-step-ahead predictor is

$$\hat{y}(t \mid t-1) = H^{-1}(\theta)G(\theta)u(t) + (1 - H^{-1}(\theta))y(t) \tag{21}$$

Equations (1) and (21) then yield the prediction error

$$\varepsilon(t) = y(t) - \hat{y}(t|t-1) = H^{-1}(\theta)((G_0 - G(\theta))u(t) + v(t)) \tag{22}$$

Let us then consider minimizing the criterion

$$V(\theta) = E[\varepsilon_F^2(t)] \tag{23}$$

where

$$\varepsilon_F(t) = L\varepsilon(t) \tag{24}$$

and where L is a stable filter. Using Parsevals relationship we can express the criterion

$$V(\theta) = \int [| G_0 - G(\theta) |^2 \Phi_u + \Phi_v] \mid \frac{L}{H(\theta)} \mid^2 d\omega \tag{25}$$

When the identification is carried out in closed loop the input signal is given by

$$u(t) = \frac{1}{1 + C_y G_0}(C_r r(t) - C_y y(t)) \tag{26}$$

where $r(t)$ is the reference signal. Assuming that $r(t)$ and $v(t)$ are uncorrelated the input spectrum becomes

$$\Phi_u = \frac{1}{| 1 + C_y G_0 |^2}(| C_r |^2 \Phi_r + | C_y |^2 \Phi_v) \tag{27}$$

Inserting (27) into the expression for $V(\theta)$ yields

$$V(\theta) = \int [\frac{| G_0 - G(\theta) |^2}{| 1 + C_y G_0 |^2} \mid C_r \mid^2 \Phi_r + \frac{| 1 + C_y G(\theta) |^2}{| 1 + C_y G_0 |^2} \Phi_v] \mid \frac{L}{H(\theta)} \mid^2 d\omega \tag{28}$$

Let us now assume that the noise model is $H(\theta) \equiv 1$, and that the disturbance term v can be neglected in the prediction error. The criterion $V(\theta)$ then becomes

$$V(\theta) = \int | G_0 - G(\theta) |^2 \left| \frac{1}{1 + C_y G_0} \right|^2 | C_r |^2 | L |^2 \Phi_r d\omega \tag{29}$$

Comparing the expression for $V(\theta)$ and $J(\theta)$, equations (29) and (19) respectively, we see that they very much resemble each other, but also that there are some important differences. First, the model $G(\theta)$ is found in several positions in the criterion $J(\theta)$ while it only shows up in the factor $G_0 - G(\theta)$ in the criterion $V(\theta)$. Second, the sensitivity function of the designed loop in $J(\theta)$ is replaced by the user chosen prefilter L in the expression for $V(\theta)$. We shall in the next sections discuss the effects of these differences, and briefly discuss some methods that have been proposed to cope with them.

4 Optimal Input Design

One method for matching the control and identification criteria is to use the ideas of optimal input design proposed in [4] and further discussed in [5]. In optimal input design the aim is to select an input spectrum for an identification experiment such that the resulting model gives optimal performance in the intended application. Here the focus is on identification for control design and more precisely the problem is to, using the model

$$y(t) = Gu(t) + He(t) \tag{30}$$

design a regulator such that the closed loop system becomes

$$y(t) = T_d r(t) + e(t) \tag{31}$$

where T_d is the desired closed loop transfer operator. This is obtained by using a regulator defined by

$$C_r = \frac{T_d H}{G} \qquad C_y = \frac{H-1}{G} \tag{32}$$

and this yields the sensitivity function

$$S_d = \frac{1}{H} \tag{33}$$

The problem posed in [4] is how the input spectrum Φ_u should be chosen when identifying the model G of the real system G_0 such that the error between the resulting output and the ideal output is minimized when the model is used for control design. In [4] it is then shown that the input spectrum and noise model should be chosen such that

$$\frac{\Phi_u}{\mid H_* \mid^2} = \frac{\mid T_d \mid^2}{\mid G_0 \mid^2} \mid S_0 \mid^2 \Phi_r \tag{34}$$

where S_0 denotes the sensitivity function obtained for the true system, i.e.

