Dynamical Effects of Time Delays and Time Delay Compensation in Power Controlled DS-CDMA

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Abstract

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Keywords: Power control, Time delay compensation, Delay analysis, Dynamics, DS-CDMA, WCDMA, cdma2000, IS-95, Oscillations, Describing functions

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I. INTRODUCTION

IN ORDER TO utilize the available resources in cellular radio systems efficiently, different radio resource management schemes are needed. One such technique is to control the output powers of the transmitters. In systems based on CDMA, this is particularly important since all terminals are communicating using the same spectrum. Most power control algorithms proposed to date strive to balance the *signal-to-interference ratio* (SIR) [2].

Fast fading has to be mitigated when possible, and therefore it is desirable to choose a high updating interval of the power control algorithm. The signaling bandwidth is kept low by utilizing only one bit for signaling, where the power is stepwise increased or decreased. Viterbi [27] proposed a scheme where the transmitter power is increased or decreased based on the comparison of received SIR and a threshold, γ^t . The scheme was further investigated by Ariyavisitakul [5].

Signaling and measuring takes time resulting in time delays in the power control loop, which in turn affects the dynamics of the closed-loop. This has primarily been considered as imperfect power control and Sim et. al. [22] concluded that power control is more sensitive to the delay than to the SIR estimator performance. Chockalingam et. al. [11] indicated, using simulations and analytically ap-

proximated second-order statistics, that the performance is degraded when subject to delays in the power control loop.

Leibnitz et. al. [20] proposed a Markov chain model to describe the power control dynamics. The horizon of transitions was, however, chosen too short to reveal the presence of oscillations.

The intuitive behavior of a power control algorithm in operation is that the received SIR oscillates up and down around the threshold γ^t as in Figure 1a. When subject to delays, however, the amplitude of the oscillations is larger as seen in Figure 1b. Primarily, time delays results in oscillations in two different ways

1. Delayed reactions to changes in external disturbances.
2. Internal dynamics of the power control loop.

In this paper, *time delay compensation* (TDC) [16] is proposed to mitigate oscillations due to internal dynamics (second item above). The first item (delayed reactions) is addressed in [23], [16]. The ambition in this paper is to illuminate the dynamical behavior of the control loop, when subject to time delays. Figure 1b describes the situation of a specific mobile in a network simulation. As seen in Figure 1a, which represents the same situation as in Figure 1b, but with TDC in operation, the oscillations are significantly reduced. This means that the capacity can be better utilized, which is critical when using variable data rate.

Fig. 1. Received SIR in a typical CDMA situation, where the power control commands are delayed by one slot. a) TDC employed, b) no TDC. (The distance between the ticks on the Y-axis is equal to the step size.)

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TDC is described using the log-linear model introduced in [8], which is similar to the model used in [11]. The dy-

namics are quantified using discrete-time describing functions to predict the different modes of oscillations.

The rest of the paper is organized as follows. Section II introduces the system models, which are used to describe the power control algorithm in operation. The closed-loop system is intuitively depicted in a block diagram capturing the dynamics. Time delay compensation is presented in Section III, and the dynamics is quantified using discrete-time describing functions in Sections IV and V. The performance improvements using TDC are further illuminated in simulations in Section VI. Recall that the ambition of the paper is to develop a model of the local loop dynamics to explain the dynamical effects of time delays and the merits of time delay compensation. The effects due to delayed reactions to disturbances such as fast fading are not addressed. Since the dynamics is in focus, simulations are primarily used to illustrate the relevance of the developed dynamical models. Finally, Section VII provides the conclusions.

A reader who only wants to grasp the most important results can peruse Sections II and III, Algorithm 3 in Section IV and Section V.

II. SYSTEM MODEL

Initially, it will be assumed that each mobile station is connected to only one base station, i.e. any soft handover scheme is not considered. This is chosen to keep the notation simple and to emphasize the core ideas. The case of soft handover will be considered in more detail in the end of this section and in Section III-B. Moreover, the nomenclature is settled to focus on the uplink (reverse link). However, the downlink (forward link) is treated analogously, except when stated.

A. Notation

All values will be represented by values in logarithmic scale (e.g., dB or dBW). Assume that m active mobile stations transmit using the powers $p_i(t)$, $i = 1, \dots, m$. The signal between transmitter i and receiver j is attenuated by the power gain $g_{ij}(t)$ (< 0). Moreover, mobile station i (transmitter i) is connected to receiver i (i.e., the connection is identified by the number of the mobile station i). Thus the corresponding connected base station will experience a desired signal power

$$C_i(t) = p_i(t) + g_{ii}(t)$$

and an interference plus noise power $I_i(t)$. The *signal-to-interference ratio* (SIR) at receiver i is defined by

$$\gamma_i(t) = p_i(t) + g_{ii}(t) - I_i(t). \quad (1)$$

One other common term for roughly the same quantity is *carrier-to-interference ratio* (C/I or CIR). These quantities are both used interchangeably, but more precisely, SIR refers to the ratio at the base band, while CIR refers to the corresponding ratio at the carrier frequency. Moreover, one differs between SIR before and after despreading, and the

difference is equal to the processing gain. The main focus in this article is the dynamical behavior of algorithms. Since the differences are just a matter of scale, they are neglectable.

