Recursive identification of physical parameters in a flexible robot arm

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Abstract

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Keywords: Robotics, Recursive identification, Physical parameters

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Abstract

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1 Introduction

This paper deals with recursive identification of the parameters in a physically parameterized model of a flexible robot arm. Recent examples of off-line robot identification in general are given in, e.g., [9], [1], and [2]. Off-line identification of the parameters in the type of physically parameterized robot models studied here is presented in [11].

Off-line identification of physical parameters in continuous time models can be carried out using commercially available software. See, for example, [5]. Recursive identification of parameters in discrete time models of black-box type is also an established area [8]. The topic of this paper, i.e. to recursively estimate the physical parameters in continuous time models, has however received less attention. The topic is important in cases like fault isolation and fault identification where the main task is to identify the continuous time parameter values as fast as possible after a fault has occurred. It is in this context the recursive identification of physical parameter values comes in. While the problem under consideration is general the attention in this paper will be concentrated to a simplified description of a flexible robot arm.

2 The Robot System

The robot that is studied in this paper is an industrial robot of the type ABB IRB 1400, and it is shown in Figure 1.

Figure 1: ABB IRB 1400.

The dynamics of the robot system when moving around axis one will be approximated by a model consisting of three masses connected via springs and dampers as shown in Figure 2. In [11] models with two and three masses were compared, and it was found that models with three masses gave considerably better results. Therefore this paper is restricted to three-mass models. The input is the torque τ generated by the electrical motor, while the output is the motor angle ϕ_m . The angles of the other masses, ϕ_g and ϕ_a respectively, are not measurable. The notations are explained in Table 1. Even though this is a considerably simplified description of a real robot arm it is a useful approximation in many situations.

Figure 2: Three-mass flexible model.

J_m, J_a, J_g	moments of inertia
k_g, k_a	spring constants
f_m, f_a, f_g	viscous friction coefficients
d_g, d_a	damping coefficients
r	gear box ratio ($r = 1/118$)
k_T	torque constant

Table 1: Notations.

Applying torque balances for the three masses and introducing the states

$$x(t) = \begin{bmatrix} r\phi_m(t) - \phi_g(t) \\ \phi_g(t) - \phi_a(t) \\ \dot{\phi}_m(t) \\ \dot{\phi}_g(t) \\ \dot{\phi}_a(t) \end{bmatrix} \quad (1)$$

the input signal $u(t) = \tau(t)$ and the output signal $y(t) = \dot{\phi}_m(t)$ gives the state space model

$$\dot{x}(t) = Ax(t) + Bu(t) \quad y = Cx(t) \quad (2)$$

where

$$A = \begin{bmatrix} 0 & 0 & r & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -\frac{r \cdot k_g}{J_m} & 0 & -\frac{r^2 d_g + f_m}{J_m} & \frac{r \cdot d_g}{J_m} & 0 \\ \frac{k_g}{J_g} & -\frac{k_a}{J_g} & \frac{r d_g}{J_g} & -\frac{f_g + d_g + d_a}{J_g} & \frac{d_a}{J_g} \\ 0 & \frac{k_a}{J_a} & 0 & \frac{d_a}{J_a} & -\frac{d_a + f_a}{J_a} \end{bmatrix} \quad (3)$$

$$B^T = \begin{bmatrix} 0 & 0 & \frac{k_T}{J_m} & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (4)$$

In the robot control system the motor angle ϕ_m is the only available output signal, but since the measurement noise is fairly small a reasonable estimate of the motor velocity is easily obtained. Therefore the motor velocity is used as output signal in the model above.

3 The recursive identification algorithm

3.1 Algorithm structure

The identification will be carried out using a recursive prediction error (RPEM) algorithm [8].

$$\hat{\theta}(t) = \hat{\theta}(t-1) + P(t|\hat{\theta})\psi(t)\varepsilon(t) \quad (5)$$

where $\varepsilon(t)$ denotes the prediction error

$$\varepsilon(t) = y(t) - \hat{y}(t, \hat{\theta}(t-1)) \quad (6)$$

and $\hat{y}(t, \hat{\theta}(t-1))$ is the prediction of the output. $P(t|t)$ is a symmetric (covariance) matrix, and $\psi(t)$ denotes the gradient of the prediction error with respect to the unknown parameters. When designing an appropriate update algorithm for the matrix $P(t|t)$ there are two main issues that have to be taken into account. First, when identifying parameters that are subject to changes it is important that the algorithm maintains its tracking ability. Second, it is important to prevent $P(t|t)$ from growing during periods of poor excitation. These issues can be dealt with in different ways and the aim here is to point out some possible solutions. The update of P is therefore carried out in three steps. The first step, sometimes denoted the measurement update, is given by

$$P(t|t) = P(t|t-1) - \frac{P(t|t-1)\psi(t)\psi^T(t)P(t|t-1)}{1 + \psi^T(t)P(t|t-1)\psi(t)} \quad (7)$$

