Avoiding Mode Pairing when Updating Finite Element Models

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AVOIDING MODE PAIRING WHEN UPDATING FINITE ELEMENT MODELS

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Abstract: Updating finite element models of complex mechanical structures requires some extra considerations. It is stressed that the two most important aspects on updating finite element models are parameter estimation properties and computational expenses. A novel mode-pairing free model updating formulation is found to have good parameter estimation properties. The computational expenses are reduced with a semi-fixed modal basis, kept fixed during several iterations.

Keywords: Parameter estimation; mechanical systems; nonlinear programming; model reduction.

1. INTRODUCTION

In aerospace applications it is for several reasons important that the FE model is accurate, i.e., that the model has a dynamical behavior close to the real structure. For instance, the security aspects on aero-planes require an experimental ground-test verification of the construction in a test set-up. The verification includes measuring of the low-frequency dynamical behavior of the structure for several configurations, e.g., for different amount of fuel in the wing-tanks. This measuring is very expensive and time-consuming. If an initial model of the structure can be updated/improved from the measurements of a single configuration, the other configurations can be simulated using the improved model. In order for these simulations to be valid, it is assumed that the improved model for the measured configuration is accurate, and that the differences between the configurations are easily accounted for in the model. This is mostly the case since the differences between the configurations are often known masses with known locations.

The specific task of improving an initial FE model using measurements is here denoted model updating. An attractive approach is to parameterize the FE model physically and estimate the numerical values of these parameters. This corresponds to the minimization of a cost function $V(\boldsymbol{\theta}) \geq 0$ describing the deviation between the model output and the measurements,

$$\tilde{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta} \in \Theta} V(\boldsymbol{\theta}) \quad (1)$$

where Θ denotes the open set of feasible values for the parameter vector $\boldsymbol{\theta}$. The minimization of V in (1) is in general a non-linear problem. An iterative procedure is therefore required in order to obtain an estimate $\hat{\boldsymbol{\theta}}$ of $\tilde{\boldsymbol{\theta}}$,

$$\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}^i + \Delta \boldsymbol{\theta}^i, \quad \boldsymbol{\theta}^0 = \boldsymbol{\theta}^{init} \quad (2)$$

where

$$\hat{\boldsymbol{\theta}} = \lim_{i \rightarrow \infty} \boldsymbol{\theta}^i \quad (3)$$

Hopefully, a consistent estimate of the parameters are achievable, $\hat{\boldsymbol{\theta}} = \tilde{\boldsymbol{\theta}}$.

Updating FE models is a task of very high complexity. This is basically due to the size of the FE models and the high number of parameters N_θ ,

but also the high number of actuators m and sensors p yielding a large amount of measurements. It is therefore important to reduce the computational expenses for model updating. For instance, model reduction is inevitable when dealing with FE models of complex industrial structures.

However, the complexity of the models is not the main obstacle for obtaining a successful updating. It is experimentally found that model updating formulations in general suffers from local minima in the parameter space. In order to alleviate the risk of ending up in a local minima, the convexity properties of the cost function describing the difference between the measurements and the model output must be enhanced.

2. BASIC THEORY

FE modeling of mechanical structures yield a mass matrix $\mathbf{M}$ and a stiffness matrix $\mathbf{K}$ of dimension $l \times l$, respectively. The normal modes ϕ_i and the resonance frequencies ω_i of the undamped FE model represented by $\mathbf{M}$, $\mathbf{K}$ are determined via the generalized eigenvalue problem

$$(\mathbf{K} - \omega_i^2 \mathbf{M})\phi_i = 0, \quad \phi_i^T \mathbf{M} \phi_i = 1, \quad i = 1, \dots, l \quad (4)$$

where the normal modes ϕ_i are mass normalized. Notice that the modal matrix $\Phi = [\phi_1, \dots, \phi_l]$ diagonalizes $\mathbf{M}$ and $\mathbf{K}$:

$$\Phi^T \mathbf{M} \Phi = \mathbf{I}, \quad \Phi^T \mathbf{K} \Phi = \text{diag}(\omega_1^2, \dots, \omega_l^2) = \Omega^2 \quad (5)$$

where **diag** denotes a diagonal matrix. Two modes ϕ_i and ϕ_j are considered to be near multiple if their corresponding resonance frequencies ω_i and ω_j are close, i.e., if $\omega_i \approx \omega_j$.