B. Closed-loop Power Control in WCDMA

In order to avoid extensive signaling in the networks, distributed power control algorithms are desirable. Furthermore, fast power updates are of interest in order to mitigate the fast fading when possible. In the WCDMA proposal, the signaling bandwidth is kept low despite the high update rate by utilizing single-bit signaling [25], [4]. The SIR is estimated at the receiver and compared to a threshold $\gamma_i^t(t)$. Then the power control command $s_i(t) = -1$ is sent to the transmitter when above the threshold and $s_i(t) = +1$ when below. The updating procedure can thus be described as

$$\begin{aligned} \text{Receiver:} \quad e_i(t) &= \gamma_i^t(t) - \gamma_i(t) \\ s_i(t) &= \text{sign}(e_i(t)) \end{aligned} \quad (2a)$$

$$\text{Transmitter:} \quad p_i(t+1) = p_i(t) + \Delta_i s_i(t) \quad (2b)$$

This algorithm will be referred to as the *Fixed Step Power Control (FSPC)* algorithm. The time index t denotes power level update instants. With this nomenclature, $\gamma_i(t)$ denotes the latest measurement available at the receiver at time instant (power level update instant) t . Furthermore, the time instants are associated with the power update interval T_s equal to a slot duration. The step size Δ_i might be adapted as well. Here, only step size updates at a much slower rate than the power level updates are considered, which means that it can be considered constant in the analysis. The effects of faster step size updates will be discussed in Section V-A.

The target value, $\gamma_i^t(t)$, is provided by an outer control loop operating at a lower update rate [25], [16], [27]. Therefore, this value may be regarded as constant on a short term. Related to the choice of target values is the following definition [28].

Definition 1

The vector of target values, $[\gamma_i^t]$, is feasible if there exist a power vector, $[p_i]$, that result in these target values.

Both measuring and signaling in cellular systems take time, which results in delayed signals. There are two main reasons for time delays. Firstly, the power control algorithm itself results in a delay of one sample, since power control commands at time t ($s_i(t)$) are used to update the power level at time $t+1$ (i.e., $p_i(t+1)$). Secondly, it takes some time to measuring perceived quality and generating power control commands. Additional delays are caused by the fact that power update commands are only allowed to be transmitted at certain time instants. Together they result in an additional delay of n samples.

Since the command signaling is standardized, these delays are known exactly in number of samples (or slots). Typical situations in WCDMA are depicted in Figure 2.

The receiver uses pilot bits in the slot (and maybe some data bits) to measure SIR (shaded area) [1]. Note that the slot synchronization is shifted between the forward and reverse links. The staggered slots enable a possibility to operate without additional delays in the control loop ($n = 0$) as in Figure 2b. Such a configuration might require the use of fewer pilot bits for estimation, to match the transmission time instant of the power control command. Conversely, if more symbols are used for estimation, the accuracy is improved but the drawback is an additional loop delay $n = 1$, see Figure 2a. Note that the estimation time is typically a fraction of the power control update interval T_s . However, since the internal dynamics will be in focus, only the (piecewise constant) power level which affect the measurement matters. Similarly, IS-95 have a delay n of 2-3 samples [24], and cdma2000 a delay $n = 1$.

Fig. 2. Typical situations in WCDMA describing time of estimation (over pilot bits and possibly some data bits), power command generation and signaling (PC), and power update. The relatively long estimation and processing time to favor estimation accuracy in a) result in a time delay $n = 1$. This is avoided in b) by considering fewer symbols in the estimation.

Time delays are conveniently represented using the *time-shift operator* q defined by

$$q^{-n}p(t) = p(t - n), \quad q^n p(t) = p(t + n).$$

Arithmetic operations of polynomials in q will be used frequently in this and subsequent section. For a more rigorous discussion on a q -operator algebra, the reader is referred to [6]. The intuitive relations to the complex variable z of the z -transform are also addressed.

Applying this scheme to the power update in the transmitter 2b yields

$$p_i(t) = \frac{\Delta_i}{q-1} s_i(t). \quad (3)$$

Using (3), the closed-loop dynamics of the power control algorithm can be depicted as in Figure 3. This *local loop* would capture the entire dynamics if the interference $I_i(t)$ can be treated as independent of the transmitter power $p_i(t)$. In a more detailed model, the cross-coupling between the local loops have to be considered. However, as will be seen further on, the local loop dynamics dominate the dynamical behavior at each receiver. When the target values $\gamma_i^t(t)$ are infeasible (see Definition 1), then the transmitter

powers will ramp up until one or several transmitters are using maximum powers. This effect is referred to as the *party effect* and is further discussed in [3]. In this article, it is assumed that the target values are feasible.

Fig. 3. WCDMA inner loop power control, where the power control commands are subject to an additional delay of n samples.

In a real systems, the power control commands may be corrupted by disturbances resulting in command errors. This is modeled by the stochastic multiplicative disturbance $x_i(t)$ with probability function

$$p_X(x) = (1 - p_{CE})\delta(x - 1) + p_{CE}\delta(x + 1),$$

i.e., the command error probability is p_{CE} . Moreover, the output powers are limited to the set $[p_{\min}, p_{\max}]$. A more detailed block diagram incorporating these components is given in Figure 4.

Fig. 4. WCDMA inner loop power control as in Figure 3, but with power command errors $x_i(t)$ and limited dynamic range.

