APPLICATIONS OF KAUTZ MODELS IN SYSTEM IDENTIFICATION

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Abstract. FIR, ARX or AR model structures can be used to describe many industrial processes. Simple linear regression techniques can be applied to estimate such models from experimental data. However, for low signal to noise ratios in combination with transfer function poles and noise model zeros close to the unit circle, a large number of model parameters are needed to generate adequate models. The Kautz model structure generalizes FIR, ARX and AR models. By using *a priori* knowledge about the dominating time constants and damping factors of the system, the model complexity is reduced, and the linear regression structure is retained. The objective of this contribution is to study an industrial example, where Kautz models have distinct advantages. The data investigated corresponds to aircraft flight flutter, which is a state when an aircraft component starts to oscillate.

Key Words. Modeling, system identification, parameter estimation, aircraft modeling, Kautz functions.

1 INTRODUCTION

To describe the behavior of real systems using dynamical models is of outmost importance in many fields of science. System identification deals with the problem of constructing dynamical models from experimental data. By far, the most used model in system identification is the ARX model

$$A(q)y(t) = B(q)u(t) + e(t), \quad (1)$$

where $u(t)$ is the input signal, $y(t)$ is the output signal, the noise process $\{e(t)\}$ is assumed to be a sequence of independent stochastic variables and

$$A(q) = 1 + \bar{a}_1 q^{-1} + \dots + \bar{a}_{na} q^{-na}, \quad (2)$$

$$B(q) = q^{-nk} [\bar{b}_1 + \dots + \bar{b}_{nb} q^{-(nb-1)}]. \quad (3)$$

Here q^{-1} denotes the shift operator: $q^{-1}u(t) = u(t-1)$. In case there are no external inputs, $B(q) = 0$, an AR time-series model structure is obtained. By taking $A(q) = 1$ the result is a FIR model structure.

The ARX, FIR and AR model structures have many nice properties and provide useful models in many applications. However, they also have some drawbacks:

Consider the following model of a general stable system

$$y(t) = G(q)u(t) + H(q)e(t). \quad (4)$$

The connection to an ARX model is

$$A(q) \approx H(q)^{-1} = 1 + \sum_{k=1}^{\infty} \bar{a}_k q^{-k}, \quad (5)$$

$$B(q) \approx H(q)^{-1}G(q) = \sum_{l=1}^{\infty} \bar{b}_l q^{-l}. \quad (6)$$

To simplify the discussion assume that $H(q) = 1$. The FIR approximation of $B(q)$ can then be viewed as a truncation of the Laurant series expansion of $G(q)$. The number of parameters, nb , needed to obtain an useful FIR description is thus determined by the rate of decrease of the impulse response. Systems with poles close to the unit circle have slowly decreasing impulse responses. Consequently, high order FIR models are then required to obtain sufficient flexible model descriptions.

For the general ARX model structure noise model zeros of $H(q)$ close to the unit circle are also a problem, since $A(q)$ and $B(q)$ can be viewed as truncations of the Laurant series expansion of $H(q)^{-1}$ and $H(q)^{-1}G(q)$, respectively.

2 KAUTZ MODEL STRUCTURES

To circumvent the problems with ARX models, the delay operator can be replaced by so-called discrete Laguerre or Kautz filters. The Laguerre

filter is defined as

$$L_k(q, a) = \frac{\sqrt{(1-a^2)}}{q-a} \left[\frac{1-aq}{q-a} \right]^{k-1},$$

$$a \in \mathbb{R}, \quad |a| < 1, \quad k \geq 1, \quad (7)$$

and the definition of the Kautz filter is

$$\Psi_k(q, b, c) = \begin{cases} \frac{\sqrt{1-c^2}(q-b)}{q^2+b(c-1)q-c} \left[\frac{-cq^2+b(c-1)q+1}{q^2+b(c-1)q-c} \right]^{\frac{(k-1)}{2}}, & k \text{ odd}, \\ \frac{\sqrt{(1-c^2)(1-b^2)}}{q^2+b(c-1)q-c} \left[\frac{-cq^2+b(c-1)q+1}{q^2+b(c-1)q-c} \right]^{\frac{(k-2)}{2}}, & k \text{ even}, \end{cases}$$

$$b, c \in \mathbb{R}, \quad |b| < 1, \quad |c| < 1, \quad k \geq 1. \quad (8)$$

The Laguerre filters are appropriate for well-damped systems, whereas Kautz filters preferably can be used for more resonant ones. To obtain adequate low order models one must choose a or b and c to correspond to the dominating pole (zero for time-series) of the true system. Observe that by taking $a = 0$ or $b = c = 0$ the original base functions $\{q^{-k}\}$ are obtained. Further properties of these filters, motivations and applications in system identification are given in Wahlberg (1991a) and Wahlberg (1991b).

