A combined state-space identification algorithm applied to data from a modal analysis experiment on a separation system

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Abstract

This paper discuss identification of state-space models from impulse response or initial value experiments. We combine Kung's realization algorithm with classical nonlinear parametric optimization in order to further refine the quality of the estimated state-space model. We apply these ideas on real data originating from a modal analysis experiment on a separation system. The results indicate that the parametric optimization step increase the model quality significantly compared with the initial model the realization algorithm provides.

Keywords: system identification, modal analysis, state-space methods, application.

1 Introduction

Identification of modal parameters of flexible mechanical structures can be carried out by estimating a state-space model given impulse response or initial value response measurements, see *e.g.* [6, 2, 4].

Consider the state-space model

$$\begin{aligned} x(t+1) &= Ax(t), \quad x(0) = x_0 \\ y(t) &= Cx(t) \end{aligned}, \quad (1)$$

where $y(t) \in \mathbb{R}^p$ and $x(t) \in \mathbb{R}^n$ with n equal the order of the system. Given data y_t , $t = 0, \dots, N-1$ from an initial value experiment the identification should yield estimates on A , C and x_0 . such that

$$V(A, C, x_0) = \frac{1}{N} \sum_{t=0}^{N-1} |y_t - y(t)|^2 \quad (2)$$

is minimized. From the state-space model (1) it is easy to derive frequencies and damping ratio for the different structural modes.

In this paper we present a combined algorithm which as a first step use Kung's realization algorithm [3] to obtain an initial estimate of a state space system matching the data. In a second step an initial state x_0 is estimated by a linear least-squares fit to the data. In the last step a nonlinear optimization is performed to further improve the criterion (2). The first step is closely related to [2] since both algorithms rely on the realization algorithm by Ho & Kalman [1]. The second and last steps are introduced since the estimates from the first step in general do not minimize (2) if the true system is of high order and/or if noise is present.

We apply the combined algorithm to real data from a modal analysis experiment of a separation system. A separation system is the connecting device between the launcher and the payload and serves the purpose of keeping the payload in place, on top of the launcher, during the launch phase and when reached the correct orbit, release the payload from the launcher. During the separation phase an explosive charge in the separation system release the payload. In order to reduce oscillations on the payload the separation system is designed to dampen out oscillations. The real data origin from a separation experiment where oscillations were measured with three accelerometers. The interesting parameters to extract are frequencies and damping ratio for the dominating modes. The data has been made available to the author by courtesy of Saab Ericsson Space AB, Linköping, Sweden.

2 A combined identification algorithm

The first estimate is obtained by using Kung's algorithm [3]: Assume we are given $N = q + r - 1$ noise free measurements y_t . Construct the block-hankel matrix

$$Y_{qr} = \begin{bmatrix} y_0 & y_1 & \cdots & y_r \\ y_1 & y_2 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ y_{q-2} & y_{q-1} & \cdots & y_{q+r-2} \end{bmatrix} \quad (3)$$

where $q > n$ and $r \geq n$ is the number of block rows and columns respectively. From (1) it follows that $y_t = CA^{t-1}x_0$ and hence Y_{qr} can be factored as

$$Y_{qr} = \mathcal{O}_q \mathcal{C}_r \quad (4)$$

where $\mathcal{O}_q$ is the extended observability matrix

$$\mathcal{O}_q = \begin{bmatrix} C^T & (CA)^T & \cdots & (CA^{q-1})^T \end{bmatrix}^T \quad (5)$$

with rank n if the system is observable and

$$\mathcal{C}_r = \begin{bmatrix} x_0 & Ax_0 & \cdots & A^{r-1}x_0 \end{bmatrix} \quad (6)$$

is a matrix of rank n if the initial state x_0 excites all modes. Calculate the singular value decomposition

$$Y_{qr} = \begin{bmatrix} \hat{U}_1 & \hat{U}_2 \end{bmatrix} \begin{bmatrix} \hat{\Sigma}_1 & 0 \\ 0 & \hat{\Sigma}_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \quad (7)$$

where the diagonal matrix $\hat{\Sigma}_1$ contains the n principal singular values. Since we assumed y_t originated from (1), Y_{qr} will be of rank n if $\min(q, r) > n$ and hence $\hat{\Sigma}_2 = 0$. Let us denote

$$J_1 = \begin{bmatrix} I_{(q-1)p} & 0_{(q-1)p \times p} \end{bmatrix} \quad (8)$$

$$J_2 = \begin{bmatrix} 0_{(q-1)p \times p} & I_{(q-1)p} \end{bmatrix} \quad (9)$$

where I_i denotes the $i \times i$ identity matrix and $0_{i \times j}$ denotes the $i \times j$ zero matrix. The system matrices are estimated as

$$\hat{A}_0 = \left(J_1 \hat{U}_1 \hat{\Sigma}_1^{1/2} \right)^\dagger \left(J_2 \hat{U}_1 \hat{\Sigma}_1^{1/2} \right) \quad (10)$$

$$\hat{C}_0 = \begin{bmatrix} I_p & 0_{p \times (q-1)} \end{bmatrix} \hat{U}_1 \hat{\Sigma}_1^{1/2}, \quad (11)$$

$$\hat{x}0_0 = \hat{\Sigma}_1^{1/2} \hat{V}_1^T \begin{bmatrix} 1 & 0_{1 \times (r-1)} \end{bmatrix}^T \quad (12)$$