C. Comparison to Carrier-based Power Control

In some systems a similar scheme is employed, but the power control commands are determined by comparing the received carrier power $C_i(t)$ to a threshold [26]. As in the previous section, the closed power control loop can also be described by the block diagram in Figure 4, except that only $g_{ii}(t)$ acts as external disturbance instead of $g_{ii}(t) - I_i(t)$ [11]. Since the local loop dynamics are equivalent [16], analysis of the SIR based power control will hold in this case as well. We can thus without loss of generality focus on the SIR-based control.

D. Soft and Softer Handover

When the quality of service is degraded, it may be beneficial to connect to several base stations. A typical situation is when the mobile station is moving from one cell to another. During a transition phase, the mobile station will

be connected to both base stations to preserve an acceptable connection. The number of connected base stations is referred to as the *active set* (AS). This active set includes the base station with the strongest received signal and the base stations within a window of size hm (*handover margin*). However, the active set contains at most $AS_{\max}$ number of base stations. For uplink power control, the mobile station should adjust the power with the largest step in the “down” direction ordered by the power control commands received from each base station in the active set [25].

A related strategy is to connect to several (normally two) sectors at the same base station. This is referred to as *softer handover*.

III. TIME DELAY COMPENSATION

As indicated by Figure 1, round-trip delays in the power control loop result in oscillations. Essentially, the core problem is that the measurements do not reflect the most recent power updates. However, these are known to the algorithm, and can be compensated for. Therefore, we adjust the measured SIR according to the power control commands that have been sent but whose effect have not yet been experienced by the receiver [16].

A. Algorithms and Implementations

Since SIR $\gamma_i(t)$ is linearly dependent of the transmission power $p_i(t)$ (see Equation (1)), an increase in the power level $p_i(t) + \Delta_i$, will result in a corresponding SIR increase $\gamma_i(t) + \Delta_i$. As discussed above, the commands not yet experienced have to be compensated for. This is accomplished by adjusting the measured SIR as

$$\check{\gamma}_i(t) = \gamma_i(t) + \Delta_i \sum_{j=1}^n q^{-j} s_i(t)$$

to include the effect of the n most recent issued commands, which are not reflected in the current measurement. Time delay compensation can thus be implemented as

ALGORITHM 1 (TIME DELAY COMPENSATION I)

1. Adjust the measured SIR:

$$\check{\gamma}_i(t) = \gamma_i(t) + \Delta_i \sum_{j=1}^n q^{-j} s_i(t)$$

2. Compute the power control command:

$$s_i(t) = \text{sign}(\gamma_i^t(t) - \check{\gamma}_i(t)).$$

Remark. In the case $n = 1$, the measurement is adjusted simply by $\check{\gamma}_i(t) = \gamma_i(t) + \Delta_i s_i(t - 1)$.

Rewrite the compensation term in order to see the effects of TDC more clearly.

$$\Delta_i \sum_{j=1}^n q^{-j} s_i(t) = \Delta_i q^{-1} \frac{1 - q^{-n}}{1 - q^{-1}} s_i(t) = (1 - q^{-n}) \cdot \frac{\Delta_i}{q - 1} s_i(t) \quad (4)$$

Introduce $\check{p}_i(t)$ to monitor the powers to be used in the transmitter (cf. (2b)). This is essentially a separate local loop running in the power control algorithm to mimic the behavior of $p_i(t)$.

$$\check{p}_i(t + 1) = \check{p}_i(t) + \Delta_i s_i(t) \iff \check{p}_i(t) = \frac{\Delta_i}{q - 1} s_i(t)$$

Together with (4), this yields

$$\Delta_i \sum_{j=1}^n q^{-j} s_i(t) = (1 - q^{-n}) \check{p}_i(t) = \check{p}_i(t) - \check{p}_i(t - n)$$

The compensation can thus be written as

$$\check{\gamma}_i(t) = \gamma_i(t) + \check{p}_i(t) - \check{p}_i(t - n).$$

Basically the compensation can be seen as operating in two phases. Monitoring the powers to be used in the transmitter, $\check{p}_i(t)$, and subtracting the old power and add the new power to the measured SIR. This can be formulated as the algorithm below. Note that when consider this as an implementation, the limited dynamic range must be considered.

ALGORITHM 2 (TIME DELAY COMPENSATION II)

1. Adjust the measured SIR:

$$\check{\gamma}_i(t) = \gamma_i(t) + \check{p}_i(t) - \check{p}_i(t - n)$$

2. Compute the power control command:

$$s_i(t) = \text{sign}(\gamma_i^t(t) - \check{\gamma}_i(t))$$

3. Monitor in the receiver the powers to be used by the transmitter:

$$\check{p}_i(t + 1) = \max(p_{\min}, \min(p_{\max}, \check{p}_i(t) + \Delta_i s_i(t))).$$

Introduce

$$H(q) = -(1 - q^{-n}).$$

Based on Figure 3, the operation of this algorithm is naturally represented by the block diagram in Figure 5. Note that limited dynamic range is not included for simplicity. Readers with a background in control recognize the relations to the Smith-predictor, discussed e.g., in [6]. The benefits of TDC are illuminated by rewriting this block diagram. After some “block diagram algebra” exercise, the diagram can be rewritten as in Figure 6. The merits of TDC are then evident, since it cancels the internal round-trip delays in the loop. However, external signals and disturbances are still delayed, and it takes some time before changes in $\gamma_i^t(t)$, $g_{ii}(t)$ and $I_i(t)$ are reflected in the measurement $\gamma_i(t)$.

