Time-domain Identification of Dynamic Errors-in-variables Systems Using Periodic Excitation Signals

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August 7, 1998

Report no.: LiTH-ISY-R-2044

Submitted to IFAC's World Congress Beijing'99

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Abstract

The use of periodic excitation signals in identification experiments is advocated. With periodic excitation it is possible to separate the driving signals and the disturbances, which for instance implies that the noise properties can be independently estimated. In the paper a non-parametric noise model, estimated directly from the measured data, is used in a compensation strategy applicable to both least squares and total least squares estimation. The resulting least squares and total least squares methods are applicable in the errors-in-variables situation and give consistent estimates regardless of the noise. The feasibility of the idea is illustrated in a simulation study.

Keywords: System identification; Least squares estimation; Errors-in-variables models.

1 Introduction

One of the most important steps in the identification process is the experiment design. This involves, for example, deciding what signals to measure, choosing the sampling interval, and designing the excitation signals. In this paper we will advocate the use of periodic excitation.

Periodic excitation has up to this point mostly been used in frequency domain identification (e.g. [7, 8]), but offer several interesting and useful advantages compared to non-periodic (random) excitation also in time domain identification. The main advantages with periodic excitation in time domain identification are:

- Data reduction. By averaging over M periods the amount of data is reduced M times.
- Improved signal-to-noise ratio. By averaging over M periods the noise variance is lowered by a factor M . This will have important consequences for both the numerical properties of the estimation algorithms and the statistical properties of the estimates.
- Separation of driving signals and noise. With periodic excitation all non-periodic signal variations over the periods will be due to random disturbances, noise. This means, for instance, that we can estimate the noise level and thus compute a priori bounds for the least squares cost function used in the identification.
- Independent estimation of non-parametric noise models. Since we can separate the signals from the noise it is possible to independently estimate the noise properties. Such a noise model can be used as a pre-whitening filter applied before the estimation or as a tool for model validation.

In this paper we will study how to identify dynamic errors-in-variables systems using time-domain data. This is a problem that has received considerable interest in the literature, see, e.g., [1, 2, 11, 14] and the more recent [3, 4, 15]. With periodic excitation a number of possibilities opens up for constructing simple, efficient methods that solves this problem. We will study some of them in this contribution. In particular, compensation methods for least squares and total least squares estimation that can handle also the errors-in-variables problem will be presented. The idea used is similar to the bias-correction technique studied in for instance [12, 14, 15]. Compared to the methods studied in these references, the proposed methods have the advantage of giving consistent estimates regardless of the properties of the noise.

2 Problem Formulation

Consider a linear, time-invariant, discrete-time system

$$y(t) = G(q)u(t) = \sum_{k=0}^{\infty} g_k q^{-k} u(t) = \sum_{k=0}^{\infty} g_k u(t-k) \quad (1)$$

where $u(t) \in \mathbb{R}$ is the input, $y(t) \in \mathbb{R}$ is the output, and q^{-1} is the delay operator ($q^{-k}u(t) = u(t-k)$). We will assume that the order of the system is finite, so that the system can be represented as

$$y(t) = -a_1 y(t-1) - \cdots - a_{n_a} y(t-n_a) + b_0 u(t-n_k) + \cdots + b_{n_b} u(t-n_k-n_b) \quad (2)$$

Here we have explicitly included the possibility of a delay n_k . The transfer operator $G(q)$ in this case becomes

$$G(q) = \frac{q^{-n_k} B(q)}{A(q)} \quad (3)$$

$$B(q) = b_0 + b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b} \quad (4)$$

$$A(q) = 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a} \quad (5)$$

The problem we consider is how to identify $G(q)$ using noisy measurements of $y(t)$ and $u(t)$. Our measured data can thus be described by

$$Z_m^N = \{z_m(1), \dots, z_m(N)\} \quad (6)$$

$$z_m(t) = z(t) + w(t) \quad (7)$$

$$z(t) = \begin{bmatrix} y(t) & u(t) \end{bmatrix}^T \quad (8)$$

$$w(t) = \begin{bmatrix} w_y(t) & w_u(t) \end{bmatrix}^T \quad (9)$$

We will also use the notation $y_m(t) = y(t) + w_y(t)$ and $u_m(t) = u(t) + w_u(t)$. The unknown signals $w_y(t)$ and $w_u(t)$ act as noise sources on the measured output and input, respectively. We will make the following assumptions about the signals $z(t)$ and $w(t)$:

- A1 $u(t)$ is periodic with period P , $P \geq 2\bar{n} + 1$ where $\bar{n}$ is an a priori given upper bound on the system order.
- A2 $u(t)$ is persistently exciting of order $\bar{n}$.
- A3 $z(t)$ and $w(t)$ are jointly quasi-stationary.

