Constrained Control Package
(version 1.1b1, April 2001)
– A Guided Tour –

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Abstract

This software package provides algorithms for analysis and design of saturation avoiding control systems. The design of linear controllers for linear, possibly multivariable or uncertain, plants is considered. To avoid a conservative design, the external signals are hard bounded in amplitude and rate. During the example sessions, only reference signals are considered as external signals, but this can be extended quite easily.

1 Introduction & Disclaimer

This manual does not intend to describe the ideas or the theoretical background behind the implemented algorithms – therefore, reference to appropriate literature is given. Moreover, only the basic usage of all functions is described, for more options and additional parameters, see the online help of each function. This manual only gives an idea of how to use all functions “in a row”.

When using this package, a citation like [9] is welcome. For recent developments of this package, check its homepage at http://www.control.isy.liu.se/~wolle/cc/. Any comments, bugs, suggestions are welcome and may be directed to wolle@isy.liu.se.

2 Installation

1. Unzip the file cc-YYMM.tgz. It extracts in a directory cc-YYMM. It was packed using gnu-tar:
gtar -czf cc-YYMM.tgz cc-YYMM. To unpack use for example: gtar -xzf cc-YYMM.tgz

2. The contents of the directory should be the following:

3. Add these directories to your MATLAB-path using the addpath-command, or add it directly in your /matlab/startup.m file.

4. The main part is coded in Matlab, but the package also contains some c-files. Executable MEX-files are present for the following environments: Solaris 5.6, Linux 2.x. The MEX-files will

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work under a not too old OS version and any Matlab 5.x version. If not or if your OS type is not present, you have to create the MEX-files: simply change to the directory hidden/ and type

mim.isy.liu.se{wolle}: mex ccBalken.c
mim.isy.liu.se{wolle}: mex ccEvalInvLaplace.c
mim.isy.liu.se{wolle}: mex ccBuschel.c

For troubleshooting see the Matlab User’s Guide. For detailed information on the precompiled MEX-files, see end of file.

5. Main developing and testing platform is Solaris 5.7 and Matlab 6.0.

3 The testdata set

The directory doc contains, additionally to this documentation, a dataset that contains some simple transfer functions and bounds to test the basis routines and the data for the aircraft example in [1], used later.

>> load testdata
>> Your variables are:
G1   U1   Udot1   aircraft   weight_s
G2   U2   Udot2   w_c        
G3   U3   Udot3   weight

4 Worst Case Output of Systems

There are four methods implemented in order to calculate the maximum possible output amplitude of an LTI system, with input bounded in amplitude and rate. They are described in [5]. The main routines are, however, ccmaxout (using Linear Programming), which handles SISO and MIMO systems in discrete and continuous time, all with the same notation. For the SISO case, there are alternatives implemented the “constructive” approach to the worst case input signal, described in [3, 2] and implemented as ccmaxoutC and ccmaxoutD respectively. These two algorithms are added to the package for “historical” reasons, and if there is any difference in the result, we believe that the Linear Programming in ccmaxout gives the more accurate result. The counterpart for uncertain SISO systems (in both discrete and continuous time) is ccsetout.

We start up with SISO, continuous time systems. We have two options:

>> [ymax,umax,time]=ccmaxout(G3,U3,Udot3,1); % using LP techniques
>> [ymaxC,tptsC]=ccmaxoutC(G3,U3,Udot3,1); % using Reichel’s algorithm

The results for the testdata set are quite the same:

<table>
<thead>
<tr>
<th>Case</th>
<th>ymax</th>
<th>ymaxC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7370</td>
<td>0.7610</td>
</tr>
<tr>
<td>2</td>
<td>0.0302</td>
<td>0.0596</td>
</tr>
<tr>
<td>3</td>
<td>0.1278</td>
<td>0.1272</td>
</tr>
</tbody>
</table>