Fig. 5. Time delay compensation (TDC) can be implemented as an internal feedback by internally monitoring the powers to be used by the transmitter, $\check{p}_i(t)$.

Fig. 6. By rewriting the diagram in Figure 5, it is evident how TDC cancels the round-trip delays in the control loop. External signals and disturbances are still delayed before they are reflected in the transmission power $p_i(t)$.

B. Applicability

TDC is applicable to both uplink and downlink power control in WCDMA, as well as systems where the transmitter powers are controlled analogously. One problem arises for the uplink when employing soft handover. While in operation, a base station is unaware of whether an issued control command is applied or not. As stated in Section II-D, the mobile station decreases the power if at least one base station has issued a down command. Therefore, TDC should be disabled in the uplink while in soft handover. The applicability to the downlink when in soft handover as well as in softer handover is depending on the combining strategy in the receiver.

A similar situation is prevalent when the power command error probability is high. The benefits of TDC will then be degraded. This is further discussed in Section VI.

IV. DESCRIBING FUNCTIONS

In this section we develop the underlying theory, first in a simple case to stress the main ideas, and then with focus to the power control local loops. The theory of *describing functions* in continuous time are further discussed by Glad [14], Atherton [7] and Phillips and Nagle [21]. De la Sen [12] provide modifications for discrete-time describing functions of some specific nonlinearities. The applicability to power control in cellular radio systems have been addressed by Gunnarsson et. al. in [8], [9], [17], [18].

A. Describing Functions with Zero Phase Assumption

Basically, we are focusing on loops that consist of a linear part with transfer function $G(q)$ and a static nonlinearity

described by the function $f(\cdot)$ resulting in a loop as in Figure 7. Note that we have assumed a zero input to the loop. Nonzero inputs are studied in Section IV-C. Essentially, the idea is to replace the static nonlinearity by a complex constant Y_f , that approximate the gain and the phase shift of the nonlinearity.

Fig. 7. Block diagram of a nonlinear system, separated into one linear and one nonlinear component.

Nonlinearities in the loop normally result in an oscillatory behavior. This will be studied by assuming that there is an oscillation in the error signal $e(t)$, and then try to verify this assumption. We proceed by making the N -periodic hypothesis

$$e(t) = E \sin(\Omega_e t) = E \sin\left(\frac{2\pi}{N}t\right),$$

where E is the amplitude of the oscillation and Ω_e is the normalized angular frequency. The hypothesis is based on a *zero phase assumption*, with the meaning that the sample at $t = 0$ is zero (slightly more general, we assume that the sampled sinus is zero for some t in every period). In order to simplify the calculations we assume for a moment that $f(\cdot)$ is odd and continuous. The computations using a general static nonlinearity are analogous, but a little bit more complicated, and we will return to a general $f(\cdot)$ further on. Since the nonlinearity is static, $w(t)$ is N -periodic as well. Using discrete time Fourier series expansion, the signal $w(t)$ is decomposed into its Fourier components as

$$w(t) = f(E \sin(\Omega_e t)) = A_1(E, N) \sin(\Omega_e t + \phi_1(E, N)) + A_2(E, N) \sin(2\Omega_e t + \phi_2(E, N)) + \dots, \quad (5)$$

where A_i and ϕ_i represents the coefficients of the Fourier expansion. Recall that a sinusoid $\hat{X} \sin(\Omega_X t)$ fed through a linear system $H(q)$ results in the output

$$\hat{X} |H(e^{i\Omega_X})| \sin(\Omega_X t + \arg(H(e^{i\Omega_X}))),$$

after the transients have decayed. Now let us make the assumption that the linear system $G(q)$ will attenuate the harmonics much more than the fundamental frequency. This is the only approximation we will make, and it yields

$$y(t) \approx A_1(E, N) |G(e^{i\Omega_e})| \sin(\Omega_e t + \phi_1(E, N) + \arg(G(e^{i\Omega_e}))). \quad (6)$$

Figure 7 yields

$$y(t) = -e(t) = -E \sin(\Omega_e t) = E \sin(\Omega_e t + \pi). \quad (7)$$

Recall that $\Omega_e = \frac{2\pi}{N}$. The loop is closed by combining Equations (6) and (7). Thus an oscillative behavior is predicted if there exists a solution to the following equations

$$A_1(E, N) \left| G(e^{i\frac{2\pi}{N}}) \right| = E \quad (8a)$$

$$\phi_1(E, N) + \arg(G(e^{i\frac{2\pi}{N}})) = \pi + 2\pi\nu, \quad \nu \in \mathbb{Z}. \quad (8b)$$

The complex Fourier series [10] is defined by

$$w(t) = \sum_{k=0}^{N-1} C_k(E, N) e^{j\Omega_e k t} \quad (9a)$$

$$C_k(E, N) = \frac{1}{N} \sum_{t=0}^{N-1} w(t) e^{-j\Omega_e k t} \quad (9b)$$

Moreover, define the complex number

$$Y_f(E, N) = \frac{A_1(E, N) e^{i\phi_1(E, N)}}{E} = \frac{2i}{E} C_1(E, N). \quad (10)$$

Then (8) can be expressed more compactly as

$$Y_f(E, N) G(e^{i\frac{2\pi}{N}}) = -1. \quad (11)$$

Equation (11) or the equations in 8 provide two equations and two unknowns. If a solution exists, it describes an approximation of the oscillation in the loop. Conversely, the lack of a solution motivates the absence of oscillative modes. Note that the defined complex constant Y_f can be seen as an complex constant approximation of the nonlinearity.

