Some Aspects of an Optimization Approach to Iterative Learning Control

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Some Aspects of an Optimization Approach to Iterative Learning Control¹

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Abstract

An optimization approach to Iterative Learning Control (ILC) is considered. The ILC algorithm is formed by minimizing a quadratic criterion in the control error and input signal. A frequency domain interpretation of the derived updating algorithm is given. Experiments carried out on an ABB IRB 1400 are presented.

1 Introduction

Iterative Learning Control (ILC) is an established control method in situations where the purpose of the system is to carry out a particular operation repeatedly. The major area of application has been industrial robots, but also other types of batch processes have been studied. Surveys of the field are given in [1] and [2], and a collection of recent results in the area can also be found in [3]. In this paper the design of the ILC algorithm will be viewed as an optimization problem where the aim is to minimize the control error while using a reasonable input signal. Previous contributions in this area can be found in e.g. [4], [5], [6], [7], and [8].

The main contribution here is that a frequency domain interpretation of the optimization based ILC will be given. The updating of the ILC input signal is expressed as a filtering operation where the filters are obtained from the model of the system and the design variables in the optimization criterion. Furthermore, the optimization based ILC algorithm will be tested in real experiments carried out using an ABB IRB 1400. The paper is organized as follows. In Section 2 the formulation of the ILC problem in general is given and in Section 3 the problem is stated as an optimization problem and an updating algorithm is derived. In Section 4 the algorithm is interpreted in the frequency domain. Section 5 and Section 6 then contain a numerical example and experiments on an ABB IRB 1400 respectively.

Finally, some conclusions are given in Section 7.

2 Problem Formulation

The discussion will be restricted to linear SISO systems working in discrete time, described by

$$y_k(t) = T_r(q)r(t) + T_u(q)u_k(t) + T_v(q)v_k(t) \quad (1)$$

where $u_k(t)$ and $y_k(t)$ denote the ILC input signal and the output signal respectively. Furthermore $r(t)$ and $v_k(t)$ denote the desired output (reference) signal and a vector of disturbance signals respectively. The subscript k denotes iteration number and equation (1) implies that the reference signal is the same in all iterations while the other signals will change from iteration to iteration. The disturbance signal $v_k(t)$ can however be composed of some repeatable component, typically a load disturbance, and some component that has random character, typically a measurement disturbance. All signals are defined on a finite time interval $t = 0, \dots, N$. Finally $T_r(q)$, $T_u(q)$, and $T_v(q)$ are stable discrete time filters.

The formulation in equation (1) is taken from [9]. One example is this structure is open loop control where $T_r(q) = T_v(q) = 0$ and $T_u(q) = G(q)$, where $G(q)$ is the transfer operator of the system to be controlled. Another example is a closed loop system operating under both feed-back and feed-forward control as depicted in Figure 1. The signals $d_k(t)$ and $n_k(t)$ denote load and measurement disturbances respectively. Figure 1 also describes the situation in which the experiments presented in Section 6 have been carried out, i.e. ILC is used as a complement to the conventional robot control system. Equation (1) also covers the common case when the ILC signal $u_k(t)$ is used as a feed-forward signal and is added to the outputs of F and F_f .

Considering linear operations a general formula for updating the input signal $u_k(t)$ is given by

$$u_{k+1}(t) = Q(q)(u_k(t) + L(q)e_k(t)) \quad (2)$$

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Figure 1: An example of a realization of the system in equation (1).

where $e_k(t) = r(t) - y_k(t)$ and $Q(q)$ and $L(q)$ are linear, possibly non-causal, filters. There are several aspects that have to be considered when $Q(q)$ and $L(q)$ are selected. First, they have to be chosen such that the ILC algorithm converges with reasonable convergence properties. Second, they have to be chosen such that the algorithm converges even if the true system differs from the model used for the design of $Q(q)$ and $L(q)$, i.e. sufficient robustness properties have to be ensured. Third, provided that the algorithm converges the filters have to be chosen such that the final error is as small as possible. There are of course other aspects of the filter design but the three factors listed above are the most important ones. The filter $Q(q)$ has a robustifying effect and an early use of this property is found in [10]. More or less systematic methods for the design of $Q(q)$ and $L(q)$ can be found in the literature, and an example of a systematic procedure is given in [11] where methods for design of robust control systems are applied. It should also be noted that $u_{k+1}(t)$ only depends on $e_k(t)$ and not on $e_{k+1}(t)$, which is the case in some papers.

3 Algorithm Derivation

The algorithm derivation presented here is not new but included for completeness. Introduce the vectors

$$\mathbf{Y}_k = (y_k(0), \dots, y_k(N))^T \quad (3)$$

$$\mathbf{U}_k = (u_k(0), \dots, u_k(N))^T \quad (4)$$

$$\mathbf{V}_k = (v_k(0), \dots, v_k(N))^T \quad (5)$$

$$\mathbf{R} = (r(0), \dots, r(N))^T \quad (6)$$

and

$$\mathbf{E}_k = \mathbf{R} - \mathbf{Y}_k \quad (7)$$

Using these notations the system can be described by the equation

$$\mathbf{Y}_k = \mathbf{T}_r \mathbf{R} + \mathbf{T}_u \mathbf{U}_k + \mathbf{T}_v \mathbf{V}_k \quad (8)$$

where $\mathbf{T}_u$ is a matrix formed by the impulse response coefficients of the transfer operator $T_u(q)$, i.e.

$$\mathbf{T}_u = \begin{pmatrix} t_0 & 0 & 0 & \dots & 0 \\ t_1 & t_0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ t_N & t_{N-1} & \dots & t_1 & t_0 \end{pmatrix} \quad (9)$$

and $\mathbf{T}_r$ and $\mathbf{T}_v$ are defined analogously. It should be noted that even though t_0 is included in $\mathbf{T}_u$ the derivation is not restricted to relative degree zero.

