Backstepping Designs for Aircraft Control – What is there to gain?

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Abstract

Aircraft flight control design is traditionally based on linear control theory, due to the existing wealth of tools for linear design and analysis. However, in order to achieve tactical advantages, modern fighter aircraft strive towards performing maneuvers outside the region where the dynamics of flight are linear, and the need for nonlinear tools arises. In this paper, backstepping is proposed as a possible framework for nonlinear flight control design. Its capabilities of handling five major issues – stability, performance, robustness, saturation, and disturbance attenuation – are investigated.

Keywords: Aircraft flight control, nonlinear control, backstepping

1. Introduction

In recent papers by the author [2, 4], backstepping has been used to design flight control laws for various control objectives. In this paper, we investigate the practical consequences of using these control laws, and outline which potential benefits they offer compared to traditional designs.

The interplay between automatic control and manned flight goes back a long time, see [9] for a historic overview. At many occasions their paths have crossed, and progress in one field has provided stimuli to the other. Today, automatic flight control systems have become a must to aid the pilot in controlling advanced fighter aircraft like JAS 39 Gripen. Due to the spectacular 1993 mishap, when a Gripen aircraft crashed over central Stockholm during the Stockholm Water Festival, it is a widely known fact, even to people outside the automatic control community, that the aircraft is designed to be unstable in certain modes. Such a design is motivated by the fact that it enables faster maneuvering. However, it also emphasizes the need for reliable control systems, stabilizing the aircraft and providing the pilot with the desired responses to his joystick inputs.

Thus, stability and performance are two main concerns when designing aircraft control systems. Another important issue is robustness, the ability to maintain stability despite an imperfect mathematical description the aircraft dynamics. Also, during a sharp turn the aircraft may not be able to produce the control surface deflections specified by the control system. This saturation obviously impairs the performance, and may even cause stability problems, and therefore calls for mathematical analysis. A final important flight controller property is the ability to attenuate external disturbances like wind gusts.

The remainder of the paper is organized as follows. Section 2 contains a brief aircraft primer and introduces the dynamics of the aircraft to be controlled. Two existing design methods are reviewed in Section 3 along with their pros and cons. The main contributions of the paper can be found in Section 4, where a backstepping control design is outlined and evaluated with respect to the properties listed above. Finally, Section 5 contains some concluding remarks.

2. Aircraft model

We will confine our discussion to controlling the longitudinal mode of a fighter aircraft, see Figure 1. For regular maneuvering, the variable to be controlled is either the pitch rate, \( q \), or the normal acceleration, \( n_z \). \( q = \dot{\theta} \) is the angular velocity of the aircraft. \( n_z \), also known as the load factor, measures the acceleration experienced by the pilot directed along his spine (“the number of \( g \)'s”), and is expressed as a multiple of the gravitational acceleration, \( g \). \( n_z \) is closely coupled to the angle of attack, \( \alpha \), which appears nicely in the longitudinal equations of motion. This motivates our choice of \( \alpha \) as the controlled variable.
Of course, speed is also a variable of interest to control. However, in this contribution we do not address this problem but simply assume $V$ to be constant (or slowly varying).

Depending on the aircraft configuration, there are different ways to control the aircraft. The elevators are the primary sources of control, often supplemented with a pair of nose wings that can be rotated. Recently, the interest in high angle of attack flight has led to the invention of thrust vectored control (TVC). Deflectable vanes are then mounted at the engine exhaust so that the engine thrust can be directed to produce a force in some desired direction. In our application, it is the net moment produced w.r.t. the aircraft center of gravity that is essential and not by which means it is produced. We will therefore lump all the aforementioned control inputs together, and denote them by $\delta$.

