Segmentation of Laser Range Images with Respect to Range and Variance

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Abstract: Segmentation is a first step towards successful tracking and object recognition in 2-D pictures. Mostly the pictures are segmented with respect to quantities as range, intensity etc. Here a method is presented for segmentation of 2-D laser range pictures with respect to both range and variance simultaneously. This is very useful since man-made objects differ from the background in the terrain by their smoothness.

The approach is based on modeling horizontal scans of the terrain as piecewise constant functions. Since the environment has a complicated and irregular structure we use multiple models for modeling different segments in the laser range image. The switching between different models, i.e., ranges belonging to different segments in a horizontal scan, are modeled by a hidden Markov model.

The method is of relatively low computational complexity and the maximal complexity can be controlled by the user. Real data is used for illustration of the method.

1 Introduction

In this paper we present an approach to the problem of laser radar range image segmentation. The approach is based on modelling horizontal scans of the terrain as piecewise constant or piecewise linear signals in random noise. Segmentation with such prerequisites has been reported previously [1], but with the difference that hidden Markov models are not used to model the switching between the different models. One approach [1] is that the transitions from one segment to the next is controlled by a two-valued stochastic process. The two values of the stochastic process determines if a jump has occurred at one specific pixel or if the pixel belongs to the same segment as the previous. The different segments have nothing in common, when a jump is detected the algorithm is “restarted” and old information is forgotten. As mentioned, in this paper we use hidden Markov models to model the switching between segments which infer that different segments can belong to one specific class (state) and hence can classes be used in a future classification of the image, e.g., background terrain vs. object etc. The other novelty in this paper is the use of not only changes in the distance but also changes in the variance of the

distance for segmentation. This is very useful since man-made objects differs from the background in the terrain by their smoothness. A natural extension of our approach is of course to use vertical scans and then combine the information into a final segmentation of the image. This is however not treated in this paper. Since the environment has a complicated and irregular structure we use multiple models for modeling different segments in the laser radar range image. The switchings between different models, i.e., parts of the horizontal scan belonging to different models, are modeled by a hidden Markov model.

In this kind of problems we are always faced with an exponential growth of complexity in search for the optimal sequence of hidden Markov states. For more information about hidden Markov models and estimation in that framework we refer to previously published comprehensive papers [2, 3]. Here we will use a suboptimal scheme named adaptive forgetting through multiple models (AFMM) [4] to limit the computational burden. There are several other schemes that can be used [5, 6, 7, 8]. The goal is to calculate the *a posteriori* probabilities of the Markov states given past measurements and to estimate the parameters in the data models. The scheme consists of running at most M Kalman filters in parallel at any

time t where M is a fixed positive number dictated by the computation and storage capabilities of the processor.

Calculation of *a posteriori* probabilities of the Markov states given past measurements is performed for three different assumptions on the measurement noise, namely

- constant known variance
- constant but unknown variance
- different and unknown variance in the different segments.

As an introduction we will first deal with the simplest case, constant known variance, and the role of priors will be discussed. In the case of unknown measurement variance the variance is considered to be a stochastic variable with a prior. Two cases of priors on the variance of the measurement noise is discussed; the flat prior, i.e., non-informative, and when the variance is assumed to be inverse Wishart distributed. The latter will be only briefly investigated although it might be the most useful case in practice. The extension to the case of inverse Wishart prior is straightforward given the discussion assuming a flat prior. This treatment of the variance results in a modified *a posteriori* distribution for the states given the past measurements. For a detailed and excellent overview of segmentation see [9] from where many ideas in this paper has originated.

The approach of considering the noise variance as a stochastic variable with a prior and using it in combination with hidden Markov models for segmentation of laser radar range images with respect to range and range variance simultaneously is new to our knowledge. Another advantage of the presented method is the low computational complexity, especially if compared to finding maximum a posteriori (MAP) estimates for Markov random fields with simulated annealing [10] and graduated non-convexity type of algorithms [11, 12].