A4 $w(t)$ has the property

$$\lim_{M \rightarrow \infty} \frac{1}{M} \sum_{k=0}^{M-1} w(t + kP) = 0 \quad \forall t \quad (10)$$

A5 $z(t)$ and $w(t)$ are uncorrelated.

Assumption A2 is required in order to uniquely identify the system. Assumptions A1 and A4 enable us to use simple averaging to remove the noise. In a stochastic setting we assume A4 to hold with probability 1. Assumption A5 implies that sample means of products of $z(t)$ and $w(t - k)$ tend to zero as the number of samples tends to infinity. In addition to the assumptions listed above, it is also assumed that an integer number of periods has been measured, that is $N = M \cdot P$, $M \geq 1$.

3 Averaging

An important step in the identification is to average the measured data. Define the averaged input and output as

$$\bar{u}(t) = \frac{1}{M} \sum_{k=0}^{M-1} u_m(t + kP), \quad t \in [1, P] \quad (11)$$

$$\bar{y}(t) = \frac{1}{M} \sum_{k=0}^{M-1} y_m(t + kP), \quad t \in [1, P] \quad (12)$$

From assumption A4 it follows that $\bar{u}(t) \rightarrow u(t)$, $t \in [1, P]$ and $\bar{y}(t) \rightarrow y(t)$, $t \in [1, P]$ as M tends to infinity. $\bar{u}(t)$ and $\bar{y}(t)$ are thus consistent estimates of noise free signals $u(t)$ and $y(t)$, respectively. In [6] this is used to derive simple, consistent methods for the identification of errors-in-variables systems. The idea in [6] was that as M tends to infinity, the noise will average out and we are effectively identifying a noise-free system. In this paper we will not generally assume that the number of periods tends to infinity, which makes the problem significantly harder.

4 Estimating the Noise Statistics

Let $\bar{z}(t) = \begin{bmatrix} \bar{y}(t) & \bar{u}(t) \end{bmatrix}$. By periodically continuing $\bar{z}(t)$ outside $t = [1, P]$ we can estimate the noise $w(t)$ as

$$\hat{w}(t) = z_m(t) - \bar{z}(t), \quad t \in [1, N] \quad (13)$$

A consistent estimate of the covariance function

$$R_{ww}(k) = Ew(t)w^T(t+k) \quad (14)$$

can now be computed as

$$\hat{R}_{ww}(k) = \frac{1}{(M-1)P} \sum_{t=1}^{MP} \hat{w}(t)\hat{w}^T(t+k) \quad (15)$$

where the convention is that all signals outside the interval $t = [1, MP]$ are replaced by 0. In practice for large data sets, the covariance function should be computed using FFT, see [10]. It is important to note that we have used P degrees of freedom for estimating the mean, so the proper normalization to get an unbiased estimate is $MP - P = (M-1)P$. How many periods do we need then? The rather precise answer provided in [9] is $M \geq 4$. The asymptotic properties $N = MP \rightarrow \infty$ of the estimate are then independent of how the excitation is divided into M and P .

An unbiased estimate of the spectrum of $w(t)$ is obtained by the periodogram

$$\hat{\Phi}_w(\omega) = \sum_{k=-MP+1}^{MP-1} \hat{R}_{ww}(k)e^{-i\omega k} \quad (16)$$

This can be used for pre-whitening of $w(t)$ prior to the estimation. It turns out that the poor variance properties of (16) does not diminish its usefulness for pre-whitening. An example of this will be shown in Section 11. We also mention that $\hat{\Phi}_w(\omega)$ can be estimated very efficiently using FFT directly from the original data.