Given the new base functions a Laguerre/Kautz model structure is defined as

$$\left[1 + \sum_{k=1}^{na} \bar{a}_k \mathcal{A}_k(q) \right] y(t) = q^{-nk} \left[\sum_{k=1}^{nb} \bar{b}_k \mathcal{B}_k(q) \right] u(t) + e(t), \quad (9)$$

where

$$\mathcal{A}_k(q) = \mathcal{B}_k(q) = \begin{cases} L_k(q, a) & \text{in the Laguerre case,} \\ \Psi_k(q, b, c) & \text{in the Kautz case.} \end{cases}$$

Note the distinction between the unknown model parameters $\bar{a}_k$ and $\bar{b}_k$ and the expected time constants specified by the filter coefficients a , b and c . As for the ARX case this model structure can be rewritten in the linear regression form

$$\begin{aligned} y(t) &= \varphi(t)' \theta + e(t), \\ \varphi(t) &= [-\mathcal{A}_1(q)y(t) \dots -\mathcal{A}_{na}(q)y(t) \\ &\quad \mathcal{B}_1(q)u(t-nk) \dots \mathcal{B}_{nb}(q)u(t-nk)]', \\ \theta &= [\bar{a}_1 \dots \bar{a}_{na} \quad \bar{b}_1 \dots \bar{b}_{nb}]'. \end{aligned} \quad (10)$$

Estimation of θ can now be done using the standard least squares method.

Due to the linear properties of these structures one can handle multiple time constants by cascading

a number of Laguerre and Kautz models. Such a general model structure can thus be defined as

$$\left[1 + \sum_{i=1}^{m^{\mathcal{L}}} \sum_{k=1}^{na_i^{\mathcal{L}}} \bar{a}_{i,k}^{\mathcal{L}} L_k(q, a_i) + \sum_{i=1}^{m^{\mathcal{K}}} \sum_{k=1}^{na_i^{\mathcal{K}}} \bar{a}_{i,k}^{\mathcal{K}} \Psi_k(q, b_i, c_i) \right] y(t) = \left[\sum_{i=1}^{m^{\mathcal{L}}} \sum_{k=1}^{nb_i^{\mathcal{L}}} \bar{b}_{i,k}^{\mathcal{L}} L_k(q, a_i) q^{-nk_i^{\mathcal{L}}} + \sum_{i=1}^{m^{\mathcal{K}}} \sum_{k=1}^{nb_i^{\mathcal{K}}} \bar{b}_{i,k}^{\mathcal{K}} \Psi_k(q, b_i, c_i) q^{-nk_i^{\mathcal{K}}} \right] u(t) + e(t), \quad (11)$$

where superscript $\mathcal{L}$ ($\mathcal{K}$) refers to the Laguerre (Kautz) part of the system and m is the number of cascaded models. Observe that this model structure still can be written in a linear regression form.

To attain a feasible low order model the dominating time constants of the real system must roughly be known. This *a priori* knowledge can, for instance, be gained through pilot step response tests or via spectral analysis. The time constants will of course only be approximately known, but based on these preliminary experiments it is possible to estimate the dominating time constants by adopting special non-linear search strategies (in this case a Gauss-Newton scheme). Notice that by using a Newton algorithm only local convergence can be guaranteed, meaning that the “optimal” time constants not necessarily are found. However, once the base functions are fixed the least squares method will always generate a global optimal solution with respect to the chosen base functions. This procedure for finding suitable base functions is followed in the next section.