The laser radar system used in this paper is described in section 2. The definitions and notations are introduced in 3, our main result is formulated in section 4 and is experimentally verified on simulated and real data in section 6. The search scheme is shortly presented in section 5. In section 7 we give a summary of the paper.

2 The laser system

In our experiments we use a coherent laser radar system [1]. The imaging laser radar system is flexible and can be optimized for Doppler or range measurements. It has a bore-sighted TV camera with the same field of view as the laser radar, allowing for a multi sensor coordination on a pixel level. The maximum field of view is 24 mrad. Laser radar images are normally obtained at a rate of 2 Hz and a field of view of 15 mrad.

The transmitter laser, a CO_2 waveguide laser emitting at a wavelength of 10.6 μm . The measurements used here typically have been obtained with a 50 ns pulse length and a peak power of 500 W. The range is measured by using

the time difference between an envelope-detected start pulse derived from the transmitted pulse on the reference detector and the similar envelope-detected received signal from the target on the signal detector. The resolution of the counter is 2.5 ns, and the measurement range is 30 ns - 163 μs . With 50 ns pulse length, the range resolution (standard deviation) is typically 3 m.

3 Problem formulation

In the first sub-section definitions, notations and the used model are introduced. In the second we shortly discuss the later used priors on the variance.

Throughout the paper we will use the following notation for sub-sets of realizations of stochastic processes. A realization of a stochastic process x from time instant t_1 to t_2 is denoted by

$$x_{t_1}^{t_2} = (x_{t_1}, \dots, x_{t_2}).$$

If $t_1 = 1$ we omit the subscript, i.e.,

$$x_1^t = x^t.$$

The value of the realization for one specific time instant is denoted by a subscript x_t .

3.1 The system

Let $z_t \in \mathcal{S} = \{1, 2, \dots, S\}$ denote a finite-state, discrete time, Markov chain with transition probabilities

$$q_{ij} \stackrel{\text{def}}{=} P(z_{t+1} = j | z_t = i) \quad i, j \in \mathcal{S}$$

and initial probability distribution $q_i \stackrel{\text{def}}{=} P(z_0 = i), i \in \mathcal{S}$.

In this paper a special case of the general linear system with coefficients which are states in a Markov chain will be used. The general linear system can be described as follows

$$\begin{aligned} x_{t+1} &= F(z_t) + v(z_t) \\ y_t &= H(z_t) + e(z_t) \end{aligned} \quad (1)$$

where $v(z)$ and $e(z)$ are independent white Gaussian noises,

$$\begin{aligned} v &\in N(0, V(z_t)) \\ e &\in N(0, R(z_t)). \end{aligned}$$

Since our approach in this report is to model the horizontal range scans of the terrain as piecewise constant signals in measurement noise, a special case of the general system (1) is used. We denote the range at time t as $\theta_t(z_t)$, where $z_t \in \mathcal{S}$ is the variable deciding which of the S models generated the measurement y_t . The special form of (1) is

$$\begin{aligned} \theta_{t+1} &= \theta_t(z_t) \\ y_t &= \theta_t(z_t) + e(z_t). \end{aligned} \quad (2)$$

The first equation in (2) is the difference equation for the constant signal and the second is the measurement equation with different measurement noise variances in the different segments. Equation (2) can be interpreted as S systems running in parallel and the variable z_t decides which model is generating the output.

We have ended up with a bank of S models each describing a class in the horizontal scan with different range and/or variance of the measurement noise. In other words, the local variations within every class, i.e. small variations in the terrain, are modeled with white Gaussian noise, while the modeling of essentially different parts of the horizontal scan is handled by using different linear models, i.e. ranges, for different classes. Note that here “class” means the set of data generated by the same model (one of S) and the class do *not* have to be connected, i.e. data in the first part of the scan and in the last can belong to the same class in spite that they belong to different segments.