5 Least Squares Estimation Using Periodic Data

Consider the linear regression model

$$\hat{y}_m(t, \theta) = \varphi^T(t)\theta \quad (17)$$

$$\varphi(t) = [-y_m(t-1), \dots, -y_m(t-n_a), \\ u_m(t-n_k), \dots, u_m(t-n_k-n_b)]^T \quad (18)$$

$$\theta = [a_1, \dots, a_{n_a}, b_0, \dots, b_{n_b}]^T \quad (19)$$

The least squares (LS) estimate of θ using N data samples can be written

$$\hat{\theta}_N = R_N^{-1} f_N \quad (20)$$

$$R_N = \frac{1}{N} \sum_{t=1}^N \varphi(t) \varphi^T(t) \quad (21)$$

$$f_N = \frac{1}{N} \sum_{t=1}^N \varphi(t) y(t) \quad (22)$$

Introduce the notation

$$\begin{aligned} \varphi_z(t) = & [-y(t-1), \dots, -y(t-n_a), \\ & u(t-n_k), \dots, u(t-n_k-n_b)]^T \end{aligned} \quad (23)$$

$$\begin{aligned} \varphi_w(t) = & [-w_y(t-1), \dots, -w_y(t-n_a), \\ & w_u(t-n_k), \dots, w_u(t-n_k-n_b)]^T \end{aligned} \quad (24)$$

Since $z(t)$ and $w(t)$ are uncorrelated we have that

$$\lim_{N \rightarrow \infty} R_N = R = R_z + R_w \quad (25)$$

$$\lim_{N \rightarrow \infty} f_N = f = f_z + f_w \quad (26)$$

$$R_z = E\varphi_z(t)\varphi_z^T(t), \quad R_w = E\varphi_w(t)\varphi_w^T(t) \quad (27)$$

$$f_z = E\varphi_z(t)y(t), \quad f_w = E\varphi_w(t)w_y(t) \quad (28)$$

If, indeed

$$y_m(t) = \varphi^T(t)\theta + e(t) \quad (29)$$

where $e(t)$ is white noise with variance λ_0 , then the least squares estimate is consistent with asymptotic covariance matrix

$$N \text{Cov } \hat{\theta} \approx \lambda_0 R^{-1} \quad (30)$$

However, with colored noise and/or noisy measurements of the input this is no longer true and the least squares estimate will be biased.

Let $\bar{R}_P$ and $\bar{f}_P$ be defined similar to R_P and f_P , respectively, except that averaged data is used. We then have that

$$\lim_{P \rightarrow \infty} \bar{R}_P = \bar{R} = R_z + \frac{1}{M} R_w \quad (31)$$

$$\lim_{P \rightarrow \infty} \bar{f}_P = \bar{f} = f_z + \frac{1}{M} f_w \quad (32)$$

The M normalization is due to the averaging which decreases the noise variance with a factor of M . The least squares estimate using averaged data will still be unbiased if the true system is given by (29), but the asymptotic covariance matrix changes to

$$MP \text{Cov} \hat{\theta}_P \approx M \frac{\lambda_0}{M} \bar{R}^{-1} = \lambda_0 \bar{R}^{-1} \quad (33)$$

The scaling factor is thus the same, but R is replaced by $\bar{R}$, and since $R \geq \bar{R}$ this means that the asymptotic covariance increases with averaged data.

6 Improving the Accuracy

If we have periodic excitation and if (29) holds then we can recover the original information in R_w and f_w using the non-parametric noise model (15). The idea is to construct non-parametric estimates $\hat{R}_w^{np}$ and $\hat{f}_w^{np}$ of R_w and f_w , respectively, from $\hat{R}_{ww}(k)$, $k = 0, 1, \dots$ and compensate for the missing terms in $\bar{R}$ and $\bar{f}$. As pointed out before, these estimates use $(M - 1)P$ degrees of freedom. Note also that $\bar{R}_P$ and $\bar{f}_P$ already contain estimates of R_w and f_w , respectively. These have P degrees of freedom (averages over P samples), and are functions of the sample mean $\bar{w}(t)$. This is important since the non-parametric estimates are based on the second-order properties of $w(t)$, and thus these two estimates of R_w and f_w are uncorrelated, and even independent if Gaussian noise is assumed. This implies that we can compensate the least squares quantities obtained from averaged data

$$R_P^c = \bar{R}_P + \hat{R}_w^{np} \quad (34)$$

$$f_P^c = \bar{f}_P + \hat{f}_w^{np} \quad (35)$$

and recover all $MP = (M - 1)P + P$ degrees of freedom. This is further discussed in [5].