This updating of P corresponds to the updating

$$R(t|t) = R(t|t-1) + \psi^T(t)\psi(t) \quad (8)$$

of the information matrix R which is the inverse of P , i.e. $P(t|t) = R^{-1}(t|t)$. In the next step, here denoted the time update, the tracking ability is ensured, and in this paper two alternatives will be considered. The first alternative is the classical forgetting factor approach, which corresponds to the updating

$$\bar{P}(t+1|t) = \frac{1}{\lambda} P(t|t) \quad (9)$$

where $0 < \lambda \leq 1$ denotes the forgetting factor. The second alternative is the Kalman filter approach, where

$$\bar{P}(t+1|t) = P(t|t) + \Delta \quad (10)$$

and Δ is a symmetric positive definite matrix. In the third step the aim is to ensure that P does not grow without bounds when the excitation is poor. This problem is equivalent to the problem that the information matrix tends to a singular matrix. The standard method for handling this problem is to use regularization, where

$$R(t+1|t) = \bar{R}(t+1|t) + \mu \cdot I \quad (11)$$

where μ is a positive scalar. Using the matrix inversion lemma this corresponds to

$$P(t+1|t) = \bar{P}(t+1|t)(I + \mu\bar{P}(t+1|t))^{-1} \quad (12)$$

i.e. the regularization of the information matrix corresponds to a normalization of the covariance matrix. See also [3]. The two algorithms under consideration

are hence obtained by combining Equations (7) and (9), giving

$$\begin{aligned}\bar{P}(t+1|t) &= \frac{1}{\lambda}(P(t|t-1) \\ &- \frac{P(t|t-1)\psi(t)\psi^T(t)P(t|t-1)}{1+\psi^T(t)P(t|t-1)\psi(t)})\end{aligned}\quad (13)$$

and combining Equations (7) and (10), yielding

$$\begin{aligned}\bar{P}(t+1|t) &= P(t|t-1) \\ &- \frac{P(t|t-1)\psi(t)\psi^T(t)P(t|t-1)}{1+\psi^T(t)P(t|t-1)\psi(t)} + \Delta\end{aligned}\quad (14)$$

together with Equation (12). The design parameters are hence λ and Δ for the tracking properties and μ for the regularization. Different aspects of the choice of these parameters will be discussed below.

3.2 Forming the prediction and its gradient

A key point when applying the RPEM algorithms above is how to determine $\hat{y}(t, \theta)$ and $\psi(t)$ for the continuous time model when only discrete time data are available. The continuous time state space model in Equation (2) can be converted to discrete time form using standard methods assuming zero order hold of the input. The resulting matrices of the discrete time state space model are however complicated functions of the physical parameters, and this makes the differentiation complicated. The approach taken here is to carry out the operations in the reversed order, i.e. to do the differentiation using the continuous time model and in a second step convert the expression for the gradient to discrete time. Consider therefore a linear state space model for which the predictor is

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) \quad \hat{y}(t, \theta) = C(\theta)x(t) \quad (15)$$

This predictor is chosen because off-line identification experiments gives good results using OE models. The equations for the gradient of the prediction with respect to a scalar parameter can be found in, e.g., [6], and they are given by

$$\psi(t) = \frac{d}{d\theta}\hat{y}(t, \theta) = C(\theta)z(t) + \bar{C}x(t) \quad (16)$$

where

$$z(t) = \frac{d}{d\theta}x(t, \theta) \quad (17)$$

and

$$\bar{C}(\theta) = \frac{d}{d\theta}C(\theta) \quad (18)$$

The time derivative of $z(t)$ is obtained from

$$\frac{d}{dt}z(t) = \frac{d}{d\theta}\dot{x}(t) = A(\theta)z(t) + \bar{A}x(t) + \bar{B}u(t) \quad (19)$$

where

$$\bar{A}(\theta) = \frac{d}{d\theta}A(\theta) \quad \bar{B}(\theta) = \frac{d}{d\theta}B(\theta) \quad (20)$$