If we for a moment forget about the Markov chain and classes then the optimal estimate of θ_t would be given by the Kalman filter equations

$$\begin{aligned}\hat{\theta}_{t+1|t} &= \hat{\theta}_{t|t-1} + K_t \varphi_t \\ \varepsilon_t &= y_t - \hat{\theta}_{t|t-1} \\ K_t &= P_{t|t-1} S_t^{-1} \\ S_t &= P_{t|t-1} + R_t \\ P_{t+1|t} &= P_{t|t-1} - P_{t|t-1} S_t^{-1} P_{t|t-1} \\ \hat{\theta}_{1|0} &= \theta_0 \\ P_{1|0} &= P_0.\end{aligned}\tag{3}$$

The estimator (3) is the well-known recursive least squares (RLS) algorithm without forgetting factor which minimizes the loss function

$$W_N(\theta) = \sum_{t=1}^N (y_t - \theta)^T R_t^{-1} (y_t - \theta).$$

Assume that the prior distribution of θ is Gaussian with mean θ_0 and covariance matrix P_0 . Then the posterior distribution $p(\theta|y^N)$ is also Gaussian with mean $\hat{\theta}$ and covariance matrix P_N , see [13] for an detailed treatment of recursive methods.

If we now assume that the state sequence z_1^N , generated by a Markov chain, is known nothing in principal changes. We have to run as many Kalman filters in parallel as there are states (S). We label the filters with 1 through S and update only one filter at each sampling instant, namely the one with the same label as the state the variable z_t is in. We end up with S estimates of θ , one for each class.

3.2 The priors

Assume the variance of the measurement noise R_t and the variance for the parameter prior P_0 is incorrectly chosen in such way so the true value differ with a scaling λ . We denote the true values of R_t and P_0 with bars,

$$\bar{P}_0 = \lambda P_0$$

$$\bar{R}_t = \lambda R_t.\tag{4}$$

The effect of this scaling on the estimated variance for the parameter $\bar{P}_{t+1|t}$ is the following

$$\bar{P}_{t+1|t} = \lambda P_{t+1|t}.$$

The value of the estimated parameter θ is however still the same. For pure filtering the actual level of the variances is not important. This is easily checked by substitution of (4) in (3). An important effect of the scaling, and this effect will be used in section 4, is that the *a posteriori* density function of z is dependent of λ .

The reason for wanting to calculate the *a posteriori* density function of z given data is because the goal is to find which value of z maximizes the density function and then pick that as an estimate (which is the definition of MAP estimate of the sequence z^N). In bayesian statistics the available information we have about the sequence or a parameter is used in the estimation. The prior information has the form of a density to the random sequence or random variable. In our case we will have two priors, one is the prior on the state sequence, which is given by the Markov transition matrix and the initial probabilities of the Markov chain, and the second is the prior on the parameter λ which we will chose as flat or inverse Wishart. For the derivation of the *a posteriori* density function of z we first need the distribution of data given past data and states. The Kalman filter theory gives the distribution for the measurement prediction as

$$y_t|y^{t-1} \in N(\hat{\theta}_{t|t-1}, S_t),$$

where $N(\mu, \sigma^2)$ denotes the Gaussian distribution with mean value μ and variance σ^2 . The density function for the complete data sequence given the states is

$$\begin{aligned}P(y^N|z^N) &= (2\pi)^{-N/2} \left(\prod_{t=1}^N \det S_t(z^N) \right)^{-1/2} \cdot \\ &\quad e^{-\frac{1}{2} \sum_{t=1}^N \varepsilon_t^T(z^N) S_t^{-1}(z^N) \varepsilon_t(z^N)}.\end{aligned}\tag{5}$$