$$S_0 = \frac{1}{1 + C_y^0 G_0} = \frac{1}{H_0} \tag{35}$$

where

$$C_y^0 = \frac{H_0 - 1}{G_0} \tag{36}$$

Furthermore H_* denotes a fixed noise model, which we also can view as a prefilter. Using a fixed noise model the criterion in equation (25) becomes

$$V(\theta) = \int \mid G_0 - G(\theta) \mid^2 \frac{\Phi_u}{\mid H_* \mid^2} d\omega \tag{37}$$

where also we have put $L = 1$. The second term in the original $V(\theta)$, equation (28), does not play any role in the minimization when a fixed noise model is used. Inserting equation (34) yields

$$V(\theta) = \int \mid G_0 - G(\theta) \mid^2 \frac{\mid T_d \mid^2}{\mid G_0 \mid^2} \mid S_0 \mid^2 \Phi_r d\omega \tag{38}$$

or equivalently

$$V(\theta) = \int \mid G_0 - G(\theta) \mid^2 \mid C_r^0 \mid^2 \mid \frac{1}{1 + C_y^0 G_0} \mid^2 \frac{1}{\mid H_0 \mid^2} \Phi_r d\omega \tag{39}$$

where

$$C_r^0 = \frac{T_d H_0}{G_0} \tag{40}$$

This means that the optimal input can be generated in closed loop using the regulator defined by the true system, and using H_0 as the fixed noise model, or equivalently taking $H_* = 1$ and using the prefilter

$$L = \frac{1}{H_0} \tag{41}$$

i.e. the true sensitivity function. Returning to the original criterion $J(\theta)$ this corresponds to replacing $G(\theta)$ by G_0 in all places except for the factor $G_0 - G(\theta)$. It has to be noted that the result in [4] was derived using a first order Taylor expansion which implies that the result is valid only for small model errors.

5 Iterative Methods

5.1 Introduction

An alternative approach for trying to match the control and identification criteria is to replace $G(\theta)$ in the control criteria with some previous estimate in all positions except in the factor $G_0 - G(\theta)$. This means that the regulator acting in the feedback loop also is based on this estimate. If we let the current estimate be denoted θ_k we can formulate the minimization problem as

$$\min_{\theta} J_k(\theta) \tag{42}$$

where

$$J_k(\theta) = \int \mid G_0 - G(\theta) \mid^2 \mid \frac{1}{1 + C_y(\theta_k) G_0} \mid^2 \mid C_r(\theta_k) \mid^2 \mid \frac{1}{1 + C_y(\theta_k) G(\theta_k)} \mid^2 \Phi_r d\omega \tag{43}$$

This is now exactly the criterion that is minimized when we carry out closed loop identification using a regulator based on the model θ_k and using the nominal sensitivity function as prefilter, i.e.

$$L = \frac{1}{1 + C_y(\theta_k) G(\theta_k)} \tag{44}$$

The idea is then that the new model θ_{k+1} from the minimization of $J(\theta_k)$ (identification) will result in a new model θ_{k+1} which is used to compute the next regulator, which will be used in a new closed loop identification experiment, etc.

The idea of iterative closed loop identification and regulator design has been proposed and discussed by several authors for different regulator design methods and different model structures. In [2] regulator design using pole placement is applied to models of ARX and ARMAX structure, while in [3] the LQG design method is used together with models of output error structure. Further approaches are given in [6] and [7]. We shall in this paper concentrate on the pole placement and LQG methods, and we start by taking a look at the pole placement method.

5.2 Pole Placement Control

Consider the ARMAX model

$$Ay(t) = q^{-k}Bu(t) + Ce(t) \tag{45}$$

i.e.

$$G = \frac{q^{-k}B}{A} \qquad H = \frac{C}{A} \tag{46}$$

and the regulator

$$u(t) = \frac{\mathcal{T}r(t) - \mathcal{S}y(t)}{\mathcal{R}} \tag{47}$$

i.e.