We look at SISO systems in discrete time and observe, that the notation for ccmaxout does not change at all:
>> G1d= c2d(G1,0.1); % etc  
>> [ymax,umax,time]=ccmaxout(G1d, U1, Udot1, 1); % using LP techniques  
>> [ymaxD,tptsD]=ccmaxoutD(G1d, U1, Udot1, 1); % using Peng’s algorithm

The results for the testdata are:

**case 1:**  
\[
\begin{align*}
\text{ymax} & = 0.7787; \\
\text{ymaxD} & = 0.7858
\end{align*}
\]

**case 2:**  
\[
\begin{align*}
\text{ymax} & = 0.0677; \\
\text{ymaxD} & = 0.0714
\end{align*}
\]

**case 3:**  
\[
\begin{align*}
\text{ymax} & = 0.1278; \\
\text{ymaxD} & = 0.1237
\end{align*}
\]

**MIMO systems** can be handled by ccmaxout in any time domain. We create system with 2 input and 3 outputs:

>> Gm=tf([[G1,G2;G3,G2;G1*G1, G3]])
>> ymaxM=ccmaxout(Gm, U1, Udot1, 1)
>> ymaxM = [1.1274; 1.7116; 1.7475]

A similar result can be obtained, when looping through all entries with ccmaxoutC (or ccmaxoutD).

**Uncertain systems** have to be parameterised as \( G + [B_1, \ldots, B_n] \cdot \theta \), where \( G \) is a SISO system, \( B = [B_1, \ldots, B_n] \) is a set of basis functions and the parameter vector \( \theta \) is living in a box. The problem is then solved by Quadratic Programming invoking ccsetout. The set of basis functions \( B \) can be any LTI object with \( n \) inputs and 1 output or, an I4CBASIS object, as used in the i4c package [6] (when choosing this option, you’ll then have to install this software).

>> BasDat.pole=1;BasDat.order=3;BasDat.Ts=0;BasDat.type='lag';
>> thbox=[0.9 1.1];
>> % using the I4CBASIS object BasDat:
>> [ymaxU,umax,time,thetamaxU]=ccsetout(G1,BasDat,thbox,U1,Udot1,1);
>> % which is equivalent to
>> Bas=basis(BasDat); % Bas is a ‘normal’ ss-object now
>> minfo(Bas)
MATLAB ss object: 3 states 1 outputs 3 inputs
>> [ymaxU,umax,time,thetamaxU]=ccsetout(G1,Bas,thbox,U1,Udot1,1);

The result, independent which description for the basis we pick, is \( \text{ymaxU} = 2.6793; \text{thetamaxU} = [1.1000; 1.1000; 1.1000] \).

### 5 Controller Design via Loop Shaping

As a first design technique, Loop Shaping [1] for SISO or MIMO systems is considered. By this, we mean clever adjustment of the design weight during “classical” Loop Shaping such that the hard bound on each control channel are met. We start with a simpler example, controller design in the SISO case, which is also outlined in [4, Sec.6.2].

>> % the plant & its input constraints
>> plant= tf([1 -1],[1 2 0]);
>> w1=ss(0.0817);
>> R=7; Rdot=2;
>> pf=1; % performance factor for the Loop Shaping procedure
>> [u_max,e_max,K]=ccls(plant,R,Rdot,w1,[],pf,[1e-5 1e5]);

3
As a result, we obtain: \( u_{\text{max}} = 0.9866 \), \( e_{\text{max}} = 0.6860 \) and \( K(s) = (0.086661s + 0.17322)/(s + 2.249) \) as controller via \( \text{tf}(K) \). A dynamical weight can be considered as well, of course:

\[
\text{Transfer function:} \\
\frac{0.0683 s^3 + 0.1417 s^2 + 0.01027 s + 0.0001235}{s^3 + 2.18 s^2 + 0.09815 s + 9.597e-05}
\]

Here, we end up with a maximum control signal \( u_{\text{max}} = 0.9213 \) and a stability margin \( e_{\text{max}} = 0.4902 \).