With the extended state vector

$$X(t) = \begin{pmatrix} z(t) \\ x(t) \end{pmatrix} \quad (21)$$

the state space description for the prediction and the gradient is given by

$$\dot{X}(t) = \begin{pmatrix} A(\theta) & \bar{A} \\ 0 & A(\theta) \end{pmatrix} X(t) + \begin{pmatrix} \bar{B} \\ B(\theta) \end{pmatrix} u(t) \quad (22)$$

$$\psi(t) = (C(\theta) \quad \bar{C})X(t) \quad (23)$$

Equations (18) and (20) are repeated for each parameter that should be identified. This means that for each parameter that is identified the state space model in Equation (22) is extended with n more states.

Depending on which parameter that is of interest the matrices $\bar{A}$, $\bar{B}$ and $\bar{C}$ will have different properties. For example, considering identification of f_m one gets

$$\bar{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{f_m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \bar{B} = 0 \quad \bar{C} = 0 \quad (24)$$

In order to generate $\hat{y}$ and ψ using discrete time data the state space model in (22) is transformed to its discrete time counterpart in each sampling point, using the current parameter estimate.

3.3 Some properties of the gradient

An important property when identifying physical parameters is that the character of the gradient $\psi(t)$ depends on which particular parameter that is identified. Consider the transfer operator of the model converted to discrete time

$$y(t) = G(q, \theta)u(t) + e(t) \quad (25)$$

where $e(t)$ is white noise, since, as mentioned above, an output error structure is considered. The prediction is hence given by

$$\hat{y}(t) = G(q, \theta)u(t) \quad (26)$$

The gradient of the prediction with respect to a scalar θ can be expressed as

$$\psi(t) = G_\theta(q, \theta)u(t) \quad (27)$$

where $G_\theta(q, \theta)$ is the derivative of the transfer operator $G(q, \theta)$ with respect to θ . The variance of $\psi(t)$ will have

big a influence on the properties of the estimates, and it can be expressed

$$E[\psi^2(t)] = \int_{-\pi/T}^{\pi/T} |G_\theta(e^{i\omega T}, \theta)|^2 \Phi_u(\omega) d\omega \quad (28)$$

The magnitude of the variance hence depends on the character of the input spectrum and the properties of the transfer function G_θ . This will be illustrated in a particular example below.

4 Identification of nominal model

First, off-line identification is used to get the parameter values of the model. The acquired parameter values are used as nominal values in the recursive algorithm. Then some of the parameters are recursively estimated.

The determination of the nominal model is done by identifying a physically parameterized model as described in [11]. The external excitation that is added to the reference signal of the robot control system is a sum of sinusoids in the range 0-30 Hz. The signal is created by setting the discrete Fourier transform of the reference signal to one with random phase and then transforming the Fourier transform to time domain. The system used for collecting data from the robot is further described in [10]. The sampling frequency of the data is 200 Hz. Since the system is operating in closed loop the applied torque signal will be affected by the feedback. The properties of the torque signal are shown in Figure 3, and it is seen that the input energy is low below 70 rad/s with peaks around 95, 125 and 185 rad/s.

Figure 3: FFT of the torque signal.

The System Identification Toolbox in Matlab [5] is used in the off-line identification. For comparison a very high order ARX model is shown together with the acquired model in Figure 4. As seen in the figure the 3-mass model is a reasonable approximation of the system. The acquired parameter values from the off-line

Figure 4: Bode plots of identified models. Solid: Physically parameterized model. Dotted: High order ARX-model.

identification will hence be used as nominal parameter values in the recursive identification below.

5 Recursive identification

5.1 Identified parameters

The choice of which parameters that should be identified recursively originates from which parameter values that are likely to change over time. An example could be a worn gear box, which indicates an increase in f_m . In these experiments the interest has been concentrated on the three parameters k_T , J_a^{-1} and f_m . To give an indication of that the particular parameter to be identified will have influence on the algorithm behavior Figure 5 shows the squared amplitude curve of the transfer operators $G_\theta(q, \theta)$ for the three parameters considered here, when the transfer operators are evaluated using the nominal parameter values. This figure together with Equation (28) and Figure 3 indicate that the variance of $\psi(t)$ will be highest for f_m , less for J_a^{-1} and lowest for k_T .