We have used that the data are independent if conditioned on the states. Bayes' law together with (5) gives the *a posteriori* distribution for the state sequence

$$\begin{aligned}P(z^N|y^N) &= (2\pi)^{-N/2} \left(\prod_{t=1}^N \det S_t(z^N) \right)^{-1/2} \cdot \\ &\quad e^{-\frac{1}{2} \sum_{t=1}^N \varepsilon_t^T(z^N) S_t^{-1}(z^N) \varepsilon_t(z^N)} \cdot \frac{P(z^N)}{P(y^N)}.\end{aligned}$$

This expression is valid if the measurement noise variance is known, or, in other words, if $\lambda = 1$. In terms of priors we could interpret λ as a random variable with the following density function (prior)

$$P(\lambda) = \delta(\lambda - 1),$$

where $\delta(x) = 1$ if $x = 0$ and zero elsewhere. The extensions of the reasoning above is of course to assume other priors on the scaling λ , and this will be discussed in section 4.

4 Results

4.1 Constant known variance

The case of constant known noise variance R_t is the simplest case of the three mentioned in section 1. We start with it and concentrate on issues other than the segmentation with respect to variance, which we leave for a later section. The idea is to first introduce how our estimator works and then it is easily extended to the case of varying variance.

If we knew the value of the sequence $z^N \stackrel{\text{def}}{=} z_1, \dots, z_N$, where N is the length of the horizontal scan, the Kalman filter would give the optimal estimate of the ranges in the different classes. The difficulty is that we do not know z^N . So, at least in the optimal case, we will have to run S Kalman filters for every possible state sequence of z^N and calculate the optimal estimates of the different ranges $\theta(z)$ and covariance matrices $P(z)$. These are later used in the calculation of the likelihoods

$$P(y^N|z^N) = (2\pi)^{-N/2} \left(\prod_{t=1}^N \det S_t(z^N) \right)^{-1/2} e^{-\frac{1}{2} \sum_{t=1}^N \varepsilon_t^T(z^N) S_t^{-1}(z^N) \varepsilon_t(z^N)}. \quad (6)$$

Using Bayes' rule we easily obtain the *a posteriori* density which we maximize. The state sequence with the highest probability is then chosen as the estimate. The estimate of θ and P following the chosen state sequence is our estimates of the range and its variance. To summarize the discussion on how to treat the case of known variance we here give the expression of the *a posteriori* probability of the sequence given data

$$P(z^N|y^N) = \frac{P(y^N|z^N)P(z^N)}{P(y^N)},$$

where $P(y^N|z^N)$ is given by (6) and $P(z^N)$ is given by the state transition matrix and the actual sequence z^N

$$P(z^N) = \prod_{i=2}^N P(z_i|z_{i-1}) \cdot P(z_1),$$

where the $P(z_1)$ is the initial probability of the Markov chain.

If we have an image containing 128×128 pixels and want to segment it into three classes we would have to run 3^{129} Kalman filters. Clearly this is impossible and a suboptimal search method has to be used. How the search for the best sequence is performed will be explained in section 5.

4.2 Constant unknown variance

In this subsection we go one step further and assume that the variance is constant over the different classes, but we do not know the value. We continue our discussion about priors in section 3.2 here. When the level of the variance is unknown, i.e., we assume that the variance is λR_t , a more fair choice than $P(\lambda) = \delta(\lambda - 1)$ as prior

is natural. The goal is to inflict as little prejudice as possible with the prior density and it should reflect our true knowledge about the random variable in question. When the random variable, here the level of the variance, is completely unknown, the best choice is a flat prior. We use the prior to modify the *a posteriori* distribution of z . When we write down the left hand side of the expression (5) we have implicitly assumed a prior on λ . It really should stay

$$P(y^N|z^N) = \int_{-\infty}^{+\infty} P(y^N|z^N, \lambda) \cdot P(\lambda) d\lambda. \quad (7)$$