where $A^\dagger = (A^T A)^{-1} A^T$ denotes the Moore-Penrose pseudo-inverse of the full-rank matrix A . If the measurements y_t are noise free the estimated system $(\hat{A}_0, \hat{C}_0, \hat{x}0_0)$ are then related to the original system $(A, C, x0)$ by a similarity transformation. If we use this algorithm on noisy data or data from a high order system Σ_2 will not be identically zero and $(\hat{A}_0, \hat{C}_0, \hat{x}0_0)$ will then be an approximation of the true system. There is no proof that this approximation minimize (2). However, in general Kung's algorithm provide models which are close to the minimum and hence we will use this estimate as an initial estimate in solving (2) using a Gauss-Newton type optimization. As an intermediate step a refined estimate of $x0$, $\hat{x}0_1$ is calculated by minimizing (2) with respect to $x0$. This minimization is easily solved by a linear least-square solution since $x0$ is linear in $y(t)$ if $\hat{A}_0$ and $\hat{C}_0$ are held constant. In the last step $(\hat{A}_0, \hat{C}_0, \hat{x}0_1)$ are converted to an observability canonical form by the use of a similarity transformation, see [5], and are then used as an initial estimate minimizing (2) with respect to $(A, C, x0)$ to obtain a final model $(\hat{A}, \hat{C}, \hat{x}0)$. The conversion to a canonical form allows us to use a sparse parameterization of the A matrix and a fix C matrix and hence save computation time. Furthermore this parameterization is also identifiable and hence each point in the parameter space represents a unique system.

The estimated model is now an abstract model of the experimental data. Particularly, the eigenvalues of the matrix $\hat{A}$ give us information about frequency and damping ratio for all identified structural modes. Denote sampling frequency with f given in Herz and $\lambda_i(A)$, $i = 1, \dots, n$ the eigenvalues of A . Each structural mode k with the principal shape

$$m_k(t) = e^{-\zeta_k \Omega_k} \sin(\omega_k t) \quad (13)$$

with $\Omega_k = \omega_k / \sqrt{1 - \zeta^2}$, is characterized by the frequency ω_k (in rad/s) and relative damping ratio ζ_k . These modal parameters are associated with a complex conjugate eigenvalue pair from the real matrix A such that

$$\Omega_k = \text{Im}(\log \lambda_k) f \quad (14)$$

$$\zeta_k = -\text{Re}(\log \lambda_k) f / \Omega_k \quad (15)$$

$$\omega_k = \Omega_k \sqrt{1 - \zeta^2} \quad (16)$$

where λ_k is the eigenvalue with positive imaginary part of the two complex conjugate eigenvalues. In order to rank the estimated modes we will apply the following procedure. Assume $\hat{A}$ has distinct

Frequency (Hz)	Damping ratio (%)	Modal influence
1116	1.73	0.0914
1129	0.38	0.0017
1138	2.92	0.2317
1186	2.31	0.1559
1235	0.66	0.0619
1309	1.94	0.1530
1344	1.20	0.1779
1385	2.85	0.1202
1493	0.82	0.0063

Table 1: Estimated frequencies and damping ratio and corresponding modal influence for the estimated model of order 18.

eigenvalues. Find the complex similarity transformation T such that $\Lambda = T^{-1}\hat{A}T$ where Λ is a diagonal matrix with the eigenvalues on the diagonal ordered in complex conjugate pairs and let $\tilde{x}0 = T^{-1}\hat{x}0$, $\tilde{C} = CT$. The output can now be written as

$$y(t) = \sum_{k=1}^n \tilde{c}_k \tilde{x}0_k \lambda_k^{t-1} \quad (17)$$

where $\tilde{c}_k$ is the k th column of $\tilde{C}$ and $\tilde{x}0_k$ is the k th element of the vector $\tilde{x}0$. For each complex conjugate eigenvalue pair and thus each mode we can define a modal influence measure M_k

$$M_k = \frac{2|\tilde{c}_{2k-1}\tilde{x}0_{2k-1}|}{\sum_{i=1}^n |\tilde{c}_i \tilde{x}0_i|} \quad (18)$$

which is a measure the influence of each mode on the output of the system. The measure M_k is a relative measure with $\sum_{k=1}^{n/2} M_k = 1$.

3 Structural mode identification: A separation system

The experimental data consists of a record of 4096 samples of the three accelerometers obtained with a sampling frequency $f = 32768$ Hz. The data is bandpass-filtered by a 5th order Butterworth filter with passband between 1000 and 1460 Hz in order to remove uninteresting frequencies. In Figure 1 the values of the criterion (2) after each identification step are plotted for some different model orders. It is quite clear that both the second and third step significantly decrease the criterion and hence the quality of the estimated model. In Table 1 the estimated frequencies, damping ratios and modal influence measure are given for an estimated model of order $n = 18$. In Figure 2 a portion of the filtered accelerometer signals are plotted together with a simulation of the estimated model of order 18. The model has captured almost all the dynamics in the signals and the two curves are almost indistinguishable.

Figure 1: The criterion (2) after each step in the identification algorithm. “o” after first subspace step. “+” after x_0 estimation by linear least-squares minimization. “*” after nonlinear minimization

Figure 2: Comparison between measured accelerometer outputs (solid line) and simulated outputs (dashed line) using the estimated model of order 18.

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