A corresponding FE technique for modeling the damping is not available and *ad hoc* approaches to construct the damping matrix $\mathbf{L}$ is therefore inevitable. If Φ also diagonalizes $\mathbf{L}$,

$$\Phi^T \mathbf{L} \Phi = \text{diag}(\gamma_1, \dots, \gamma_l) = \mathbf{\Gamma}, \quad \gamma_i = 2\xi_i \omega_i \quad (6)$$

the damping is so-called proportional. In (6), the ξ_i is the damping ratio of the normal mode ϕ_i . The assumption of proportional damping is found to be valid for flexible structures in practice as well as in theory, see (Géradin and Rixen, 1994).

The input/output (I/O) behavior of a mechanical structure may be modeled

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{L}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) &= \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \quad (7)$$

where $\mathbf{x}$ is the vector of the l degrees of freedom (dof.), $\mathbf{u}$ is the vector of m input forces, and $\mathbf{y}$ the vector of p displacement measurements. In (7), $\mathbf{B}$ (of dimension $l \times m$) describes the positions of the actuators yielding the input forces and $\mathbf{C}$ ($p \times l$)

the positions of the sensors measuring the output. Equivalently, (7) may be written

$$\begin{aligned} \ddot{\mathbf{q}}(t) + \mathbf{\Gamma}\dot{\mathbf{q}}(t) + \Omega^2 \mathbf{q}(t) &= \Phi^T \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\Phi(t) \end{aligned} \quad (8)$$

where $\mathbf{q} = \Phi\mathbf{x}$ is a transformation to the modal coordinates $\mathbf{q}$.

The frequency response function (FRF) of (7) is defined as the transfer function $\mathbf{G}(s)$ evaluated on the imaginary axis, $s = j\omega$,

$$\mathbf{G}(\omega) = \mathbf{C} (\mathbf{K} + j\omega \mathbf{L} - \omega^2 \mathbf{M})^{-1} \mathbf{B} \quad (9)$$

where ω denotes frequency, and where $\mathbf{G}(\omega)$ denotes $\mathbf{G}(j\omega)$ for simplicity. The FRF of (8) may be conveniently written as

$$\mathbf{G}(\omega) = \sum_{i=1}^l \frac{\mathbf{C}\phi_i \phi_i^T \mathbf{B}}{\omega_i^2 - \omega^2 + j2\xi_i \omega_i \omega} \quad (10)$$

showing the interpretation of the triple $\{\phi_i, \xi_i, \omega_i\}$ of modal parameters in the frequency domain.

An efficient reduction of the system matrices $\mathbf{M}$, $\mathbf{L}$, $\mathbf{K}$ to the low-frequency area can be performed as

$$\mathbf{M}_r = \Phi_r^T \mathbf{M} \Phi_r, \quad \mathbf{L}_r = \Phi_r^T \mathbf{L} \Phi_r, \quad \mathbf{K}_r = \Phi_r^T \mathbf{K} \Phi_r \quad (11)$$

using a reduced modal matrix $\Phi_r = [\phi_{i_1}, \dots, \phi_{i_r}]$. It remains to determine the i_j 's. Traditionally, ϕ_{i_j} are chosen through a pairing of the normal modes ϕ_i of the model and the corresponding estimates of the structure $\hat{\phi}_j^x$,

$$\phi_{i_j} \leftrightarrow \hat{\phi}_j^x, \quad j = 1, \dots, N_q^x \quad (12)$$

where N_q^x denotes the number of estimated modes of the structure. The actual pairing of modes often relies on the correlation between the ϕ_i and the $\hat{\phi}_j^x$. The pairing of modes also implies a pairing of the corresponding triples of modal parameters,

$$\{\phi_{i_j}, \xi_{i_j}, \omega_{i_j}\} \leftrightarrow \{\hat{\phi}_j^x, \hat{\xi}_j^x, \hat{\omega}_j^x\}, \quad j = 1, \dots, N_q^x \quad (13)$$

giving the possibility to construct a reduced damping matrix $\mathbf{L}_r$, and incorporating deviation in the resonance frequencies in the cost function.