$$C_r = \frac{\mathcal{T}}{\mathcal{R}} \qquad C_y = \frac{\mathcal{S}}{\mathcal{R}} \tag{48}$$

where $\mathcal{R}, \mathcal{S}$ and $\mathcal{T}$ are polynomials. The regulator polynomials are designed such that the closed loop system gets desired poles, which means that

$$P_C P_O = A\mathcal{R} + q^{-k}B\mathcal{S} \tag{49}$$

where P_C determines the closed loop poles and P_O is the observer polynomial. The sensitivity function then becomes

$$S_d = \frac{A\mathcal{R}}{P_C P_O} \tag{50}$$

In [2] it is proposed that, when an ARX-model is estimated (equation error estimation), data should be prefiltered using

$$L = \frac{\mathcal{R}}{P_C P_O} \tag{51}$$

This prefilter is the nominal sensitivity function except for the polynomial A. However, identifying an ARX-model includes estimating a noise model

$$H(\theta) = \frac{1}{A(\theta)} \tag{52}$$

so the weighting in the identification criterion becomes

$$LH^{-1}(\theta) = \frac{A(\theta)\mathcal{R}}{P_C P_O} \tag{53}$$

which can be seen as a sensitivity function consisting of nominal and estimated quantities. This filtering strategy will be tested in a simulation example later in the report.

The same arguments can be applied when an ARMAX-model is estimated, and the estimated C-polynomial is used as observer polynomial. In [2] it is proposed to use

$$L = \frac{\mathcal{R}}{P_C} \tag{54}$$

and with the estimated noise model

$$H(\theta) = \frac{C(\theta)}{A(\theta)} \tag{55}$$

this gives the weighting

$$LH^{-1}(\theta) = \frac{A(\theta)\mathcal{R}}{C(\theta)P_O} \tag{56}$$

which also can be considered as a sensitivity function containing nominal and estimated quantities.

5.3 LQG Control

The derivation of the data filter in the LQG case presented in [3] is also based on a comparison of two control loops. Letting y and u denote the output and control signals when the system G_0 is controlled by some fixed regulator we have

$$y(t) = \frac{C_r G_0}{1 + C_y G_0} r(t) + \frac{1}{1 + C_y G_0} v(t) \tag{57}$$

and

$$u(t) = \frac{C_r}{1 + C_y G_0} r(t) - \frac{C_y}{1 + C_y G_0} v(t) \tag{58}$$

By furthermore letting $y_d(t)$ and $u_d(t)$ denote the output when the model G is controlled by the same regulator in a noise free loop we have

$$y_d(t) = \frac{C_r G}{1 + C_y G} r(t) \tag{59}$$

and

$$u_d(t) = \frac{C_r}{1 + C_y G} r(t) \tag{60}$$

The aim is then to minimize

$$J_N = \frac{1}{N} \sum_{t=1}^{N} (y(t) - y_d(t))^2 + \lambda (u(t) - u_d(t))^2 \tag{61}$$

When N tends to infinity the criterion can be expressed

$$J = \int \left[\frac{| G_0 - G(\theta) |^2 \, (1 + \lambda \mid C_y |^2)}{| 1 + C_y G_0 |^2 | 1 + C_y G |^2} \mid C_r |^2 \, \Phi_r + \frac{(1 + \lambda \mid C_y |^2)}{| 1 + C_y G_0 |^2} \Phi_v \right] d\omega \tag{62}$$

or

$$J = \int \left[\frac{| G_0 - G(\theta) |^2}{| 1 + C_y G_0 |^2} \mid C_r |^2 \, \Phi_r + \frac{| 1 + C_y G |^2}{| 1 + C_y G_0 |^2} \Phi_v \right] \frac{(1 + \lambda \mid C_y |^2)}{| 1 + C_y G |^2} d\omega \tag{63}$$

Comparing this integral with the prediction error criterion $V(\theta)$, equation (28), implies that the identification should be done in closed loop using the prefilter

$$L = \frac{M}{1 + C_y G} \tag{64}$$

where M satisfies

$$| M |^2 = (1 + \lambda \mid C_y |^2) \tag{65}$$

Since the prefilter here contains the model to be estimated it is replaced by the current estimate. Apart from the factor M the prefilter then becomes the nominal sensitivity function.