7 Consistent Least Squares Estimation of Errors-in-Variables Systems

A similar idea can be used to remove the bias in the least squares estimate due to (colored) noise $w(t)$ acting on the input and the output: we simply have to subtract away the terms in $\bar{R}_P$ and $\bar{f}_P$ that are due to the noise $w(t)$ using the non-parametric

estimates $\hat{R}_w^{np}$ and $\hat{f}_w^{np}$. By equating the degrees of freedom it can be shown that

$$\hat{R}_z = \bar{R}_P - \frac{1}{M-1} \hat{R}_w^{np} \quad (36)$$

$$\hat{f}_z = \bar{f}_P - \frac{1}{M-1} \hat{f}_w^{np} \quad (37)$$

are consistent estimates of R_z and f_z , respectively. We have thus removed all effect of the noise $w(t)$ in $\bar{R}_P$ and $\bar{f}_P$ by a simple subtraction operation and the resulting least squares estimate

$$\hat{\theta}_P = \hat{R}_z^{-1} \hat{f}_z \quad (38)$$

will be consistent regardless of $w(t)$. The method (36)-(38) will be referred to as the compensated least squares (CLS) method. Due to its simplicity and general applicability, this method is a very interesting alternative to other methods that are applicable in the errors-in-variables situation. Note that with the CLS method no iterations are required to find the estimate – a clear advantage compared to most other errors-in-variables methods which frequently use singular value decompositions (SVDs) and to most other time-domain identification schemes which often use Gauss-Newton type search algorithms for finding the estimates.

8 The Total Least Squares Solution

For simplicity, assume that $n_a = n_b = n$ and $n_k = 0$. The relation (2), $t \in [1, N]$, can in this case be restated as

$$T^N \theta' = 0 \quad (39)$$

where

$$T^N = \begin{bmatrix} -Y^N & U^N \end{bmatrix} \quad (40)$$

$$Y^N = \begin{bmatrix} y(n+1) & \dots & y(1) \\ \vdots & \ddots & \vdots \\ y(N) & \dots & y(N-n) \end{bmatrix} \quad (41)$$

$$U^N = \begin{bmatrix} u(n+1) & \dots & u(1) \\ \vdots & \ddots & \vdots \\ u(N) & \dots & u(N-n) \end{bmatrix} \quad (42)$$

$$\theta' = [1, a_1, \dots, a_n, b_0, \dots, b_n]^T \quad (43)$$

The non-trivial right null space of the data matrix T^N describes the system.

With noisy measurements $z_m(t)$ of $z(t)$, a total least squares (TLS) solution is natural to apply. Denote the noisy variant of T^N by T_m^N . The total least squares solution $\hat{\theta}'_{TLS}$ can be stated as

$$T_{TLS}^N = \arg \min_T \|T_m^N - T\|_F^2 \quad (44)$$

subject to

$$T_{TLS}^N \hat{\theta}'_{TLS} = 0 \quad (45)$$

The solution is easily calculated by a singular value decomposition of the data matrix T_m^N . Introduce the error

$$W^N = T_m^N - T^N \quad (46)$$

With periodic data, averaged over M periods ($N = MP$), we have that

$$\|W^P\|_F^2 \rightarrow 0, \quad \text{as } M \rightarrow \infty \quad (47)$$

using Assumption A4. Under these conditions the TLS estimate is a consistent estimate of the system.

Let R'_w be the covariance matrix of

$$\varphi'_w(t) = [-w_y(t), \dots, -w_y(t-n), w_u(t), \dots, w_u(t-n)]^T \quad (48)$$

(cf. (24)). To improve the efficiency of the total least squares estimator one can use R'_w which lead to the generalized total least squares (GTLS) solution [13]. The GTLS solution $\hat{\theta}'_{GTLS}$ is

$$T_{GTLS}^N = \arg \min_T \|(T_m^N - T)(R'_w)^{-1/2}\|_F^2 \quad (49)$$

subject to

$$T_{GTLS}^N \hat{\theta}'_{GTLS} = 0 \quad (50)$$

To understand the effect of the scaling $(R'_w)^{-1/2}$ it is instructive to study the product of T_m^N and its transpose. Introduce the notation

$$R'_N = \frac{1}{N} (T_m^N)^T T_m^N \quad (51)$$

$$R'_z = E \varphi'_z(t) (\varphi'_z(t))^T \quad (52)$$

$$\varphi'_z(t) = [-y(t), \dots, -y(t-n), u(t), \dots, u(t-n)]^T \quad (53)$$

The solution to the TLS (GTLS) problem is given by the right null space of T_m^N or alternatively by the null space of R'_N . Using Assumption A5 we see that

$$R'_N \rightarrow R'_z + R'_w, \quad \text{as } N \rightarrow \infty \quad (54)$$

If we include the scaling $(R'_w)^{-1/2}$ the covariance matrix R'_w is replaced by the identity matrix, which does not affect the directions of the singular vectors of R'_z . This means that the GTLS solution can be computed by finding the singular vector corresponding to the smallest singular value of the matrix $(R'_w)^{-T/2} R'_N (R'_w)^{-1/2}$. If the true R'_w is known, or if a consistent estimate of it can be computed, the GTLS estimator is consistent even if the variance of the noise $w(t)$ does not tend to zero.