The main aircraft dynamics\(^\text{1}\) are described by the following differential equations:

\begin{align*}
\dot{\alpha} &= -\frac{L(\alpha)}{mV} + \frac{g}{V} + q \\
\dot{q} &= \frac{M(\alpha, q, \delta)}{I}
\end{align*}

Here, $\alpha$ = angle of attack, $L$ = aerodynamical lift force (the force opposing gravity, enabling aircraft to fly at all), $m$ = aircraft mass, $V$ = aircraft speed, $q$ = angular velocity of the aircraft body, $M$ = net moment acting on the aircraft w.r.t. its center of gravity, and $I$ = aircraft moment of inertia.

$L$ and the part of $M$ that spurs from aerodynamics are nonlinear in their arguments and both are proportional to the aerodynamic pressure,

$$\dot{q} = \frac{1}{2} \rho(h)V^2$$

where $\rho$ = air density is a function of $h$ = altitude. See Figure 2 for an illustration of the lift force.

For simplicity, the thrust force and the dependence of the gravitational force on the aircraft orientation have been neglected.

3. Angle of attack control of today

Given the equations of motion (1), how do we select $\delta$ to bring the aircraft from its current state, $\alpha$, to the state commanded by the pilot, $\alpha_{\text{ref}}$?

Let us first review the prevailing linear technique, upon which the control laws of JAS 39 Gripen rely, and then look at the nonlinear technique which has been appointed as its successor [1].

3.1. Gain-scheduling

Gain-scheduling is based on the divide-and-conquer strategy that is common to many engineering disciplines. Its application to aircraft control design can be outlined as follows:

1. Divide the flight envelope, given by the part of the altitude-speed space one wants to conquer, into smaller regions termed flight cases. These are selected such that within each region, the aircraft dynamics (1) vary insignificantly with $V$ and $h$.

2. For each region, linearize (1) at the steady state that corresponds to level flight given by $\alpha = \alpha_0$, $q = 0$, $\delta = \delta_0$. This yields a system of the form

$$\frac{d}{dt} \begin{bmatrix} \alpha - \alpha_0 \\ q \end{bmatrix} = A \begin{bmatrix} \alpha - \alpha_0 \\ q \end{bmatrix} + B(\delta - \delta_0)$$

where $A$ and $B$ are constant matrices.
3. For each linearized system (2), use linear control design methods to determine a stabilizing controller
\[ \delta = \delta_0 - K \left[ \frac{\alpha - \alpha_0}{q} \right] + k_0(\alpha_{\text{ref}} - \alpha_0) + k_0(\alpha_{\text{ref}} - \alpha_0) \]
that brings \( \alpha \) to \( \alpha_{\text{ref}} \) in the desired fashion.

4. Use a gain-scheduler to blend the control laws for the different regions together so that the transitions between different regions are smooth and transparent to the pilot.

Let us review the pros and cons of gain-scheduling.

+ Arriving at the linear model (2) allows the control designer to utilize all the classical tools for control design and robustness and disturbance analysis.

− The complexity of the divide-and-conquer approach is very high since for each region, a controller must be designed. The number of regions may be over 50.

− Only the nonlinear system behavior in speed and altitude are considered. Stability is therefore only guaranteed for low angles of attack and low angular rates.

### 3.2. Dynamic inversion (feedback linearization)

The idea behind gain-scheduling was to provide the pilot with the same aircraft response irrespectively of the aircraft speed and altitude. This philosophy is even more pronounced in dynamic inversion, which is the term used in the aircraft community for what is known as feedback linearization [8] in control theory. As the name implies, the natural aircraft dynamics are “inverted” and replaced by the desired linear ones through the wonders of feedback.

A dynamic inversion design for (1) goes as follows:

1. Introduce
   \[ \dot{\alpha} = -\frac{L(\alpha)}{mV} + \frac{q}{V} + q \equiv z \quad (3) \]
   to replace \( q \) in the system description.

2. Compute \( \dot{z} \).
   \[ \dot{z} = -\frac{1}{mV} \frac{dL(\alpha)}{d\alpha} z + \frac{M(\alpha, q, \delta)}{I} \equiv u \quad (4) \]
   Here we have also introduced the “virtual” input variable \( u \).