The correct expression for $P(y^N|z^N, \lambda)$ is the following

$$P(y^N|z^N, \lambda) = (2\pi)^{-N/2} \left(\prod_{t=1}^N \det S_t \right)^{-1/2} \lambda^{-N/2} e^{-\frac{V_N}{2\lambda}}, \quad (8)$$

where $V_N = \sum_{t=1}^N \varepsilon_t^T(z^N) S_t^{-1}(z^N) \varepsilon_t(z^N)$. If we assume the variance to be known or, which is equivalent, assume the prior on λ as $P(\lambda) = \delta(\lambda - 1)$ and insert that together with (8) into (7) we obtain the expression (5). If we instead use the more fair prior $P(\lambda) = 1$ on λ , i.e., we assume all values of λ to be equally probable, we obtain the following *a posteriori* likelihood function

$$P(z^N|y^N) = \frac{\Gamma(\frac{N-2}{2})}{2\pi^{N/2} (\prod_{i=1}^N \det S_t)^{1/2} (V_N)^{\frac{N-2}{2}}} \cdot \frac{P(z^N)}{P(y^N)}. \quad (9)$$

Notice that the dependence of z^N has been suppressed in the expression (9). The derivation of equation (9) is included in Appendix A.1. Γ is the gamma-function

$$\Gamma(a+1) = \int_0^\infty x^a e^{-x} dx \quad a > -1.$$

4.3 Unknown variance varying over the different segments

The next step is to assume unknown variance but allow different variance in the different classes. This assumption on the variance is the most interesting of the three mentioned. It is this *a posteriori* likelihood function we try to maximize when the image in section 6 is segmented with respect to variance. If we assume the noise variance in the different classes to be $\lambda(i)R_t$, where R_t is known, $\lambda(i)$ is unknown but considered as a stochastic variable with a flat prior, i.e., $P(\lambda) = 1$, and $i = 1 \dots S$, the expression for the *a posteriori* likelihood is the following

$$P(z^N|y^N) = \frac{1}{\pi^{N/2} 2^S} \prod_{k=1}^S \frac{\Gamma(\frac{N(k)-2}{2}) P(z^N)}{(D(k) V(k)^{N(k)-2})^{1/2} P(y^N)} \quad (10)$$

where

$$D(k) = \prod_{t \in S_k} \det S_t.$$

In words the expression above means taking the product of $\det S_t$ over the data points t which belong to class $k \in \mathcal{S}$. The number of data points summed over the classes k is of course N , i.e., $\sum_{k=1}^S N(k) = N$. We will not present the calculations leading to expression (10), they are similar to those for expression (9).

4.4 Inverse Wishart prior

As already mentioned in section 1 three cases of degree of knowledge about the variance are treated in this paper; known, unknown but constant over the classes and, finally, unknown and varying over the classes. In the case of unknown variance we have so far modified the *a posteriori* probability of the states using a non-informative prior on the parameter λ . In practice we often know something about the variance and that information should not be thrown away. We will in this section present a useful choice of prior on λ if beforehand information on λ is available. We will assume an inverse Wishart density of λ . We will first discuss the case of constant variance over the classes, and then shortly mention the expression for the case of varying variance over the classes. First, let us take a look on the inverse Wishart distribution. The inverse Wishart distribution has two parameters and will in this paper be denoted by $W^{-1}(m, \sigma)$. The probability density function is

$$P(\lambda) = \frac{\sigma^{m/2} e^{-\frac{\sigma}{2\lambda}}}{2^{m/2} \Gamma(m/2) \lambda^{(m+2)/2}}, \quad (11)$$

The mean and the variance of this distribution is given

Figure 1: The probability density function of the inverse Wishart distribution.

by

$$\begin{aligned} E(\lambda) &= \frac{\sigma}{m-2} \\ Var(\lambda) &= \frac{2\sigma^2}{(m-2)^2(m-4)}. \end{aligned}$$

Fig 1. shows the inverse Wishart density function with mean 1 and variance 1. The point with introducing the

inverse Wishart distribution on the prior is its usefulness if the measurement noise variance is not exactly known, but rather we have a vague conception about the value of the measurement noise variance. In that case an inverse Wishart distributed prior is assumed with the mean value as the expected noise variance and the variance of the prior chosen according to the certainty about the value of the noise variance.