For adjustment of the design weights, \textit{magshape} (\( \mu \) Toolbox) for instance is an appropriate tool and \textit{sys2lti} (i4c-package) is a nice tool to convert the mutools SYSTEM variable to an LTI object.

For a controller design in the MIMO case, see [4, Sec.6.4] or [7], an aircraft model and some weights is already contained in \textit{testdata}:

\[
\text{MATLAB ss object:} \quad 5 \text{ states} \quad 3 \text{ outputs} \quad 3 \text{ inputs}
\]

As pointed out, there are different ways to produce the weighting functions. Define as transfer functions:

\[
\text{MATLAB ss object:} \quad 3 \text{ states} \quad 3 \text{ outputs} \quad 3 \text{ inputs}
\]

Having these weights at hand, the actual controller design runs as in the SISO case (from the syntax point-of-view):

\[
\text{MATLAB ss object:} \quad 3 \text{ states} \quad 3 \text{ outputs} \quad 3 \text{ inputs}
\]

The results are somewhat different to those given in [4, Sec.6.4], as we use another implementation for determining the maximum output amplitude \( u_{\text{max}} \) here:

\[
u_{\text{max}} = [26.5273; 10.4946; 61.4195]; e_{\text{max}} = 0.3786; \\
u_{\text{max},s} = [20.3149; 8.1607; 49.1611]; e_{\text{max},s} = 0.3525.
\]

6 Design of Optimal Controllers

In this section, we would like to discuss a method for designing optimal controllers. Optimality refers to the fact that we achieve the smallest possible worst case error (i.e. difference between reference signal and plant output) during runtime. Moreover, the presented framework allows to assess feasibility of the constraint control problem. For details, we refer to [8]. This approach relies upon parameterisation of all stabilising controllers via Youla parameterisation. Suppose a plant to be controlled, along with bounds on amplitude and rate of the reference signal:
>> R=1; Rdot=0.8;
>> Gnom=tf([1 -1],[1 -2 0]);
>> Qorder=6;   % define order of the Youla parameter
>> Alpha=[0:0.1:1];% grid on alpha
>> [Result]=optidesign (Gnom,R,Rdot,Qorder,Alpha);

The structure Result then contains the solution of the Pareto-optimal problem on the grid alpha: Youla parameter, controller and the values for maximum control amplitude and maximum error amplitude. Solving this problem involves non-linear optimisation and will take some time (in this example about 30 minutes per grid point in order to arrive at a reasonable stage).

References


A  Compiler options

A.1 Solaris: *.mexsol

------------------------------------------------------------------
Matlab Version: 5.3.0.10183 (R11)
OS Version: SunOS 5.6 Generic_105181-13 sun4u sparc SUNW,Ultra-1
Host: mim.isy.liu.se

5
Compiled with mex-options:

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<th>Option</th>
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<tr>
<td>CC</td>
<td>cc</td>
</tr>
<tr>
<td>CC flags:</td>
<td>-dalign -KPIC</td>
</tr>
<tr>
<td>CFLAGS</td>
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<td>f77</td>
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<tr>
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<tr>
<td>LD</td>
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<td>LDFLAGS</td>
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<td>arguments</td>
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</table>

A.2 Linux: *.mexlx

Matlab Version: 5.2 (R10)
OS Version: Linux 2.0.27 #1 Sat Dec 21 23:44:11 EST 1996 i586
Host: magnani.upb.de
Compiled with mex-options:

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<td>CC</td>
<td>gcc</td>
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<td>CFLAGS</td>
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<tr>
<td>FC</td>
<td>f2c</td>
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<td>FC flags:</td>
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<tr>
<td>arguments</td>
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</tr>
<tr>
<td>LD</td>
<td>gcc</td>
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<tr>
<td>Link flags:</td>
<td></td>
</tr>
<tr>
<td>LDFLAGS</td>
<td>-shared -rdynamic</td>
</tr>
</tbody>
</table>

6
LDDEBUGFLAGS =
LDOPTIMFLAGS =
arguments =

---------------------------------------------------------------