B. Discrete-Time Describing Functions

The derivation in the previous section is based on a zero phase assumption. This is not so important when the static nonlinearity is continuous, since phase shifts in $e(t)$ result in corresponding phase shifts in $w(t) = f(e(t))$. The situation is different when considering discontinuous static nonlinearities. For example, consider a relay (sign function) defined by

$$f(e) = \begin{cases} 1, & e \geq 0 \\ -1, & e < 0 \end{cases} \quad (12)$$

It is discontinuous at $e = 0$. Therefore, there will be some problems with the previous definition of the describing function at the zero-crossings. This is evident in Equation (5) with $f(\cdot)$ as a sign function

$$w(t) = \text{sign}(E \sin(\frac{2\pi}{N} t)).$$

Both $t = 0$ and $t = N/2$ will result in $w(t) = 1$, and thereby cause asymmetry in (9b). This asymmetry cannot remain persistent together with the relay and the integrator ($\frac{1}{q-1}$). Therefore we introduce the unknown time shift δ_e to avoid sampling at the zero-crossings.

$$w(t) = \text{sign}(E \sin(\frac{2\pi}{N}(t + \delta_e))). \quad (13)$$

Note that $w(t)$ will remain the same independent of time shifts in the range between 0 and 1, $\delta_e \in (0, 1)$. Note that a time shift of an entire sample is the same as a time delay and should therefore be a part of the linear transfer function $G(q)$. The unknown time shift is thus among the parameters characterizing an oscillation. Its effect depends the nonlinearity and the period N .

Recall the following central assumption from the previous section

Assumption 1

The linear part $G(q)$ in Figure 7 attenuates the harmonics much more than the fundamental frequency Ω_e . More formally, we assume that

$$\left| \frac{G(e^{ik\Omega_e})}{G(e^{i\Omega_e})} \right| \Big|_{\Omega_e = \frac{2\pi}{N}} < 1, \quad k = 2, \dots, \frac{N}{2}$$

For monotonically decreasing $G(e^{i\Omega_e})$, this criterion only has to be verified for $k = 2$. Therefore, the assumption is

$$r_G = \left| \frac{G(e^{i2\Omega_e})}{G(e^{i\Omega_e})} \right| \Big|_{\Omega_e = \frac{2\pi}{N}} < 1 \quad (14)$$

If Assumption 1 above holds, the complex representation of the output from the system G can be approximated by

$$y(t) \approx C_1(E, N, \delta_e) G(e^{i\Omega_e})$$

where the complex Fourier coefficient $C_1(E, N, \delta_e)$ with $\delta_e \in (0, 1)$ is given by

$$\begin{aligned} C_1(E, N, \delta_e) &= \frac{1}{N} \sum_{t=0}^{N-1} f\left(E \sin\left(\frac{2\pi}{N}(t + \delta_e)\right)\right) e^{-i(\frac{2\pi}{N}t)}. \end{aligned}$$

Analogously, the complex representation of the error signal $e(t) = E \sin(\Omega_e(t + \delta_e))$ is

$$e(t) = \frac{E}{2i} e^{i\Omega_e \delta_e}$$

Since the input is assumed zero, we have $e(t) = -y(t)$, which yields the following equality, which has to be fulfilled by an oscillation

$$-1 = G(e^{i\Omega_e}) \underbrace{\frac{2i}{E} C_1(E, N, \delta_e) e^{-i\Omega_e \delta_e}}_{Y_f(E, N, \delta_e)}$$

The discussion motivates the following definition of the discrete-time describing function

Definition 2 (Discrete-Time Describing Function)

The discrete-time describing function of the static nonlinearity $f(\cdot)$ is defined (cf. (10)) by

$$\begin{aligned} Y_f(E, N, \delta_e) &= \frac{2i}{NE} \sum_{t=0}^{N-1} f(E \sin(\Omega_e(t + \delta_e))) e^{-i(\Omega_e(t + \delta_e))}. \end{aligned}$$

With an analogous reasoning as in the previous section, we are interested in the solution to

$$Y_f(E, N, \delta_e)G(e^{i\frac{2\pi}{N}}) = -1, \quad \delta_e \in (0, 1). \quad (15)$$

This is essentially two equations and a constraint, which is more clear by separating the magnitude and the phase of each side.

$$|Y_f(E, N, \delta_e)| |G(e^{i\frac{2\pi}{N}})| = 1 \quad (16a)$$

$$\arg Y_f(E, N, \delta_e) + \arg G(e^{i\frac{2\pi}{N}}) = \pi + 2\pi\nu, \quad \delta_e \in (0, 1), \quad \nu \in \mathbb{Z} \quad (16b)$$

We thus have two equations and four unknowns (E , N , δ_e and ν), which enables several possible solutions.

This far we have assumed that Assumption 1 holds. If this is not the case, the analysis may still not be in vain. In such a case, the higher frequencies contribute to the waveform of the error signal $e(t)$. Describing functions focus on the fundamental frequency of the oscillations. Therefore, the estimated fundamental period N obtained from describing functions analysis is still informative. Instead, different waveforms of the error signal may be assumed, depending on the application, the nonlinearity and the system.