3. In terms of the new variables, \( z \) and \( u \), the problem is now linear!
   \[ \dot{\alpha} = z \]
   \[ \dot{z} = u \]

Again, linear techniques can be used to determine a control law,
\[ u = -K \left[ \frac{\alpha}{z} \right] + k_0 \alpha_{\text{ref}} \quad (5) \]
that satisfies the given specifications regarding stability and performance. To implement (5) we combine it with (4) which yields
\[ M(\alpha, q, \delta) = -K(\alpha - z)^T + k_0 \alpha_{\text{ref}} + \frac{1}{mV} \frac{dL(\alpha)}{d\alpha} z \quad (6) \]
with \( z \) as in (3), which implicitly defines \( \delta \). The problem of solving (6) for \( \delta \) is known as control allocation.

Let us turn to the pros and cons of dynamic inversion.

+ Using a single controller, the pilot is provided with the same aircraft response irrespectively of the flight condition.

+ Stability is guaranteed even for high angles of attack given that (6) can be satisfied.

− The control law expression explicitly involves the lift force, \( L \), as well as its derivative w.r.t. \( \alpha \). In practice, \( L \) comes with an uncertainty in the order of 10% and thus, the nonlinear behavior cannot be completely cancelled. The effects of these nonlinear remainders are difficult to analyze [10] and robustness is therefore the Achille’s heel of dynamic inversion.

### 4. Angle of attack control using backstepping

A major advantage of using nonlinear control is that one controller can be used for all flight cases as demonstrated in the preceding dynamic inversion design.

A major concern about the dynamic inversion control law (6) is that it relies on exact knowledge of the lift force. This problem can, at least partially, be circumvented using backstepping.

Backstepping [7] is a fairly young nonlinear control design method based on Lyapunov theory. During the last decade it has received a lot of attention, with numerous theoretical as well as practical results reported. A key feature of backstepping is that it allows for a more flexible way of dealing with system nonlinearities than simply cancelling them, as in dynamic inversion. If we do not need to cancel the nonlinearities, we obviously also do not have to know them exactly.

Let us now outline a backstepping design for the system (1). For technical details we refer to [4]. The ideas on how to benefit from a useful nonlinearity spur from [6].
The key step of this design is to realize that $L(\alpha)$ is not a very harmful nonlinearity – in most cases it naturally stabilizes $\alpha$! All the control law has to do is to

(a) Dominate $L$ in regions where $L$ is harmful.
(b) Make $\alpha = \alpha_{ref}$ the steady state of (1).

Let us try to satisfy these two demands, considering only Equation (1a) and for a minute regarding $q$ as the input variable.

Starting with (b), we see that

$$q = \frac{L(\alpha_{ref})}{mV} = \frac{g}{V} = q_0$$  \hspace{1cm} (7)

yields

$$\dot{\alpha} = \frac{L(\alpha) - L(\alpha_{ref})}{mV}$$  \hspace{1cm} (8)

making $\alpha = \alpha_{ref}$ an equilibrium.

Now turn to (a). The right hand side of (8) is shown in Figure 3 for $\alpha_{ref} = 5^\circ$ and $\alpha_{ref} = 25^\circ$, respectively. For $\alpha < \alpha_{ref}$, $\dot{\alpha}$ is positive, driving $\alpha$ towards $\alpha_{ref}$. For $\alpha > \alpha_{ref}$, $\dot{\alpha}$ is initially negative, again driving $\alpha$ towards $\alpha_{ref}$.

However, for large enough values of $\alpha$, $\dot{\alpha}$ starts to increase and eventually becomes positive, driving $\alpha$ further away from its commanded value, $\alpha_{ref}$. This effect is due to the physical fact that the lift force increases only up to a certain angle of attack, the stall angle, whereafter it decreases, see Figure 2.