The mode-pairing in (12) is a combinatorial optimization problem of rather high complexity. For large flexible structures with many modes in the model/structure, sub-optimal algorithms must be used in order to decrease the time for the mode-pairing.

The estimate $\hat{\mathbf{G}}^x$ of $\mathbf{G}^x$ at the frequencies $\omega_{R,i}$, $i = 1, \dots, N_\omega$ can be obtained as

$$\hat{G}_{kl}(\omega_{R,i}) = \frac{y_k(\omega_{R,i})}{u_l(\omega_{R,i})} \quad (14)$$

where $\hat{G}_{kl}$ is the entry of row k and column l of $\hat{\mathbf{G}}^x$. In (14), $y_k(\omega_{R,i})$ denotes the Fourier

transform of the output from sensor k , and where $u_l(\omega_{R,i})$ denotes the Fourier transform of the sinusoidal input force applied with actuator l .

In (Williams *et al.*, 1985), the multi-variate mode indicator function (MMIF) is presented. It is used to detect (multiple) modes of the structure as well as compute an input force vector $\mathbf{U}_i$. The input signal $\mathbf{u}(t) = \mathbf{U}_i \sin(\hat{\omega}_i^x t)$ is then applied to the structure in order to obtain the estimate $\hat{\phi}_i^x$,

$$\hat{\phi}_i^x = \text{Im}\{\mathbf{G}^x(\hat{\omega}_i^x)\mathbf{U}_i\} \quad (15)$$

for $i = 1, \dots, N_q^x$, see (Andersson, 1997). The $\mathbf{U}_i$ is the eigenvector $\lambda_{i,min}$

$$\lambda_{i,min} = \min_j \lambda_{i,j}, \quad i = 1, \dots, N_q^x \quad (16)$$

where

$$(\mathbf{A}_i - \lambda_{i,j}(\mathbf{A}_i + \mathbf{B}_i))\mathbf{U}_{i,j} = 0, \quad j = 1, \dots, m \quad (17)$$

where $\mathbf{A}_i = \text{Re}\{\hat{\mathbf{G}}^x(\hat{\omega}_i^x)\}^T \text{Re}\{\hat{\mathbf{G}}^x(\hat{\omega}_i^x)\}$ and $\mathbf{B}_i = \text{Im}\{\hat{\mathbf{G}}^x(\hat{\omega}_i^x)\}^T \text{Im}\{\hat{\mathbf{G}}^x(\hat{\omega}_i^x)\}$.

3. MODEL UPDATING FORMULATION

In ground vibration testing, the estimate $\hat{\mathbf{G}}^x$ of $\mathbf{G}^x$ is obtained in the low-frequency area. The estimated triple of modal parameters $\{\hat{\phi}_i^x, \hat{\xi}_i^x, \hat{\omega}_i^x\}$ are also commonly available.

It is experimentally found that the previous described mode-pairing technique may introduce severe local minima in the parameter space of the cost function V . In particular, the assumption of a valid mode pairing is critical whenever the dynamical behavior of the model is not close to the structure's, i.e., when $\theta^i \not\approx \bar{\theta}$, or when near multiple modes of the model and/or the structure are present. An objective have therefore been to develop a mode-pairing free technique to update FE models of complex mechanical structures.

It is also found that a combined modal/frequency domain cost function based on both deviations in resonance frequencies and deviations in properties defined by the the FRF has good parameter estimate properties (Andersson, 1997), i.e., an estimate $\hat{\theta}$ of θ where

$$\hat{\theta} \approx \bar{\theta}, \quad V(\hat{\theta}) \ll V(\theta^0) \quad (18)$$

where θ^0 is the initial value of the parameter vector θ .