If we now go through similar calculations as in section 4.2 in the case when the prior on λ is inverse Wishart distributed the resulting *a posteriori* distribution is the following

$$P(z^N | y^N) = \frac{\Gamma(\frac{N+m}{2})(2\sigma)^{m/2}}{\pi^{N/2} \Gamma(\frac{m}{2})(\prod_{i=1}^N \det S_t)^{1/2}} \cdot \frac{P(z^N)}{(V_N + \sigma)^{\frac{N+m}{2}} P(y^N)}. \quad (12)$$

Derivation of equation (12) is found in Appendix A.2.

4.4.1 Different inverse Wishart distributed priors for each class

Here we assume that different values of mean and variance of the prior on the measurement noise variance is used in the different classes. The likelihood of the complete data sequence given the state sequence expressed in the likelihoods of the individual class is

$$P(y^N | z^N) = \prod_{i=1}^S P(y_{t \in i}), \quad (13)$$

where the factors on the right side of the equality (13) is given by

$$\begin{aligned} \int_0^\infty P(y_{t \in i} | \lambda(i)) \cdot P(\lambda(i)) d\lambda &= \\ \int_0^\infty (2\pi)^{-N(i)/2} \lambda(i)^{-N(i)/2} (\prod_{t \in i} \det S_t)^{-1/2} \cdot \\ e^{-\frac{1}{2} \sum_{t \in i} \varepsilon_t^T S_t^{-1} \varepsilon_t} \cdot W^{-1}(m(i), \sigma(i)) d\lambda(i) &= \\ \frac{\Gamma(\frac{N(i)+m(i)}{2})(2\sigma(i))^{m(i)/2}}{\pi^{N(i)/2} \Gamma(\frac{m(i)}{2})(\prod_{t \in i} \det S_t)^{1/2}} \cdot \\ \frac{1}{(V_N(i) + \sigma(i))^{\frac{N(i)+m(i)}{2}}}. \end{aligned} \quad (14)$$

The likelihood (13) is possible to express as a product because it is assumed that data in the different classes are independent. This is a natural assumption since different classes in an image often belong to different objects in the image. Note that this independence assumption would be valid in the case of more complex models, e.g. dynamic models, describing the different classes. The final expression for the *a posteriori* likelihood is given by combination of (13), (14) and Bayes' rule

$$P(z^N|y^N) = \prod_{i=1}^S \left(\frac{\Gamma(\frac{N(i)+m(i)}{2})(2\sigma(i)^{m(i)/2})}{\pi^{N(i)/2}\Gamma(\frac{m(i)}{2})(\prod_{t \in i} \det S_t)^{1/2}} \cdot \frac{P(z^N)}{(V_N(i) + \sigma(i))^{\frac{N(i)+m(i)}{2}} P(y^N)} \right).$$

5 Search scheme

Since we are faced with the problem of exponential growth of the possible paths of z^N in the search of the maximum of $P(z^N|y^N)$, some suboptimal search strategy has to be used. We have in this report chosen the AFMM method, but slightly modified. In the AFMM algorithm the number of Kalman filters (in our case, due to the specific model of the signal, we use RLS algorithms) is limited to M . At each pixel step the *a posteriori* probabilities for the M branches, i.e., different paths of z_1^n , is produced by the M Kalman filters. The most probable branch is allowed to split, and the $M - S$ branches with the lowest probabilities are cut off (forgotten). The modification used here is that branches are not allowed to be cut off if they are younger than a specified age. The additional parameter is called *life length*. The objective with the life length parameter is to assure that branches live long enough so a change in variance can be detected. For that at least 4-5 data points are needed.

6 Examples

To demonstrate the performance of our method we will apply it on two sets of data, one containing simulated data and the other a data set collected with a laser range radar system.

Figure 2: The signal used for testing segmentation with respect to variance. There is a drop in noise variance in samples 35 through 75.