To counteract these destabilizing tendencies, we add a term dominating the growth of $\dot{\alpha}$. We have great freedom in selecting this term. An interesting choice is to select

$$q = q_0 - k_1(\alpha - \alpha_{ref})$$

$$k_1 > \max_{\alpha} \frac{1}{mV} \frac{dL(\alpha)}{d\alpha}$$  \hspace{1cm} (9)

which is linear in $\alpha$. To determine $k_1$ we only need to know a bound on the lift force slope past the stall angle, according to Figure 2.

2. The control law (9) is expressed in terms of $q$, and therefore referred to as a “virtual” control law in back-stepping. Can (9) be converted into a realizable control law, expressed in terms of the actual input $\delta$? Yes.

In [4] it was shown that if

$$M(\alpha, q, \delta) = -k_2(q - q_0 + k_1(\alpha - \alpha_{ref}))\delta$$

$$k_2 > k_1$$  \hspace{1cm} (10)

with $q_0$ as in (7), can be solved for $\delta$, then $\alpha = \alpha_{ref}$ is a globally stable equilibrium of (1).

Let us now evaluate the performance of the backstepping control law with special focus on the five important properties listed in the introduction: stability, performance, robustness, saturation, and disturbance attenuation.

**Stability** Stability is guaranteed for all flight conditions, including high angles of attack. This holds under the restriction that (10) can be satisfied.

**Performance** Since the closed loop system is not linear in, e.g., the angle of attack, the control system does not provide the same aircraft response for all situations. However, the nominal performance, valid around a low angle of attack operating point, can be tuned according one’s requirements.

To do this, linearize the closed loop system, given by the aircraft dynamics (1) combined with the feedback law (10), at the operating point. Then select $k_1$ and $k_2$ using, e.g., pole placement och linear quadratic techniques.
Robustness  Robustness profits greatly from the backstepping design. The lift force nonlinearity does not appear in
the feedback loop but only in the feedforward link from $\alpha_{\text{ref}}$. A lift force model error therefore only shifts the equi-
librium but does not jeopardize the stability [5].

The effects of a model error in the pitching moment $M$
are not as clear. However, two methods to adapt to such an
error were proposed in [3].

Saturation  Let us now consider the case where (10) can-
not be satisfied due to saturation, i.e., when there are no
control surface deflections that will produce the desired mo-
ment. Can stability be guaranteed for a certain amount of
saturation? Yes.

Rearranging (10) to eliminate $k_2$ we see that stability is
guaranteed as long as

$$-\frac{M(\alpha, q, \delta)}{(q - q_0 + k_1(\alpha - \alpha_{\text{ref}}))} I > k_1$$

Thus, depending on how $k_1$ was chosen, we can tolerate
a certain amount of degradation the moment actually pro-
duced compared to the demanded moment. In control ter-
minality, our design has an gain margin of $(k_1/k_2, \infty)$.

Disturbance attenuation  How well does the control law
attenuate the effects of wind gusts, and how does sensor
noise propagate through the system? A main weakness of
nonlinear control design, including backstepping, is the lack
of tools to quantitatively analyze the effects such external
disturbances. Fortunately, as reported in [1], a flight con-
troller giving a properly selected bandwidth generally does
a good job of suppressing disturbances.

5. Conclusions

Judging from the evaluations in Section 5, we conclude
that backstepping offers a framework well suitable for air-
craft flight control design. Stability and robustness are is-
 issues which backstepping inherently handles well due to its
Lyapunov foundation. In this paper we have also shown
backstepping to be compatible with saturation analysis and
performance tuning.

Two things, for which satisfactory solutions are yet to
be found, are disturbance analysis and how to incorporate
some sort of integral control to asymptotically reach the
commanded state despite uncertain nonlinearities and con-
stant disturbances.

Let us finally reformulate our conclusions in terms of the
question posed in the paper title – what is there to gain using
backstepping instead of existing techniques?

- One controller handles all flight cases.
- Guaranteed global stability even for high angles of at-
tack.
- Robustness against lift force modeling errors.
- Guaranteed stability even in the case of moderate input
saturation, due to the controller gain margin.

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