3.1 Model reduction

Defining $\hat{\phi}_i$ as

$$\hat{\phi}_i = \mathbf{\Phi}_q \hat{\mathbf{a}}_i, \quad i = 1, \dots, N_q^x \quad (19)$$

where

$$\mathbf{\Phi}_q = [\phi_1, \dots, \phi_{N_q}] \quad (20)$$

$$\hat{\mathbf{a}}_i = \arg \min_{\mathbf{a} \in \mathbb{R}^{N_q}} \|\mathbf{\Phi}_q \mathbf{a}_i - \hat{\phi}_i^x\|_2 \quad (21)$$

where N_q denotes the number of computed normal modes. The matrix of reduction $\bar{\mathbf{\Phi}}_r$ is then defined as

$$\bar{\mathbf{\Phi}}_r = [\bar{\phi}_1, \dots, \bar{\phi}_{N_q^x}], \quad \bar{\phi}_i = \frac{1}{((\hat{\phi}_i^x)^T \mathbf{M} \hat{\phi}_i^x)^{1/2}} \hat{\phi}_i \quad (22)$$

where $\bar{\phi}_i$ is the mass-normalized version of $\hat{\phi}_i^x$. Notice that the mass-normalization of $\hat{\phi}_i^x$ in (22) is based on the assumption that the mass matrix $\mathbf{M}$ of the model is approximating the mass distribution of the structure sufficiently accurate.

The reduced system matrices $\bar{\mathbf{M}}_r$ and $\bar{\mathbf{K}}_r$ become

$$\bar{\mathbf{M}}_r = \bar{\mathbf{\Phi}}_r^T \mathbf{M} \bar{\mathbf{\Phi}}_r, \quad \bar{\mathbf{K}}_r = \bar{\mathbf{\Phi}}_r^T \mathbf{K} \bar{\mathbf{\Phi}}_r \quad (23)$$

3.2 Damping model design

In order to obtain a reduced damping matrix $\bar{\mathbf{L}}_r$, some assumptions about the damping distribution in the structure must be made. Let $\mathbf{\Phi}_x = [\phi_1^x, \dots, \phi_{N_q^x}^x]$ denote the "true" modes of the structure, and let $\mathbf{L}_x$ denote the "true" damping matrix of the structure. If the damping distribution in the structure are proportional, then $\mathbf{\Phi}_x^T \mathbf{L}_x \mathbf{\Phi}_x$ will be diagonal. Assuming that $\hat{\phi}_i^x$ is parallel with ϕ_i^x , and that $\bar{\phi}_i$ is parallel with $\hat{\phi}_i^x$:

$$\hat{\phi}_i^x \parallel \phi_i^x, \quad \text{and} \quad \bar{\phi}_i^r \parallel \hat{\phi}_i^x, \quad i = 1, \dots, N_q^x \quad (24)$$

then also $\bar{\mathbf{\Phi}}_r^T \mathbf{L}_x \bar{\mathbf{\Phi}}_r$ will be diagonal. These assumptions indicate that $\bar{\mathbf{L}}_r$ may be modeled as

$$\bar{\mathbf{L}}_r = \text{diag}(\dots, 2\hat{\xi}_i^x \bar{\omega}_i, \dots) \quad (25)$$

where $\bar{\omega}_i$ is the square root of the Rayleigh quotient R in

$$R(\bar{\phi}_i) = \frac{\bar{\phi}_i^T \mathbf{K} \bar{\phi}_i}{\bar{\phi}_i^T \mathbf{M} \bar{\phi}_i} = (\bar{\omega}_i)^2, \quad i = 1, \dots, N_q^x \quad (26)$$

see (Craig, Jr, 1981).