The discrete-time describing functions analysis is summarized in the following algorithm

ALGORITHM 3 (DISCRETE-TIME DESCRIBING FUNCTIONS ANALYSIS)

Consider the situation in Figure 7, where the loop is separated in a linear ($G(q)$) and a nonlinear ($f(\cdot)$) part. Then the oscillation in the error signal $e(t)$ is approximated by the procedure

1. Determine the discrete-time describing function of the nonlinearity as

$$Y_f(E, N, \delta_e) = \frac{2i}{NE} \sum_{t=0}^{N-1} f(E \sin(\Omega_e(t + \delta_e))) e^{-i(\Omega_e(t + \delta_e))},$$

where $\Omega_e = \frac{2\pi}{N}$ and $\delta_e \in (0, 1)$.

2. Compute $\hat{G}(q)|_{q=e^{2\pi i/N}}$.

3. Solve the following equation for E , N and δ_e .

$$Y_f(E, N, \delta_e)G\left(e^{i\frac{2\pi}{N}}\right) = -1 \quad (17)$$

If one solution (E , N , δ_e) exists, then the oscillation is approximated by

$$e(t) = E \sin\left(\frac{2\pi}{N}(t + \delta_e)\right).$$

If several solutions exist, then several modes of oscillation are possible.

4. Investigate the correctness of Assumption 1. If it does not hold, the estimated periods are still informative, but alternative waveforms may be discussed.

C. Simplified Nonzero Input Case

When considering nonzero inputs in general, the analysis becomes more complex. In some cases, however, such as the power control case, some approximative simplifications are readily available. Recall from the introduction that the control error

$$e_i(t) = \gamma_i^t(t) - \gamma_i(t)$$

depend on the one hand on bad SIR tracking due to delayed reactions to external disturbances, i.e., $g_{ii}(t)$ and $I_i(t)$. On the other hand, it also depend on the internal dynamics of the local control loop which is the main issue in this paper. To focus on the dynamical effects we assume that the power gain and the interference power are constant over the delay horizon (n samples).

Consider the FSPC algorithm in (2) and include the delays as in Figure 3. This yields

$$p_i(t+1) = p_i(t) + \Delta_i \text{sign}(\gamma_i^t(t) - p_i(t-n) - g_{ij}(t) + I_i(t)).$$

Introduce $\dot{p}_i(t) = p_i(t) - \gamma_i^t(t) + g_{ij}(t+n) - I_i(t+n)$. The assumptions of constant power gain and interference power hence yield

$$\dot{p}_i(t+1) = \dot{p}_i(t) + \Delta_i \text{sign}(-\dot{p}_i(t-n)),$$

which is the zero input case. This will only be an approximation. A speculative proposition is that small deviations essentially effect the unknown time shift δ_e . Therefore if the resulting oscillative behavior comprises several modes, the mode switching is stimulated by these deviations.

Note that the constant power gain and interference power assumption enables us to describe the oscillative behavior due to the internal dynamics. When subject to faster variations, the control error is also affected by the tracking variance.

D. Describing Function of a Sign Function

An ideal relay or sign function (see (2)) is defined in (12)

$$f(e) = \begin{cases} 1, & e \geq 0 \\ -1, & e < 0 \end{cases}$$

The corresponding describing function is obtained by applying Definition 2. In the sign function case, the complex Fourier coefficient $C_1(E, N, \delta_e)$ can be computed as

$$\begin{aligned} C_1(E, N, \delta_e) &= \frac{1}{N} \sum_{t=0}^{N-1} f(E \sin(\Omega_e(t + \delta_e))) e^{-i(\Omega_e t)} \\ &= \frac{1}{N} \sum_{t=0}^{N/2-1} e^{-i(\Omega_e t)} - \frac{1}{N} \sum_{t=N/2}^{N-1} e^{-i(\Omega_e t)} \\ &= \frac{1}{N} [1 - e^{-i\pi}] \sum_{t=0}^{N/2-1} e^{-i\Omega_e t} \\ &= \frac{2}{N \sin(\frac{\pi}{N})} e^{i(\frac{\pi}{N} - \frac{\pi}{2})} \end{aligned}$$

The discrete-time describing function is thus

$$Y_f(E, N, \delta_e) = \frac{2i}{E} C_1(E, N, \delta_e) e^{-i\Omega_e \delta_e} = \frac{4}{NE \sin\left(\frac{\pi}{N}\right)} e^{i\left(\frac{\pi}{N} - \delta_e \frac{2\pi}{N}\right)} \quad (18)$$

V. DESCRIBING FUNCTION ANALYSIS

The analysis in this section is focused on the dynamical behavior of the FSPC algorithm in (2), which more compactly is given by

$$p_i(t+1) = p_i(t) + \Delta_i \text{sign}(\gamma_i^t(t) - \gamma_i(t)), \quad (19)$$

In operation, the corresponding local loop can be associated with the block diagram in Figure 8. With the sign function as the static nonlinearity, and the remaining parts all linear, this is in full analogy with the block diagram in Figure 7. Hence, discrete-time describing functions analysis is applicable.

Fig. 8. The decision feedback of the FSPC algorithm in (19) is visualized as a sign function block, which is a static nonlinearity.