6 Simulation Example

We shall now illustrate and evaluate some of the proposed methods for matching the control and identification criteria using a simulation example. In the design of a test example there are several things that have to be chosen. We have to specify the "true system", the structure and order of the model, the control design method, the closed loop specifications and the character of the signals involved. In the example we shall study the third order system

$$Y(s) = \frac{2}{(s+1)} \cdot \frac{229}{(s^2 + 30s + 229)} U(s) \tag{66}$$

which, when it is sampled using sampling interval $T = 0.04$, has the discrete-time transfer operator

$$G_0 = \frac{q^{-1}(0.0036 + 0.0107q^{-1} + 0.0019q^{-2})}{(1 - 2.0549q^{-1} + 1.3524q^{-2} - 0.2894q^{-3})} \tag{67}$$

The system will be identified using ARX- and OE-models of first and second order, and the control design will be done using the pole placement method. The closed loop system will be specified by a desired bandwidth ω_B as follows. For a first order continuous time system

$$G(s) = \frac{a}{s+a} \tag{68}$$

we have $\omega_B = a$. A pole for the continuous time system in a corresponds to a pole for the discrete time system in e^{-Ta}, where T is the sampling interval. This will not give exactly the correct bandwidth for the discrete time system for values of ω_B close to the Nyquist frequency, but a reasonable approximation in the frequency range we will consider.

For the second order model there will be two closed loop poles to place and we will for simplicity choose two real poles. The continuous time system

$$G(s) = \frac{a^2}{(s+a)^2} \tag{69}$$

has bandwidth

$$\omega_B = a\sqrt{\sqrt{2} - 1} \tag{70}$$

and we will hence place the (continuous time) closed loop poles in

$$s = -\omega_B/\sqrt{\sqrt{2} - 1} \tag{71}$$

The observer pole is placed in ω_B.

The reference signal is in all experiments chosen as white noise with unit variance filtered through a second order Butterworth low pass filter with bandwidth ω_B. The input and output data sequences contain 30000 data each.

As mentioned we will identify models using both ARX- and output error model structures. Apart from the difference in the computation of the estimate the two model structures will have two main differences. First, computing the ARX model involves the estimation of a disturbance model, which will affect the frequency domain fit of the model, while the OE structure does not include any disturbance model. Second, since the predictor for an output error model is the transfer function from input to output this always has to be stable. This is means that the OE function always produces stable models, while the ARX function can deliver unstable models. This subject will be further discussed below.

6.1 First Order Model

6.1.1 Total Minimization

Since we deal with a simulation experiment where we know the true system it is in fact possible to minimize the control criterion $J(\theta)$ and to actually obtain the model that gives the optimal control performance. This problem can be expressed as

$$\min_{\theta} E[\tilde{y}^2(\theta)] \tag{72}$$

where

$$\tilde{y}(\theta) = (T_d - T_0(\theta))r \tag{73}$$

is the difference between the designed and achieved output, and r has some specified character. In this first order case T_d will be the same for all models so we can consider this as independent of θ. Furthermore the first order model implies

$$\mathcal{R} = 1 \qquad \mathcal{S} = s_0 \qquad \mathcal{T} = t_0 \tag{74}$$

and hence

$$T_0(\theta) = \frac{t_0(\theta)G_0}{1 + s_0(\theta)G_0} \tag{75}$$

The minimizing θ and the resulting loss function depends on the specifications of the closed loop system, and we will carry out the minimization for different values of the desired closed loop bandwidth. The minimization is carried out using the MATLAB function `fminu`, and the results are given in Table 1 below. These values can now be used for comparison when we evaluate some different filtering strategies. It can also be noted that the minimizing model for ω_B larger than 4 is unstable.

6.1.2 Identification Using Optimal Input

Another possibility in a simulated experiment is to evaluate the optimal input design. The results for the optimal input design in [4] were derived for a slightly different formulation of the pole placement problem, but we will apply the interpretation of this result to our problem. The interpretation was that in optimal input design the model G in the criterion $J(\theta)$ was replaced by the true system. This idea can of course also be applied here, and results in a closed loop experiment with the regulator calculated for the true transfer function and the true sensitivity function acting as prefilter. The results are given in Table 1 below.In the table we see that this method for low bandwidths gives only slightly higher value of J. For larger $\omega_B 4$ the criterion starts to increase and for bandwidths larger than 5 the closed loop system becomes unstable. The reason is that the optimal input design, derived using the assumption that the model errors are small, in this case emphasizes a fit at high frequencies.