The point is now that with periodic excitation we can obtain a consistent estimate of R'_w very easily using the non-parametric noise model (15). In the rest of the paper we shall refer to this variant of the general algorithm as the GTLS algorithm.

9 A Compensation Method for Total Least Squares Estimation

Let $\bar{R}'_P$ be defined as R'_P except that periodic data is used. With periodic data, averaged over M periods, we have that

$$\bar{R}'_P \rightarrow R'_z + \frac{1}{M} R'_w, \quad \text{as } N \rightarrow \infty \quad (55)$$

Let $\hat{R}_w^{mp}$ be a non-parametric estimate of R'_w obtained using (15). Similar arguments as in Section 7 will show that

$$\bar{R}'_P - \frac{1}{M-1} \hat{R}_w^{mp} \quad (56)$$

is a consistent estimate of R'_z . This holds regardless of the noise, which implies that the total least squares estimator with the compensation (56) gives consistent estimates regardless of the noise even though the number of periods, M , does not tend to infinity. This method will be referred to as the compensated total least squares (CTLS) estimator.

10 Pre-whitening of the Noise

As mentioned in Section 4 the spectrum of the noise signal $w(t)$ can be estimated very efficiently using FFT when periodic data is used. Similarly we can pre-filter the data

very easily in the frequency domain simply by multiplying the Fourier transformed data sequences and the inverse of a square-root factor of the estimated noise spectrum. The corresponding time-domain signals are then obtained through IFFT. To preserve the relation between $u(t)$ and $y(t)$ it is important that $u_m(t)$ and $y_m(t)$ are pre-filtered using the same filter. We thus have the two choices: either we compute the pre-filter that will whiten the noise in $y_m(t)$ ($w_y(t)$), or compute the pre-filter that will whiten the noise in $u_m(t)$ ($w_u(t)$). In many cases it is most natural to whiten the noise on the output, but in other cases the choice is more arbitrary. The former would for instance be the case if we know that the measurement noise on $y(t)$ and $u(t)$ is negligible, so that $w(t)$ basically is due to process noise acting on the output, or if the system operates in closed-loop and the measurement noise is negligible, which typically leads to similar spectra of $w_u(t)$ and $w_y(t)$. If the choice is less obvious, one can benefit from whitening the noise which has the highest variance. This will of course distort the spectrum of the other noise signal, but since the variance is smaller the net effect will be positive.

Pre-whitening of the noise can also be used to derive simplified estimation algorithms. Consider for instance the least squares estimator (20)-(22). If $w_y(t)$ is white noise and if $w_y(t)$ is uncorrelated with $w_u(t - n_k - k)$, $k \geq 0$, then f_w defined in (28) will be zero. This means that the CLS algorithm (36)-(38) can be simplified since the second compensation (37) may be skipped.

11 Example

Consider the system

$$y(t) = 1.5y(t-1) - 0.7y(t-2) + u(t-1) + 0.5u(t-2) + v(t) \quad (57)$$

$$v(t) = \frac{1}{1 + 0.9q^{-1}}e(t) \quad (58)$$

where $e(t)$ is white Gaussian noise with variance σ_e^2 . Apart from the noise $v(t)$, (57) is of the form (2) with $a_1 = -1.5$, $a_2 = 0.7$, $b_0 = 1$, $b_1 = 0.5$, $n_a = 2$, $n_b = 2$, and $n_k = 1$. This system was simulated using the control law

$$u(t) = r(t) - 0.25y(t) \quad (59)$$

where $r(t)$ is a periodic reference signal with period P . In the simulations $r(t)$ was taken as a unit binary random signal. We also added colored measurement noise on both $y(t)$ and $u(t)$. These noise sources were independent but with equal spectra. The measurement noises were realized as Gaussian, white noise sequences filtered

through a second-order, high-pass Butterworth filter with cut-off frequency 0.3. The variance of the white noise was σ_n^2 .