3 AIRCRAFT FLIGHT FLUTTER

Flight flutter is when an aircraft component at a specific airspeed starts to oscillate. Above this critical or “flutter” speed disturbances will grow uncontrolled until they are finally limited by nonlinearities, whereas at lower speeds disturbances will be damped out. The experiments are usually carried out by attaching special transducers to the airframe, in this case to the wings, thereby introducing artificial mechanical vibrations. When flying at constant speed data is often analyzed offline, giving information about whether to allow the aircraft to fly faster or not. To a certain extent the flutter behavior is therefore a measure of the maximum performance of the aircraft.

Data gained during flight flutter tests is typically

Fig. 1. Filtered flight flutter data used for modeling.

noisy with a rather low SNR, and for safety and economical reasons only short data records can be permitted. Thus Kautz or Laguerre models are likely to perform well. The data that will be used here originates from Schoukens and Pintelon (1991) where the identification was performed in the frequency domain using the ELiS (Estimation of Linear Systems) algorithm.

Flutter data is obtained using burst swept sine (4 Hz - 40 Hz) excitations generating a force input, $u(t)$, leading to an acceleration response, which is taken as the measured output, $y(t)$. The data is sampled at 100 Hz and consists of three different data sets each containing 2048 samples. The first data set is exclusively used for modeling, while the other two data sets are reserved for validation tests. As in Schoukens and Pintelon (1991) the goal of the identification is to model the frequencies which falls into the frequency band 4 to 11 Hz. The raw input-output data is therefore filtered through a fifth-order Butterworth band-pass filter with this pass-band. The energy of the filtered time domain signals is concentrated below 8 seconds, and thus only the first 800 samples are used for modeling. The resulting excitation and response signals are shown in Fig. 1.

Since the smooth amplitude plot of the spectral estimate in Fig. 2 clearly shows four peaks, a model of at least order 8 should be searched for. A natural first choice would then be to combine 4 dif-

Fig. 2. Bode plot of the eighth order Kautz model together with the spectral analysis estimate.

ferent second order Kautz model structures, each describing one of the peaks. However, as depicted in Fig. 2, this model cannot capture the dynamics of the 6 Hz peak, even though non-linear search strategies for estimating the poles are used. By increasing the model order and using 2 second and 2 fourth order Kautz structures, i.e. 12 estimated parameters, a much better frequency approximation around 6 Hz is obtained, which is indicated by Fig. 3. In Fig. 4 the locations of the poles and the zeros of this model, along with a portion of a time domain simulation using fresh data, are shown. As can be seen the agreement is quite striking and the Kautz model of order 12 clearly has the ability to describe most of the flutter dynamics.

For comparison a number of ordinary ARX, OE and ARMAX models were fitted to the data. Using the ARX approach no model of order 40 or lower has the ability to satisfactory describe the 4 resonant modes. Concerning OE and ARMAX modeling a large number of model structures (of maximum order 40) have been evaluated, but so far none has had the capacity of reflecting the 6 Hz peak. In fact, the best models derived show a frequency behavior very similar to the eighth order Kautz model, although the number of estimated parameters is much larger (20 or more). Apparently, due to low SNR and a high sampling rate, it is here very hard to determine accurate ARX, OE or ARMAX models.

Fig. 3. Bode plot of the twelfth order Kautz model together with the spectral analysis estimate.

Finally, it should be remarked that the frequency response of the Kautz model of order 12 is very similar to the one obtained in Schoukens and Pintelon (1991) using the ELiS method. Yet the number of estimated parameters is larger (22) in the latter case.

4 PROGRAM PACKAGE

To extend and simplify the use of Laguerre and Kautz models in system identification a package of MATLAB m-files has been developed. The package is designed to utilize the features provided by the System Identification Toolbox (Ljung, 1991) and is freely available on request.

5 CONCLUSION

System identification using Kautz models has been applied to aircraft flight flutter data. Traditional models (ARX, OE and ARMAX) require high model orders to cope with the resonant behavior and low signal to noise ratio of the data. The relative short experimental time also limits the number of parameters that can be accurately estimated. The idea of Kautz models is to use *a priori* knowledge about the time constants and damping factors of the system to reduce the model complexity. It is shown that the flutter data can be accurately described by a low-order Kautz model,

Fig. 4. Location of the discrete poles (x) and zeros (o) of the twelfth order Kautz model (top). Comparison between measured output and simulated model output (bottom).

whereas ARX, OE, and ARMAX models fail to describe the peaks in the data.

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