ω_B	Total minimization	Optimal input
1	$2.4 \cdot 10^{-5}$	$2.5 \cdot 10^{-5}$
2	$2.1 \cdot 10^{-4}$	$2.2 \cdot 10^{-4}$
3	$8.2 \cdot 10^{-4}$	∞
4	$2.3 \cdot 10^{-3}$	$3.3 \cdot 10^{-3}$
5	$4.8 \cdot 10^{-3}$	$9.7 \cdot 10^{-3}$
6	$8.8 \cdot 10^{-3}$	∞
7	$1.4 \cdot 10^{-2}$	∞
8	$2.2 \cdot 10^{-2}$	∞
9	$3.1 \cdot 10^{-2}$	∞
10	$4.2 \cdot 10^{-2}$	∞

Table 1: Loss function in total minimization and identification using optimal input

6.1.3 Iterative Design

The test of the iterative design schemes starts by identifying a model from open loop data. The model is then used in the design of a pole placement regulator which is used for collecting a new set of data from the system, now acting in closed loop. This procedure is repeated ten times and the value of the criterion after the tenth iteration is used for the comparison.

The iterative design method is tested in three different version for each model structure. First, the input output data collected from the closed loop system are used directly without any extra processing. Second, the input and output signals are filtered through a fifth order Butterworth low pass filter with bandwidth equal to the closed loop bandwidth. The idea behind this filtering is to keep the fit of the low order to the higher order system in the low frequency range. Third, data are filtered using the nominal sensitivity function using the current regulator and the model from the previous identification experiment. When we use OE-models this means that data before computing the model θ_k are filtered through the filter

$$L(q) = \frac{1}{1 + C_y(\theta_{k-1})G(\theta_{k-1})} \tag{76}$$

For ARX-models the filter is given by

$$L(q) = \frac{\mathcal{R}(\theta_{k-1})}{P_C P_O} \tag{77}$$

as discussed above.

The results from the simulations are summarized in Tables 2 and 3 below. Using these results the following observations can be made:

- The OE-model allows larger bandwidths than the ARX-model. This is related to the fact that OE-models has to be stable, and this will be further discussed below.
- For ARX-models low pass filtering allows larger bandwidth than the other two alternatives. Since a first order model gives a poor model of a third order system at high frequencies the low pass filtering is beneficial since it keeps the fit at low frequencies.

ω_B	No filtering	Low pass filter	Sensitivity function
1	$2.6 \cdot 10^{-5}$	$2.5 \cdot 10^{-5}$	$2.3 \cdot 10^{-5}$
2	$2.7 \cdot 10^{-4}$	$2.2 \cdot 10^{-4}$	$2.3 \cdot 10^{-4}$
3	$1.1 \cdot 10^{-3}$	$9.2 \cdot 10^{-4}$	$9.0 \cdot 10^{-4}$
4	$3.4 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$2.6 \cdot 10^{-3}$
5	$9.9 \cdot 10^{-3}$	$5.3 \cdot 10^{-3}$	$6.5 \cdot 10^{-3}$
6	∞	$1.1 \cdot 10^{-2}$	$1.8 \cdot 10^{-2}$
7	∞	$2.1 \cdot 10^{-2}$	∞
8	∞	$4.2 \cdot 10^{-2}$	∞
9	∞	∞	∞
10	∞	∞	∞

Table 2: Loss function for iterative design using first order ARX-model

ω_B	No filtering	Low pass filter	Sensitivity function
1	$2.5 \cdot 10^{-5}$	$2.7 \cdot 10^{-5}$	$2.4 \cdot 10^{-5}$
2	$2.3 \cdot 10^{-4}$	$2.6 \cdot 10^{-4}$	$2.6 \cdot 10^{-4}$
3	$1.1 \cdot 10^{-3}$	$1.0 \cdot 10^{-3}$	$9.8 \cdot 10^{-4}$
4	$3.2 \cdot 10^{-3}$	$2.9 \cdot 10^{-3}$	$2.6 \cdot 10^{-3}$
5	$8.1 \cdot 10^{-3}$	$7.2 \cdot 10^{-3}$	$8.8 \cdot 10^{-3}$
6	$1.9 \cdot 10^{-2}$	$1.4 \cdot 10^{-2}$	$7.7 \cdot 10^{-2}$
7	$9.4 \cdot 10^{-2}$	$2.8 \cdot 10^{-2}$	$1.0 \cdot 10^{-1}$
8	$1.2 \cdot 10^{-1}$	$1.2 \cdot 10^{-1}$	$1.2 \cdot 10^{-1}$
9	$1.4 \cdot 10^{-1}$	$1.4 \cdot 10^{-1}$	$1.5 \cdot 10^{-1}$
10	$1.8 \cdot 10^{-1}$	$1.6 \cdot 10^{-1}$	$1.7 \cdot 10^{-1}$