A number of identification approaches were considered:

1. **LS, LS-A, LS-AF** Least squares estimation using raw data, averaged data, and averaged and pre-filtered data, respectively.
2. **CLS, CLS-F** Least squares estimation with compensation, cf. Eqs. (36)-(38), using averaged data and averaged and pre-filtered data, respectively.
3. **TLS, TLS-A, TLS-AF** Total least squares estimation using raw data, averaged data, and averaged and pre-filtered data, respectively.
4. **GTLS, GTLS-F** Generalized total least squares estimation with estimated noise statistics using averaged data and averaged and pre-filtered data, respectively.
5. **CTLS, CTLS-F** Total least squares estimation with compensation, cf. Eq. (56), using averaged data and averaged and pre-filtered data, respectively.

With colored noise on both the input and the output the LS and TLS method will be biased. LS-A, LS-AF, TLS-A, and TLS-AF will be consistent as the number of periods tends to infinity, otherwise these methods will also give biased results. The other methods give consistent estimates regardless of the noise and the number of periods used (as long as $M \geq 4$).

In the simulation we used $P = 64$, $M = 32$, $\sigma_e^2 = 0.09$, $\sigma_n^2 = 0.01$. In the pre-filtering of the data using a non-parametric noise model we chose to whiten the data on the output. The results of a Monte Carlo simulation consisting of 16 different runs are summarized in Table 1. The numbers shown are the means and standard deviations of the estimated parameter values for each method.

Studying Table 1, we can first note that the LS and TLS methods perform very badly, while the results are quite good when averaged data is used. Focusing on the proposed methods, CLS and CTLS, we see that these compare well with the other methods, both with averaged data and with averaged and pre-filtered data. In this example, the improvement in the accuracy with pre-filtered data is substantial despite the poor variance properties of the periodogram (16). This holds for all methods, as can be seen from Table 1.

Table 1: Summary of identification results.

Parameter	a_1	a_2	b_0	b_1
True value	-1.5000	0.7000	1.0000	0.5000
LS	-1.0799	0.2033	0.9969	0.9158
	± 0.1340	± 0.1079	± 0.1037	± 0.0634
TLS	-0.9891	-0.5340	1.7171	3.2591
	± 1.6012	± 1.5505	± 3.0464	± 4.6528
LS-A	-1.4798	0.6766	0.9963	0.5240
	± 0.0251	± 0.0247	± 0.0565	± 0.0637
CLS	-1.4991	0.6992	0.9970	0.5058
	± 0.0246	± 0.0238	± 0.0586	± 0.0647
TLS-A	-1.5026	0.6874	1.0412	0.5245
	± 0.0259	± 0.0244	± 0.0656	± 0.0670
GTLS	-1.5095	0.7087	1.0209	0.4788
	± 0.0266	± 0.0252	± 0.0602	± 0.0666
CTLS	-1.5102	0.7131	1.0170	0.4698
	± 0.0255	± 0.0235	± 0.0636	± 0.0634
LS-AF	-1.4992	0.6991	0.9991	0.4994
	± 0.0030	± 0.0028	± 0.0116	± 0.0126
CLS-F	-1.5006	0.7006	0.9988	0.4984
	± 0.0032	± 0.0029	± 0.0118	± 0.0127
TLS-AF	-1.5002	0.7001	1.0013	0.4981
	± 0.0032	± 0.0029	± 0.0118	± 0.0127
GTLS-F	-1.5009	0.7008	1.0013	0.4957
	± 0.0033	± 0.0030	± 0.0118	± 0.0128
CTLS-F	-1.5013	0.7012	1.0013	0.4944
	± 0.0032	± 0.0029	± 0.0118	± 0.0126

12 Conclusions

We have studied the problem of identifying dynamic errors-in-variables systems using periodic excitation signals. Two new algorithms, the CLS and the CTLS algorithms, have been presented that gives consistent estimates regardless of the noise on the input and output. With the CLS algorithm the estimate is found without iterations by solving a standard least squares problem, which can be done very efficiently using FFT. This method can therefore be an interesting alternative to existing time- and frequency-domain methods for this problem. The CTLS algorithm is an alternative to the GTLS algorithm where the noise statistics are estimated from data. The performance of the CTLS method and the GTLS method is similar.

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