The first signal used is white Gaussian noise with different variances. The signal is generated as $y_t = e_t$, where e_t

is white Gaussian noise with variance 1, except for samples number 35 through 70 where the signal is generated as $y_t = 0.5e_t$. There is a drop of measurement noise variance in the middle of the signal. The test signal is shown in Fig. 2.

Two states are used in the hidden Markov model and the transition probability matrix is the following

$$P = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix}.$$

We chose to use 7 RLS schemes in the search for the optimal path, the life length parameter is set to 5, i.e., no branch younger than five samples is allowed to be cut off.

The result of the simulation is shown in Fig. 3. The algorithm finds the transitions almost exactly even though it is difficult to exactly determine the jumps by simply looking at the signal. The other part of the verification

Figure 3: The resulting segmentation of the test signal. The dashed line shows the partition of the signal in classes, class one are the parts of the signal where the measurement noise has the variance 1 and class two the part with variance 0.25. The dotted line shows the true segmentation.

of our method is the test on real laser range radar system. The range image obtained from the system is shown in Fig. 4. The image has been pre-processed so there are no drop-outs. Drop-outs in the image is an effect of diffuse targets which give rise to speckle effects or an effect of pointing the laser beam into non-target areas (e.g. the sky). The light pulse sent out do not return within a pre-set time interval, the value of the measured range in such pixels are set to a pre-determined value, in our case -1. In this report the pre-processing is performed in an *ad hoc* manner since it is not the main topic in this study. Drop-outs are simply replaced by the median value of the surrounding pixels not being a drop-out. For the scene studied here, the drop-out probability is about 25 %.

Some additional explanation of Fig. 4 may be needed. In the middle of the image there is a vertical sign (approximately 400 m from the laser system) and therefore

Figure 4: The range image obtained from the laser range radar. The drop-outs has been removed.

there is an area of constant range in the middle of Fig. 4. On the sides of the sign there is relatively smooth ground, and that gives the approximately linearly increase in distance as the laser beam scans higher and higher up, closer to the horizon. When the sign is passed there are some bushes on the right hand side, and again that part of the picture has roughly constant distance from the laser system.

Again two states are used in the hidden Markov model to segment the image. In the 2-D case we use our method to segment the image row by row. When the segmentation of one row is finished we use the estimated values of range in the different classes as the initial values for the segmentation of the next row. The resulting segmentation is shown in Fig. 5. Class two is associated with the

Figure 5: The row-by-row segmented image. Two classes are used.

sign in the middle of the image and with the bushes in the upper right corner. In the lower part of the image there are un-regular distribution between the two classes. This is due to that the method tries to use all the degrees of freedom when segmenting, i.e., two classes are used al-

though only one is needed. The effect is that the signal is divided between the two classes in an unregular way. A way out of this problem could be to estimate, from data, the optimal number of states on beforehand. This will increase the computational complexity, but the effects of spurious jumping will be avoided. This is however a topic for further research.

7 Summary

In this paper a new method for segmentation of laser range radar images is presented. The segmentation method detects segments with different variances and ranges.

Our method is based on multiple models and a row by row segmentation of the image, i.e. it is basically a 1-D method but some information is taken along from row to row. The information is the values of the previous row's ranges in the individual segments. This ranges are used as initial values for the segmentation of the next row. Multiple models are used to model different parts of a row, parts with different variance or range, and the switching between the models is governed by a hidden Markov chain. In the search for the optimal state sequence we use a sub-optimal search algorithm.

Our experiments show that we can detect changes of a factor four in variance of the measured range, which is useful since man-made objects often differs from its natural environment just by their smoothness. The method is also exemplified on a measured laser range radar image and shows good results. What is needed is an estimation of the number of states in the underlying Markov chain. If the number is overestimated we get the undesirable effect of spurious jumping between states as the method uses all the available degrees of freedom. This effect is seen in the first rows of Fig. 5. How to avoid this will be treated in a subsequent paper.