However, the matrices $\bar{\mathbf{M}}_r$ and $\bar{\mathbf{K}}_r$ are in general not diagonal, i.e., the damping is not proportional since $\bar{\mathbf{L}}_r$ is diagonal. If a proportional damping is required during the model updating, a diagonalization of $\bar{\mathbf{M}}_r$ and $\bar{\mathbf{K}}_r$ can first be made as

$$\bar{\mathbf{M}}_r^d = \mathbf{\Psi}^T \bar{\mathbf{M}}_r \mathbf{\Psi}, \quad \bar{\mathbf{K}}_r^d = \mathbf{\Psi}^T \bar{\mathbf{K}}_r \mathbf{\Psi} \quad (27)$$

where

$$\mathbf{\Psi} = [\psi_1, \dots, \psi_{N_q^x}] \quad (28)$$

In (28), the basis vectors ψ_i is defined by

$$(\bar{\mathbf{K}}_r - \omega_i^2 \bar{\mathbf{M}}_r) \psi_i = 0, \quad \psi_i^T \bar{\mathbf{M}}_r \psi_i = 1 \quad (29)$$

for $i = 1, \dots, N_q^x$, compare with (4)-(5). The proportional damping matrix $\bar{\mathbf{L}}_r^d$ may be approximated by the diagonal of $\Psi^T \bar{\mathbf{L}}_r \Psi$:

$$\bar{\mathbf{L}}_r^d = \text{diag}(\Psi^T \bar{\mathbf{L}}_r \Psi) = \text{diag}(\dots, \bar{\zeta}_i, \dots) \quad (30)$$

Notice that the damping of the updated model becomes proportional if the parameter estimation is consistent ($\hat{\boldsymbol{\theta}} = \tilde{\boldsymbol{\theta}}$), and if the mode estimates are unbiased, i.e., if $\hat{\phi}_i^x \parallel \phi_i^x$. This is true even if non-proportional damping is used during the model updating.

The reduced model (for proportional damping) becomes

$$\begin{aligned} \bar{\mathbf{M}}_r^d \ddot{\mathbf{q}} + \bar{\mathbf{L}}_r^d \dot{\mathbf{q}} + \bar{\mathbf{K}}_r^d \mathbf{q} &= (\bar{\Phi}_r^d)^T \mathbf{B} \mathbf{u} \\ \mathbf{y} &= \mathbf{C} \bar{\Phi}_r^d \mathbf{q} \end{aligned} \quad (31)$$

where

$$\bar{\Phi}_r^d = \bar{\Phi}_r \Psi \quad (32)$$

The FRF corresponding to (31) becomes

$$\bar{\mathbf{G}}_r^d(\omega) = \mathbf{C} \bar{\Phi}_r^d (\bar{\mathbf{Z}}_r^d(\omega))^{-1} (\bar{\Phi}_r^d)^T \mathbf{B} \quad (33)$$

where

$$\bar{\mathbf{Z}}_r^d(\omega) = \text{diag}(\dots, \omega_i^2 - \omega^2 + j\bar{\zeta}_i \omega, \dots) \quad (34)$$

3.3 Cost function

Using all the assumed measurements available, a mode-pairing free cost function can be obtained as

$$V(\boldsymbol{\theta}) = \nu_1^2 V_1(\boldsymbol{\theta}) + \nu_2^2 V_2(\boldsymbol{\theta}) + \nu_3^2 V_3(\boldsymbol{\theta}) \quad (35)$$

where ν_i are cost function weights, and where

$$V_1(\boldsymbol{\theta}) = \sum_{i=1}^{N_\omega} \|\bar{\mathbf{G}}_r^d(\boldsymbol{\theta}, \omega_{R,i}^x) - \hat{\mathbf{G}}^x(\omega_{R,i}^x)\|_F^2 \quad (36)$$

$$V_2(\boldsymbol{\theta}) = \sum_{i=1}^{N_q^x} \left| \frac{\bar{\omega}_i(\boldsymbol{\theta}) - \hat{\omega}_i^x}{\hat{\omega}_i^x} \right|_2^2 \quad (37)$$

$$V_3(\boldsymbol{\theta}) = \sum_{i=1}^{N_q^x} \|\bar{\mathbf{G}}_r^d(\boldsymbol{\theta}, \hat{\omega}_i^x) \mathbf{U}_i^x - \mathbf{Y}_i^x\|_2^2 \quad (38)$$