In this case with a sign function and an integrator, is easy to realize that primarily modes with even periods N are dominating the oscillations. This is formulated in the following conjecture.

Conjecture 1 (Even-period Oscillations of FSPC)

An ideal relay (sign function) together with an integrator as in the FSPC case, cannot have oscillations of odd periods. While in operation, single cycles of odd periods might be present, but they will be regarded as transitions between even-period cycles.

Consider the process outlined in Algorithm 3. The discrete-time describing function of the sign function is provided in (18)

$$Y_f(E, N, \delta_e) = \frac{4}{NE \sin\left(\frac{\pi}{N}\right)} e^{i\left(\frac{\pi}{N} - \delta_e \frac{2\pi}{N}\right)}, \quad \delta_e \in (0, 1) \quad (20)$$

The linear part of the block diagram in Figure 8 is

$$G(q) = \frac{\Delta_i}{q^n(q-1)}$$

On the unit circle $G(q)$ is equal to

$$G(q)|_{q=e^{2\pi i/N}} = \frac{\Delta_i}{2 \sin(\pi/N)} e^{-i\left(\frac{\pi}{2} + \frac{\pi}{N} + \frac{2\pi}{N}n\right)} \quad (21)$$

Equations (17), (18) and (21) yield

$$\frac{2\Delta_i}{NE \sin^2 \frac{\pi}{N}} e^{-i\left(\frac{\pi}{2} + \frac{2\pi}{N}(\delta_e + n)\right)} = -1 = e^{-i(\pi + 2\pi\nu)}, \quad \delta_e \in (0, 1). \quad (22)$$

Separating the phase and magnitude equalities, results in the two equations

$$E = \frac{2\Delta_i}{N \sin^2 \frac{\pi}{N}} \quad (23a)$$

$$\frac{\pi}{2} + \frac{2\pi}{N}(\delta_e + n) = \pi + 2\pi\nu, \quad \delta_e \in (0, 1), \quad \nu \in \mathbb{Z} \quad (23b)$$

Thus, it is the phase equality (23b) that allow different solutions. It can be rewritten as

$$N = \frac{4(\delta_e + n)}{1 + 4\nu}, \quad \nu \in \mathbb{Z}, \quad \delta_e \in (\mathbb{K}, \mathbb{K}). \quad (24)$$

Since N is positive, $\nu \geq 0$. The denominator describes an integer, thus also the numerator has to be an integer. Therefore, only δ_e -values in the set $\{0.25, 0.5, 0.75\}$ are plausible. Moreover, N is even according to Conjecture 1, which excludes all values but the second possible δ_e -values. Moreover, as commented in Section IV-B, the describing functions analysis focuses on the fundamental frequency. This corresponds to the longest period N , which excludes $\nu > 0$ in (24). Hence, the possible periods are obtained from

$$N = 2 + 4n. \quad (25)$$

For each N , the corresponding amplitude is computed using (23a).

Finally, we have to verify the correctness of Assumption 1. If it does not hold, the estimated periods are still informative, as pointed out in Section IV-B. With the step-wise updates of the sign function, an intuitive alternate waveform is a triangular wave. Such a waveform of period N has the amplitude

$$E' = \frac{N\Delta_i}{4}. \quad (26)$$

We exemplify the analysis by a concrete example

EXAMPLE 1 (ANALYSIS OF FSPC IN A TYPICAL WCDMA SETTING) Consider the typical delay situation in a WCDMA system (see Figure 1b) $n = 1$. The possible oscillation modes (identified by the period N) are obtained from (25). Thus

$$N_0 = 6.$$

To verify Assumption 1, compute the ratio

$$r_G = \frac{|G(e^{i2\Omega_e})|}{|G(e^{i\Omega_e})|} \Big|_{\Omega_e = \frac{2\pi}{N_0}} = \frac{\sin(\pi/N_0)}{\sin(2\pi/N_0)} = \frac{1}{2 \cos(\pi/N_0)} \approx 0.58,$$

where the expression in (21) is used in the second equality. It is therefore more relevant to use the expression in (26) than in (23a) to compute the mode amplitude. This gives $E'_0 = 1.5\Delta_i$. Compare the result to the introductory simulation in Figure 1b. The estimated mode describes the simulated oscillations well.

□

Corresponding oscillation analyses with respect to other delay situations are summarized in Table I.

n	Oscillation modes	E_0	E'_0	r_G
0	$N_0 = 2$	Δ_i	Δ_i	-
1	$N_0 = 6$	$1.33\Delta_i$	$1.5\Delta_i$	0.58
2	$N_0 = 10$	$2.1\Delta_i$	$2.5\Delta_i$	0.53
3	$N_0 = 14$	$2.9\Delta_i$	$3.5\Delta_i$	0.51

TABLE I

PREDICTED OSCILLATION MODES FOR VARIOUS DELAYS. THE QUANTITY r_G IS DEFINED IN (14) IN ASSUMPTIONS 1. SINCE r_G IS NOT SMALL ENOUGH, E'_0 IS PROBABLY A BETTER AMPLITUDE APPROXIMATION THAN E_0 .

A. Discussion

The benefits of employing TDC is further illuminated by Table I. Since TDC cancels the internal round-trip delays in the loop, as concluded in Section III-A, the period of oscillations is reduced to a minimum.