5.2 Design variables

The recursive identification will be carried out using both the forgetting factor and the covariance matrix modification versions of the RPEM algorithm. The design parameters to choose are the initial values of θ and P respectively and also the forgetting factor λ and the matrix Δ . The choices of λ and Δ are trade offs between tracking ability of the algorithm and the variance of the parameter estimates. In this application $\lambda = 0.995$ has been found to be an appropriate value. Since the input can be chosen almost freely it can be designed to give sufficient excitation to avoid windup problems for the matrix P . Hence the regularization parameter can be put to zero in this application.

Figure 5: Plot of $|G_\theta(e^{j\omega T}, \theta)|^2$ for the three parameters. Solid line: k_T . Dashed line: J_a^{-1} . Dash-dotted line: f_m .

The choice of $P(0)$ can be made from different viewpoints. In case the algorithm is going to be used with some kind of change detection it is realistic to assume that $P(0)$ can be chosen to give a fast convergence after sudden changes in the true parameters. Without change detection the tracking properties will be determined by the current values of P , which depend on the choice of λ and the properties of the input signal via $\psi(t)$. Assuming $\psi(t)$ to be quasi-stationary, see [4], and λ to be close to one the matrix P can in steady state, see [7], be approximated by $\mathbf{P}$ given by

$$\mathbf{P} = (1 - \lambda)Q^{-1} \quad (29)$$

and

$$\mathbf{P}Q\mathbf{P} = \Delta \quad (30)$$

respectively, where

$$Q = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N \psi(t)\psi^T(t) \quad (31)$$

and μ has been put to zero. As discussed above the variance of the components of $\psi(t)$ will be of different magnitude and hence will also the elements of $\mathbf{P}$ be of different magnitude. This will then give different tracking and variance properties of the different parameter estimates. Using forgetting factor there is only one design variable available to affect the trade off between tracking and variance. Using the covariance modification the matrix Δ offers more freedom for dealing with this problem.

The aim here is to show the algorithm properties without any change detection involved. In order to select an initial value of P that represents the steady state behavior the update Equation for P is first run using the nominal parameter values in the computation of $\psi(t)$. The mean value of the diagonal elements in P are then used to form the initial value $P(0)$ in the

actual identification. The initial value of the parameters in θ is set to 20% above the nominal values of the parameters to see how well the algorithm adapts.

5.3 Results

The parameter adaptation for the forgetting factor case is shown in Figure 6, and it can be seen that all parameters converge to their nominal values. The convergence rate is approximately the same for the estimates of J_a and f_m respectively, while it is somewhat higher for the estimate of k_T . In Figure 7 the diagonal elements of P are shown. The big difference in magnitude is caused by the big difference in the magnitude of the elements in $\psi(t)$.

Figure 6: Parameter estimates using forgetting factor.

Figure 7: Diagonal elements of P . Solid: P_{11} . Dashed: P_{22} . Dotted: P_{33} .

An identification experiment is also carried out where the covariance matrix modification is used. The aim in this experiment is only to illustrate that this freedom is available. How to use it in a suitable way is left for further work. For example, assume that it is desirable that the diagonal elements of P are of the same order of magnitude. Due to the difference in magnitude in the elements of $\psi(t)$ it is then necessary to let the elements of Δ be of different magnitude. In the experiments the choice

$$\Delta = \rho \begin{pmatrix} 5 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 500 \end{pmatrix} \quad (32)$$

is used, where $\rho = 10^{-8}$. Figures 8 and 9 show the results from this experiment. Figure 9 shows that the diagonal elements of P now have the same magnitude. The changes in algorithm properties are illustrated in Figure 8. The convergence rate for k_T is lower and the variance is lower than for the forgetting factor case. The variance of the estimate of f_m is much higher due to the increase in the corresponding element in P . The results indicate that it is possible to handle the tracking and variance trade off more or less individually for the different parameters by suitable choices of the elements in Δ .

Figure 8: Parameter estimates using covariance matrix modification.

Figure 9: Diagonal elements of P . Solid: P_{11} . Dashed: P_{22} . Dotted: P_{33} .

In these experiments the excitation could be chosen almost freely. In practice it can not be expected that the input signal is sufficiently exciting all the time during normal robot operation. How to deal with this problem using the regularization procedure presented above is left for further work.

6 Conclusions

Recursive identification of continuous time parameters of a flexible robot arm has been considered. It has been illustrated how physical parameters of continuous time models can be recursively identified using discrete

time data. Some aspects of the choice of method for obtaining tracking abilities of the algorithm have been illustrated.

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