Table 3: Loss function for iterative design using first order OE-model

6.1.4 Unstable Models

As was seen in the simulations there can be a significant difference between the results using ARX- and OE-models due to the stability properties of the identified model. The system is identified using a first order model

$$G(\theta) = \frac{bq^{-1}}{1 + aq^{-1}} \tag{78}$$

and using the first order model we compute a pole placement regulator that gives the closed loop system

$$T_d = \frac{(1+\alpha)q^{-1}}{1+\alpha q^{-1}} \tag{79}$$

For the first order this is obtained by the controller

$$u(t) = t_0 r(t) - s_0 y(t) \tag{80}$$

where

$$s_0 = \frac{\alpha - a}{b} \qquad t_0 = \frac{1+\alpha}{b} \tag{81}$$

Consider now the third order system G_0 presented above. Assume that the input to the system is a pure sinusoid with angular frequency ω, and that we identify a first order model

$$G(\theta) = \frac{bq^{-1}}{1 + aq^{-1}} \tag{82}$$

from the sinusoidal input output data. Furthermore assume that we repeat this experiment for a number of frequencies between zero and the Nyquist frequency. The results of this experiment are then given in Figures 1 and 2 below, which show the estimated $a-$ and $b-$parameter from the ARX- and OE-model respectively. As can be seen in the figure the $a-$parameter always has magnitude less than one, while the same parameter in the ARX-case for some frequencies attains very large values. Looking at the $b-$parameter we see that these too attains large values at some frequencies.

The reason for this behavior is of course the fit of a low order model to a higher order system. When the input is a pure sinusoid we try to fit the model exactly to the true system at that particular frequency. In some frequency ranges such a fit requires the model to be unstable. The most critical case is when the phase of G_0 is $-180°$. For this particular example this happens at around 12 rad/s. The behavior of the parameters is explained by by just computing the gain and phase of the first order system. This gives

$$\arg G(e^{i\omega}) = \arg b - \arctan \frac{\sin \omega}{a + \cos \omega} \tag{83}$$

and

$$\mid G(e^{i\omega}) \mid = \frac{\mid b \mid}{\sqrt{1 + a^2 + 2a\cos\omega}} \tag{84}$$