A Appendix

A.1 Derivation of the *a priori* probability in the case of a flat prior on λ

Here the derivation of equation (9) is shown. The difficulty is to calculate the *a priori* probability, the *a posteriori* probability is simply obtained by applying Bayes' rule. Assume the prior of λ is flat, $P(\lambda) = 1$. The *a priori* probability is then given by

$$\begin{aligned}
 P(y^N | z^N) &= \int_0^\infty P(y^N | z^N, \lambda) \cdot P(\lambda) d\lambda \\
 &= \int_0^\infty (2\pi)^{-N/2} \left(\prod_{t=1}^N \det S_t \right)^{-1/2} \lambda^{-N/2} e^{-\frac{y_N}{2\lambda}} d\lambda \\
 &= (2\pi)^{-N/2} \left(\prod_{t=1}^N \det S_t \right)^{-1/2} 2^{\frac{N-2}{2}} \Gamma\left(\frac{N-2}{2}\right).
 \end{aligned}$$

$$= V_N^{-\frac{N-2}{2}} \int_0^\infty \frac{V_N^{\frac{N-2}{2}} e^{-\frac{V_N}{2\lambda}}}{2^{\frac{N-2}{2}} \Gamma(\frac{N-2}{2}) \lambda^{\frac{N-2+2}{2}}} d\lambda, \quad (15)$$

where the last factor in expression (15) is recognized as the inverse Wishart distribution (11) integrated from 0 to ∞ . Finally we obtain

$$P(y^N | z^N) = (2\pi)^{-N/2} \left(\prod_{t=1}^N \det S_t \right)^{-1/2} 2^{\frac{N-2}{2}} \Gamma\left(\frac{N-2}{2}\right) V_N^{-\frac{N-2}{2}},$$

which multiplied by $\frac{P(z^N)}{P(y^N)}$ gives equation (9).

A.2 Derivation of the *a priori* probability in the case of inverse Wishart prior λ

In a similar way as in the previous section we derive the *a posteriori* probability density for the case of a inverse Wishart distribution on the prior of λ .

$$\begin{aligned} P(y^N | z^N) &= \int_0^\infty P(y^N | z^N, \lambda) \cdot P(\lambda) d\lambda \\ &= \int_0^\infty (2\pi)^{-N/2} \left(\prod_{t=1}^N \det S_t \right)^{-1/2} \lambda^{-N/2} \cdot \\ &\quad e^{-\frac{V_N}{2\lambda}} \frac{\sigma^{m/2} e^{-\frac{\sigma}{2\lambda}}}{\lambda^{(m+2)/2} 2^{m/2} \Gamma(m/2)} d\lambda \\ &= \frac{\sigma^{m/2} \pi^{-N/2}}{\Gamma(m/2) \left(\prod_{t=1}^N \det S_t \right)^{1/2}} \cdot \\ &\quad \int_0^\infty 2^{-\frac{N+m}{2}} \lambda^{-\frac{N+m+2}{2}} e^{-\frac{1}{2\lambda} (V_N + \sigma)} d\lambda \\ &= \frac{\Gamma\left(\frac{N+m}{2}\right) (2\sigma)^{m/2} \pi^{-N/2}}{\Gamma(m/2) \left(\prod_{t=1}^N \det S_t \right)^{1/2} (V_N + \sigma)^{\frac{N+m}{2}}} \cdot \\ &\quad \int_0^\infty \frac{(V_N + \sigma)^{\frac{N+m}{2}} e^{-\frac{V_N + \sigma}{2\lambda}}}{2^{m/2} \Gamma\left(\frac{N+m}{2}\right) \lambda^{\frac{N+m+2}{2}}} d\lambda, \end{aligned}$$

where the integrand in the last equation is recognized as the inverse Wishart distribution with parameters $W^{-1}(N+m, V_N + \sigma)$, and thus it integrates to one.

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