A Subspace Approach for Approximation of Rational Matrix Functions to Sampled Data¹

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Abstract

Algorithms for approximation of rational matrix factors to data is described. The method is based on a subspace based multivariable frequency domain state-space identification, canonical and spectral factorization and parametric optimization. The algorithms can be used for identifying spectral factors and factors of positive real functions from frequency data. The methods are directly applicable in the $D - K$ algorithm for complex μ -synthesis and the $Y - Z - K$ algorithm for mixed μ -synthesis.

1 Notation

$\mathcal{RL}_\infty$ is the set of real-rational matrix functions with no poles on the imaginary axis. Let $\mathcal{RH}_\infty = \{X(s) : X(s) \in \mathcal{RL}_\infty, X(s) \text{ analytic in } \operatorname{Re}(s) > 0\}$ be the set of stable real-rational matrix functions. Let A^* denote the conjugate transpose of A and let $\|\cdot\|$ denote any matrix norm and $G^{\sim}(s) = G^T(-s)$.

2 Introduction

Realization and approximation of matrix functions plays an important role in many automatic control and signal processing applications. In this paper subspace based approximations are proposed and discussed. The methods presented are focused on state-space realizations of scalings and multipliers in connection with $D - K$ iterations (spectral factorization) [2, 3] and $Y - Z - K$ iterations [6, 5] used for robust controller design. The method proposed in this paper is also applicable when finding a rational spectral factor from a measured power spectrum of a multivariable disturbance. For optimal filtering such a factor of the disturbance spectrum is needed [1].

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The problem treated is to find a state-space realization which approximates given multivariable frequency data $W_k \in \mathbb{C}^{p \times p}$ sampled at frequencies ω_k .

In the spectral factorization problem $W_k = W_k^* > 0$ and a spectral factor $\hat{G}(s)$ is sought which minimizes

$$\sum_k \|W_k - \hat{G}(j\omega_k)^* \hat{G}(j\omega_k)\|^2 \quad (1)$$

where $\hat{G}, \hat{G}^{-1} \in \mathcal{RH}_\infty$, i.e. $\hat{G}$ is a stable and inversely stable real rational matrix.

The second problem which arises in the $Y - Z - K$ algorithm deals with positive real data W_k , i.e. $W_k + W_k^* > 0$ and two factors $\hat{Y}(s)$ and $\hat{Z}(s)$ are sought such that

$$\sum_k \|W_k - \hat{Y}(j\omega_k)^* \hat{Z}(j\omega_k)\|^2 \quad (2)$$

is minimized and $\hat{Y}, \hat{Y}^{-1}, \hat{Z}, \hat{Z}^{-1} \in \mathcal{RH}_\infty$.

By parametrizing the unknown factors (G, Y, Z) a non-linear constrained parametric optimization problem results which have to be solved by iterative methods. In order for such a method to be successful for any non-trivial case, high quality initial estimates of the unknown factors have to be determined. This paper will focus on algorithms for deriving these initial estimates.

3 Preliminaries

First we revise some known results on factorization of square rational matrices.

Lemma 1 (Spectral factorization [4]) *Assume that $W(s) = W^{\sim}(s) > 0$, $W(\infty) > 0$ and $W, W^{-1} \in \mathcal{RL}_\infty$. Then there exist matrix functions G such that $W(s) = G^{\sim}(s)G(s)$ and $G, G^{-1} \in \mathcal{RH}_\infty$.*

Lemma 2 (Positive real factorization [6, 4]) *Assume that $W(j\omega) + W^{\sim}(j\omega) > 0$, $\forall \omega \in \mathbb{R} \cup \{\infty\}$*

and $W, W^{-1} \in \mathcal{RL}_\infty$. Then there exist matrix functions Y, Z , such that $W(s) = Y^\sim(s)Z(s)$ and $Y, Y^{-1}, Z, Z^{-1} \in \mathcal{RH}_\infty$.

Constructive state-space algorithms exist for these factorizations, see [4, 8].

4 Basic Algorithm

The solution to the approximation problem can be split into three main steps

Step 1 Approximation of a rational matrix $\hat{W}(s) \in \mathcal{RL}_\infty$ such that $\sum_k \|W_k - W(j\omega_k)\|^2$ is small. This is done with a subspace based frequency domain state-space identification algorithm [7]. In this first approximation step we impose no restrictions on $\hat{W}(s)$.

Step 2 First it is checked if $\hat{W}(s) + \hat{W}(s)^* > 0$. If not a modification $\gamma > 0$ is introduced

$$\hat{\hat{W}}(s) := \hat{W}(s) + \gamma I$$

such that $\hat{\hat{W}}(j\omega) + \hat{\hat{W}}(j\omega)^* > 0$. To find a suitable γ is a convex problem which can be solved by an LMI using the Kalman-Yakubovich-Popov lemma or by a simple bisection technique checking the eigenvalues of the associated Hamiltonian matrix.

A factorization $\hat{\hat{W}} = \hat{Y}^\sim \hat{Z}$ according to Lemma 2 is then well defined.

Step 3 The obtained factors are converted to some state-space basis suitable for parametrization and the iterative parametric optimization of (1) or (2) can be performed.

This basic algorithm can directly be used to the problem given in equation (2). The spectral factorization problem can be solved using some variations of this basic algorithm. If modifications are necessary in step 2 ($\gamma > 0$) the quality of the approximation obtained by the subspace method in step 1 becomes degraded and step 3 is instrumental in order to obtain good results.

5 Spectral Factorization

Since Step 1 in the algorithm finds a rational approximation to the given data without imposing any constraints it is most likely that the obtained approximation does not satisfy $\hat{W} = \hat{W}^\sim$, and consequently Lemma 1 cannot be applied. Two possibilities immediately emerges: 1) Use Lemma 2 to obtain a factorization $\hat{W} = \hat{Y}^\sim \hat{Z}$ and let $\hat{G} := \hat{Z}$ be the spectral factor.

However for some data it can happen that the orders of Y and Z are not equal which gives an indication of a potentially bad approximation. A second approach can then be applied: 2) Let $\hat{\hat{W}} := \frac{1}{2}(\hat{W} + \hat{W}^\sim)$ which is Hermitian and Lemma 1 applies. For computational reasons it is better to use the result in [8, Theorem 13.25] which directly uses $\hat{W}$ and never forms $\hat{\hat{W}}$. The drawback of this second approach is that the order of the factor $\hat{G}$ is doubled. Prior to the optimization we recommend to reduce the order by a balanced truncation. The truncation preserves the stability of $\hat{G}$. Inverse-stability of the reduced factor can be recovered, if necessary, by a second spectral factorization.

6 Conclusions

This paper presents algorithms for realization of state-space representations of matrix functions given as frequency sampled data. The algorithm is based on subspace identification. The method is geared towards solving realization problem in D - K and Y - Z - K iterations when designing robust controllers.

The subspace method combined with canonical factorization show promising results for D - K iterations. It is general in the respect that it can be used for multi-variable scaling as well as real- μ synthesis.

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