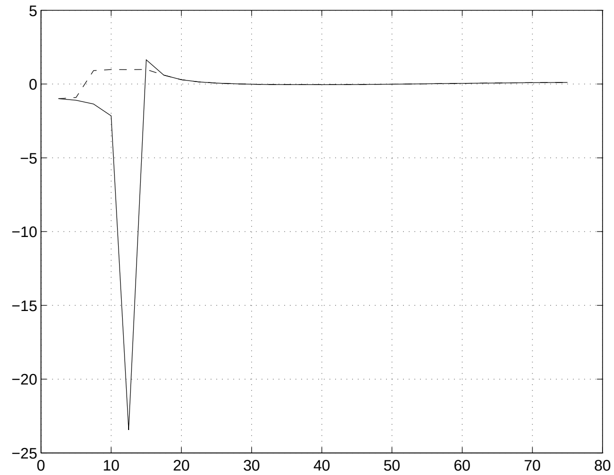

Figure 1: a-parameter as function of frequency. Solid ARX-model. Dashed OE-model

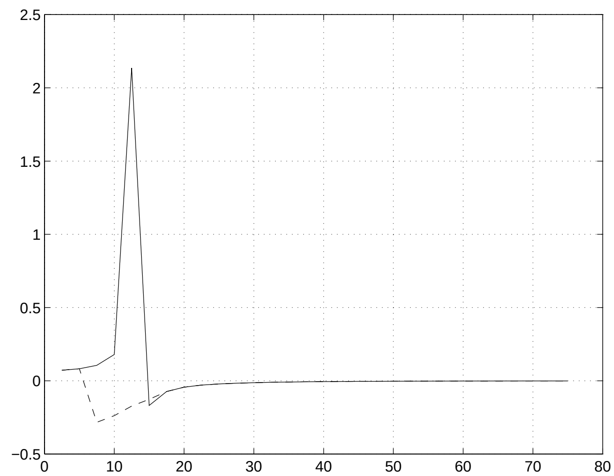

Figure 2: b-parameter as function of frequency. Solid ARX-model. Dashed OE-model

In the first expression we see that the only way of obtaining phase equal to -180° is to let a tend to infinity. However, to obtain the right gain b must then also tend to infinity.

Let us then consider to consequences of stable and unstable models for the control design. Recall the expression for the feedback gain

$$s_0 = \frac{\alpha - a}{b} \tag{85}$$

When the model fit is pushed to higher frequencies we obtain different properties of s_0 depending on the model structure. For the OE-structure the $b-$parameter starts growing while a remains between -1 and 1. Hence s_0 will decrease when we approach the frequency where the true system has phase -180°. For the ARX-structure the feedback gain will behave like

$$s_0 = \frac{-a}{b} \tag{86}$$

a and b will then tend to infinity in a way such that s_0 tend to the inverse of the gain of G_0, i.e the closed loop system tends to the boundary of the stability region. From this viewpoint the OE-model is to be preferred.

6.2 Second order model

6.2.1 Total minimization and identification using optimal input

We now increase the model order and carry out experiments similar to those above for second order models of ARX and OE structure. To start with we do the direct minimization experiment and identification using optimal input. The results from these simulations are summarized in Table 4 below. Using a second order model the results from identification using optimal input are very close to those from the direct minimization. The explanation is that, using a second order model, the model error is small, such that the optimal input design are applicable.

ω_B	Total minimization	Optimal input
1	$4.8 \cdot 10^{-9}$	$4.8 \cdot 10^{-9}$
2	$3.9 \cdot 10^{-8}$	$4.0 \cdot 10^{-8}$
3	$2.9 \cdot 10^{-7}$	$2.9 \cdot 10^{-7}$
4	$1.4 \cdot 10^{-6}$	$1.4 \cdot 10^{-6}$
5	$4.9 \cdot 10^{-6}$	$4.9 \cdot 10^{-6}$
6	$1.3 \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$
7	$3.0 \cdot 10^{-5}$	$3.0 \cdot 10^{-5}$
8	$6.0 \cdot 10^{-5}$	$6.1 \cdot 10^{-5}$
9	$1.1 \cdot 10^{-4}$	$1.1 \cdot 10^{-4}$
10	$1.8 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$

Table 4: Loss function in total minimization and identification using optimal input

6.2.2 Iterative Design

The results from the iterative design procedure using second order models are presented in Tables 5 and 6. The observations can be summarized as follows:

- The closed loop system is stable for all bandwidths.
- Filtering using the sensitivity function gives the best performance in almost all cases.

ω_B	No filtering	Low pass filter	Sensitivity function
1	$1.4 \cdot 10^{-7}$	$9.2 \cdot 10^{-9}$	$5.4 \cdot 10^{-8}$
2	$1.2 \cdot 10^{-7}$	$4.9 \cdot 10^{-8}$	$3.5 \cdot 10^{-8}$
3	$1.3 \cdot 10^{-6}$	$3.6 \cdot 10^{-7}$	$2.6 \cdot 10^{-7}$
4	$1.0 \cdot 10^{-5}$	$1.8 \cdot 10^{-6}$	$1.4 \cdot 10^{-6}$
5	$3.7 \cdot 10^{-5}$	$6.2 \cdot 10^{-6}$	$4.9 \cdot 10^{-6}$
6	$9.3 \cdot 10^{-5}$	$1.6 \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$
7	$1.9 \cdot 10^{-4}$	$3.6 \cdot 10^{-5}$	$3.0 \cdot 10^{-5}$
8	$3.5 \cdot 10^{-4}$	$7.4 \cdot 10^{-5}$	$6.2 \cdot 10^{-5}$
9	$5.7 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	$1.1 \cdot 10^{-4}$
10	$8.5 \cdot 10^{-4}$	$2.1 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$