where F denotes the Frobenius norm, and $\omega_{R,i}^x$ denotes the measured frequency points of $\hat{\mathbf{G}}^x$. Minimizing V_1 corresponds to the maximum-likelihood (ML) estimate of $\boldsymbol{\theta}$, see (McKelvey, 1995). The V_2 is included in order to increase the convexity properties of the cost function, see (Abrahamsson *et al.*, 1996) where the enhanced parameter estimation properties using the Rayleigh quotient in the cost function are studied. In (38), $\mathbf{Y}_i^x$ corresponds to the output from the structure for the input $\mathbf{u}(t) = \mathbf{U}_i^x \sin(\hat{\omega}_i^x t)$,

$$\mathbf{Y}_i^x = \mathbf{G}^x(\hat{\omega}_i^x) \mathbf{U}_i^x \quad (39)$$

3.4 Implementation

Using numerical difference approximations of $\partial \bar{\mathbf{M}} / \partial \boldsymbol{\theta}$ and $\partial \bar{\mathbf{K}} / \partial \boldsymbol{\theta}$, approximating derivatives of $\bar{\mathbf{M}}_r$, $\bar{\mathbf{K}}_r$ are obtained as

$$\frac{\partial \bar{\mathbf{M}}_r}{\partial \theta_k} \approx \bar{\Phi}_r^T \frac{\mathbf{M}(\theta_k + \delta \theta_k) - \mathbf{M}(\theta_k)}{\delta \theta_k} \bar{\Phi}_r \quad (40)$$

$$\frac{\partial \bar{\mathbf{K}}_r}{\partial \theta_k} \approx \bar{\Phi}_r^T \frac{\mathbf{K}(\theta_k + \delta \theta_k) - \mathbf{K}(\theta_k)}{\delta \theta_k} \bar{\Phi}_r \quad (41)$$

If the modal basis of reduction $\bar{\Phi}_r$ and the linearization of $\bar{\mathbf{M}}_r$ and $\bar{\mathbf{K}}_r$,

$$\bar{\mathbf{M}}_r^{lin}(\boldsymbol{\theta} + \Delta \boldsymbol{\theta}) \approx \bar{\mathbf{M}}_r(\boldsymbol{\theta}) + \sum_k \frac{\partial \bar{\mathbf{M}}_r(\boldsymbol{\theta})}{\partial \theta_k} \Delta \theta_k \quad (42)$$

$$\bar{\mathbf{K}}_r^{lin}(\boldsymbol{\theta} + \Delta \boldsymbol{\theta}) \approx \bar{\mathbf{K}}_r(\boldsymbol{\theta}) + \sum_k \frac{\partial \bar{\mathbf{K}}_r(\boldsymbol{\theta})}{\partial \theta_k} \Delta \theta_k \quad (43)$$

are semi-fixed, i.e., kept fixed for several iterations, the computational expenses can be reduced, see (Andersson, 1997).

In Fig. 1, a flow-chart of a general model updating strategy is shown.

Fig. 1. A flow chart of the general model updating procedure.

Let $V(\boldsymbol{\theta}) = \mathbf{v}(\boldsymbol{\theta})^T \mathbf{v}(\boldsymbol{\theta})$ where $\mathbf{v}(\boldsymbol{\theta}) = \mathbf{v}_m(\boldsymbol{\theta}) - \mathbf{v}_s$ is the deviation vector between the output of the model $\mathbf{v}_m$ and the measurements of the structure $\mathbf{v}_s$. Linearizing $\mathbf{v}$ in $\boldsymbol{\theta}$,

$$\mathbf{v}(\boldsymbol{\theta} + \delta \boldsymbol{\theta}) \approx \mathbf{v}(\boldsymbol{\theta}) + \mathbf{J}(\boldsymbol{\theta}) \delta \boldsymbol{\theta} \quad (44)$$

the parameter difference vector $\Delta \boldsymbol{\theta}^i$ in (2) can be determined as