Assume that the power gain and the interference are relatively slowly varying compared to the updating rate of the power control algorithm. Then the dominating oscillation originates from the internal dynamics together with the sign function. This oscillative behavior is typical when nonlinear elements are present [7]. Therefore, thresholding with sign functions should be avoided when possible. In the power control algorithm implementation, however, the benefits of a low command signaling bandwidth justifies the use of thresholding.

One possible way of utilizing two bits is to use one for power up/down commands and one for step size up/down commands. This will incorporate yet another sign function in the control loop resulting in an even more complex oscillative behavior for which compensation is complicated. From a dynamics point of view, it is better to increase/decrease the step size more seldom.

VI. SIMULATIONS

In this section, system simulations will back-up and illuminate the obtained dynamical models and approximations from the previous sections. The simulations are network simulations, where the mobiles are mutually disturbing each other via the cross-couplings. Despite these interconnections, the dynamical behavior at each receiver is dominated by the local loop, as will be indicated in the simulations. The focus is on the power control error fluctuations, and therefore, the evaluation is focused on a specific

user in the multi-user environment. Power gain is modeled as proposed by Hata [19] (path loss), Gudmundson [15] (shadow fading) and [25] (fast fading, Vehicular A). Some of the important parameters are summarized in Table II. The simulation model is described in more detail in [13].

Antennas	Sectorized
Cell radius	2000 m
Control sample interval	$T_s = 0.667$ ms
Slots per frame	15
Power command delay	1 slot
Shadow fading std.dev.	8 dB
Shadow fading corr. dist.	100 m
Fast fading model	Vehicular A
Mean mobile station speed	10 m/s

TABLE II

SYSTEM SIMULATION PARAMETERS FOR A TYPICAL WCDMA CASE.

First, fast fading is omitted to emphasize the influence of the internal dynamics. Consider the system in operation without TDC. As seen in Figure 9a, the oscillation is fairly stable and the correspondence with the predicted oscillation in Table I is good. When employing TDC, the oscillations are more or less mitigated, and the algorithm tracks the target value much better. Less fading margin is needed to reside permanently above some critical level, which is turn increases the capacity. Furthermore, a more stable operation result in a more stable overall system.

Fig. 9. Control error $e_i(t) = \gamma_i^t(t) - \gamma_i(t)$ for the typical case of one slot delay. The correspondence to the predicted oscillations in Table I is good. a) No TDC, b) TDC.

As stated in Section III-B, power command errors may reduce the performance of TDC. Consider the case described in Figure 4 with command error probability $p_{CE} = 0.05$. The error signals together with the multiplicative disturbance $x_i(t)$ are depicted in Figure 10. As expected, command error will result in bursty errors in SIR, but the algorithm recovers fast.

Fig. 10. Same case as in Figure 9, but with command errors ($p_{CE} = 0.05$). a) Multiplicative disturbance $x_i(t)$, b) error without TDC c) error with TDC.

Increased power command error probability degrades the performance gradually. By using the error standard deviation, the variations in the error signals can be quantified. Figure 11 captures the degradation in the performance of TDC with increased command error probability, but it is still beneficial compared to without TDC.

As stated in Section IV-C, the control error $e_i(t)$ is also affected by the tracking variance, which is more evident when considering fast fading. To illustrate its effect, the same scenario as above is considered, but with fast fading. With the focus on the same mobile station, the results are summarized in Figure 12. The error variance is now more affected by tracking errors since the power control algorithm cannot track the fast fading. However, TDC still has its merits.

From the simulations we conclude that TDC is beneficial to employ despite power command errors and tracking errors due to bad tracking of the fast fading. The benefits are more emphasized when the command error probability is low. Furthermore it was illuminated that describing functions provide relevant predictions of oscillations in the local loops.

VII. CONCLUSIONS

A common implementation of power control in many DS-CDMA cellular systems is to utilize a power up-down command device. This device together with round-trip delays in the power control loops result in oscillations in output powers as well as in received SIR. By utilizing a log-linear

Fig. 11. Standard deviation of the received SIR error in a typical WCDMA situation, where the power control commands are delayed by one slot. a) TDC employed, b) no TDC.

model and the introduced discrete-time describing functions we quantify the oscillation mode. Both period and amplitude of the oscillation can be determined with good accuracy.

More importantly, Time Delay Compensation (TDC) is proposed to mitigate the oscillations. When employing TDC, the power control algorithm operates more stable, which is important from a network perspective. The scheme is simple to implement, and the main idea is to adjust the measured SIR according to the power control commands that have been sent but whose effect have not yet been experienced by the receiver. Moreover, the fading tracking capability is improved and thus less fading margin is needed.

Simulations illustrate the oscillations and the accuracy of predicted oscillations obtained from describing function analysis. TDC is beneficial to employ despite power command errors and fading tracking errors. Furthermore it was illuminated that describing functions provide relevant predictions of oscillations in the local loops.

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Fig. 12. The impact of TDC when subject to fast fading is illustrated by the resulting SIR error $e_i(t) = \gamma_i^t(t) - \gamma_i(t)$. For comparison, the error standard deviation $\sigma_\gamma = \text{Std}\{e_i(t)\}$ is computed. a) No delay, $\sigma_\gamma = 0.43$ b) delay of one slot, $\sigma_\gamma = 0.98$ c) same as in b), but with TDC in operation, $\sigma_\gamma = 0.64$.

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