Table 5: Loss function for iterative design using second order ARX-model

ω_B	No filtering	Low pass filter	Sensitivity function
1	$3.4 \cdot 10^{-8}$	$6.3 \cdot 10^{-9}$	$8.3 \cdot 10^{-9}$
2	$3.6 \cdot 10^{-8}$	$4.9 \cdot 10^{-8}$	$3.3 \cdot 10^{-8}$
3	$2.8 \cdot 10^{-7}$	$4.0 \cdot 10^{-7}$	$2.6 \cdot 10^{-7}$
4	$1.5 \cdot 10^{-6}$	$2.1 \cdot 10^{-6}$	$1.3 \cdot 10^{-6}$
5	$5.2 \cdot 10^{-6}$	$7.4 \cdot 10^{-6}$	$5.0 \cdot 10^{-6}$
6	$1.5 \cdot 10^{-5}$	$2.0 \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$
7	$3.4 \cdot 10^{-5}$	$4.7 \cdot 10^{-5}$	$3.0 \cdot 10^{-5}$
8	$6.9 \cdot 10^{-5}$	$1.0 \cdot 10^{-4}$	$6.0 \cdot 10^{-5}$
9	$1.3 \cdot 10^{-4}$	$1.9 \cdot 10^{-4}$	$1.0 \cdot 10^{-4}$
10	$2.2 \cdot 10^{-4}$	$3.2 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$

Table 6: Loss function for iterative design using second order OE-model

6.3 Second order model, fifth order system

Finally we apply the iterative methods with second order models to a system of fifth order, simply obtained by appending two more time constants to the third order system used previously. The continuous system is now given by

$$Y(s) = \frac{2}{(s+1)} \cdot \frac{229}{(s^2+30s+229)} \cdot \frac{1}{(s+1)} \frac{10}{(s+10)} U(s) \tag{87}$$

The results from simulations of the iterative design schemes are given in Tables 7 and 8. Similar to what was found for first order models we see that when the modeling error is large the low pass filtering allows higher bandwidths than the sensitivity function filtering, since the model fit is kept in the low frequency range.

ω_B	No filtering	Low pass filter	Sensitivity function
1	$4.8 \cdot 10^{-6}$	$2.7 \cdot 10^{-6}$	$3.2 \cdot 10^{-6}$
2	$1.1 \cdot 10^{-4}$	$1.9 \cdot 10^{-4}$	$8.4 \cdot 10^{-4}$
3	∞	$1.5 \cdot 10^{-3}$	$1.1 \cdot 10^{-3}$
4	∞	$8.8 \cdot 10^{-3}$	∞
5	∞	$1.6 \cdot 10^{-2}$	∞

Table 7: Loss function for iterative design using second order OE-model

ω_B	No filtering	Low pass filter	Sensitivity function
1	$5.0 \cdot 10^{-5}$	$2.7 \cdot 10^{-6}$	$2.2 \cdot 10^{-6}$
2	$1.5 \cdot 10^{-3}$	$1.1 \cdot 10^{-4}$	$9.6 \cdot 10^{-5}$
3	$2.6 \cdot 10^{-2}$	$9.0 \cdot 10^{-4}$	$9.1 \cdot 10^{-4}$
4	∞	$4.0 \cdot 10^{-3}$	$4.8 \cdot 10^{-3}$
5	∞	$1.5 \cdot 10^{-2}$	∞

Table 8: Loss function for iterative design using second order ARX-model

7 Conclusions

We have in this report carried out some comparative experiments of methods that aim at matching the control and identification criteria. We have illustrated how the ideas of optimal input design and iterative design can be seen as different approximation in order to match these criteria. The methods have also been tested experimentally in simulations and the conclusions from these are that both identification using optimal input design and iterative design with data prefiltering using the nominal sensitivity function perform well when the model errors are small. For larger model errors better performance can be obtained by just low pass filtering of data before identification in the iterative schemes.

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