$$\Delta \boldsymbol{\theta}^i = \arg \min_{\Delta \boldsymbol{\theta}} \left\{ \|\mathbf{J}(\boldsymbol{\theta}^i) \Delta \boldsymbol{\theta} - \mathbf{d}(\boldsymbol{\theta}^i)\|_2^2 \right. \quad (45)$$

$$\left. + \mu^2 \|\Delta \boldsymbol{\theta}\|_2^2 \right\} \quad (46)$$

where $\mathbf{d}(\boldsymbol{\theta}^i) = \mathbf{v}_m(\boldsymbol{\theta}^i) - \mathbf{v}_s$, and μ denotes the regularization term, see (Moré, 1983). In

(Andersson, 1997), a robust strategy for determining the value of μ is proposed. It is a modified version of Fletcher's strategy in (Wolfe, 1978), being more robust against ending up in undesired local minima.

4. NUMERICAL SIMULATION EXAMPLE

Fig. 2 shows a simple aircraft-like structure.

Fig. 2. The Garteau model.

In the examples below, 30 sensors are used to collect measurements. The cost function weights ν_i are determined so that

$$V_1 = \frac{10}{14}V, \quad V_2 = \frac{1}{14}V, \quad V_3 = \frac{3}{14}V \quad (47)$$

in (36)-(38) for the initial values of the parameters, $\theta = \theta^{init}$.

4.1 Example 1

If the actuators are located to $\{12_z, 14_x, 15_z, 20_y\}$, the 9 low-frequency modes are estimated appropriately, since

$$\min_i \frac{|(\phi_i^x)^T \hat{\phi}_i^x|^2}{|\phi_i^x|^2 |\hat{\phi}_i^x|^2} = 0.99 \quad (48)$$

The parameter history then become as in Fig. 3. It is seen that the estimated parameter values $\hat{\theta}$ have converged to the global minima in 13 iterations. The cost function history is shown in Fig. 4. It is seen that the linearization $\bar{M}_r^{lin}, \bar{K}_r^{lin}$ is valid since the circles (o) is close to the solid line. The * denotes the values of the cost function for a new computation of the modal basis of reduction Φ_r .

4.2 Example 2

If the actuators are located to $\{12_z, 15_z, 20_y\}$, the 9 low-frequency modes are not estimated appropriately, since

Fig. 3. The history of the relative parameter values, Example 1.

Fig. 4. The history of the cost function, Example 1.

$$\min_i \frac{|(\phi_i^x)^T \hat{\phi}_i^x|^2}{|\phi_i^x|^2 |\hat{\phi}_i^x|^2} = 0.87 \quad (49)$$

The parameter history becomes as in Fig. 5. It can be stated that not appropriately estimated modes may turn up as bias in the parameter estimate.

Fig. 5. The history of the relative parameter values, Example 2.

However, even if the mode estimates $\hat{\phi}_i^x$ are not parallel with the true modes of the structure ϕ_i^x , the cost function decrease is substantial:

$$V(\hat{\theta}) = 6.7 \cdot 10^{-5} \cdot V(\theta^{init}) \quad (50)$$

The relative large deviation in the estimated parameter values $\hat{\theta}$ from θ is in view of (50) due to that the parameters are dependent, i.e., the deviation in a parameter value is compensated by the deviations in the other parameter values.

5. CONCLUSIONS

It was pointed out that the two most important aspects on a model updating formulation is its parameter estimation properties and its computational complexity. It was also noted that model updating in general suffers from undesired local minima in the cost function. In order to reduce the risk of ending up in a local minima, the traditional technique of pairing modes was circumvented when constructing the reduced size model. A novel combined frequency domain/modal domain mode-pairing free cost function was in simulation examples shown to yield good parameter estimates if the modes of structure are appropriately estimated. If the modes of the structure are not appropriately estimated, bias in the parameter estimates emerged. The value of the cost function are reduced considerably, though. A modal reduction basis was used in several iterations in order to reduce the time-consuming computation of the normal modes.

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