This representation of the system can also be found in e.g. [5] and [8]. In equation (9) the matrix elements along the diagonals are constant, but in case the system dynamics change during one iteration it is straightforward to let the coefficients in $\mathbf{T}_u$ vary along the diagonals. See e.g. [7]. It is however assumed that the same $\mathbf{T}_u$ is valid in each iteration.

It is also worth noticing that a system representation similar to (8) is used in [12], where a so called run-to-run control problem is considered. Then the “input” vector $\mathbf{U}$ contains (corrections of) recipe variables and the “output” vector $\mathbf{Y}$ contains (normalized) product quality variables.

Consider now the criterion

$$\mathbf{J} = \mathbf{E}_{k+1}^T \mathbf{W}_e \mathbf{E}_{k+1} + \mathbf{U}_{k+1}^T \mathbf{W}_u \mathbf{U}_{k+1} \quad (10)$$

where $\mathbf{W}_e$ and $\mathbf{W}_u$ are weight matrices determining the trade off between performance and input energy. The weight matrices can be used for both time and frequency weighting. Criteria like (10) can also be found in e.g. [5] and [8]. In order to achieve an iterative procedure the criterion is minimized subject to the constraint

$$(\mathbf{U}_{k+1} - \mathbf{U}_k)^T (\mathbf{U}_{k+1} - \mathbf{U}_k) \leq \delta \quad (11)$$

Equation (8) implies that

$$\mathbf{E}_{k+1} = (\mathbf{I} - \mathbf{T}_r) \mathbf{R} - \mathbf{T}_u \mathbf{U}_{k+1} - \mathbf{T}_v \mathbf{V}_{k+1} \quad (12)$$

and for simplicity the discussion will be confined to the case when $\mathbf{T}_r = 0$ and $\mathbf{T}_v = 0$, i.e.

$$\mathbf{E}_{k+1} = \mathbf{R} - \mathbf{T}_u \mathbf{U}_{k+1} \quad (13)$$

where $\mathbf{T}_u$ denotes a nominal model of the system.

Introduction of a Lagrange multiplier, differentiation, and use of equation (13) gives

$$\mathbf{U}_{k+1} = (\mathbf{W}_u + \lambda \cdot \mathbf{I} + \mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u)^{-1} (\lambda \mathbf{U}_k + \mathbf{T}_u^T \mathbf{W}_e \mathbf{R}) \quad (14)$$

Using

$$\mathbf{R} = \mathbf{E}_k + \mathbf{T}_u \mathbf{U}_k \quad (15)$$

the equation can be reformulated into

$$\mathbf{U}_{k+1} = \mathbf{Q}(\mathbf{U}_k + \mathbf{L}\mathbf{E}_k) \quad (16)$$

where

$$\mathbf{Q} = (\mathbf{W}_u + \lambda \cdot \mathbf{I} + \mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u)^{-1} (\lambda \cdot \mathbf{I} + \mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u) \quad (17)$$

and

$$\mathbf{L} = (\lambda \cdot \mathbf{I} + \mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u)^{-1} \mathbf{T}_u^T \mathbf{W}_e \quad (18)$$

The updating matrices $\mathbf{Q}$ and $\mathbf{L}$ hence depend on the nominal model $\mathbf{T}_u$ which is considered to be given and the weight matrices $\mathbf{W}_u$ and $\mathbf{W}_e$. The Lagrange multiplier λ is not computed explicitly but instead used as a design variable.

A number of interesting observations can be made by studying equations (16) - (18) closer. First, by putting $\mathbf{W}_u = 0$, i.e. removing the penalty on the input signal equation (17) implies that $\mathbf{Q} \equiv \mathbf{I}$, i.e. an update equation without any weighting of the previous input signal. This updating is also obtained by minimizing a criterion of the type

$$\mathbf{J} = \mathbf{E}_{k+1}^T \mathbf{E}_{k+1} + \mu (\mathbf{U}_{k+1} - \mathbf{U}_k)^T (\mathbf{U}_{k+1} - \mathbf{U}_k) \quad (19)$$

as studied in e.g. [6]. Since the use of $\mathbf{Q} \neq \mathbf{I}$ can have a robustifying property this is an important difference. Second, by instead putting $\lambda = 0$, i.e. removing the constraint on the update step $\mathbf{U}_{k+1}$ becomes

$$\mathbf{U}_{k+1} = (\mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u + \mathbf{W}_u)^{-1} \mathbf{T}_u^T \mathbf{W}_e \mathbf{R} \quad (20)$$

i.e. the computation of the input vector becomes a one-step procedure. Third, choosing both $\mathbf{W}_u = 0$ and $\lambda = 0$ results in the choice (assuming that the inverse exists)

$$\mathbf{U}_{k+1} = \mathbf{T}_u^{-1} \mathbf{R} \quad (21)$$

In equation (16) the error signal is formed using the nominal model of the system, while in real use the actual error signal from the system is used. Introducing

$$\mathbf{E}_k^0 = \mathbf{R} - \mathbf{Y}_k^0 \quad (22)$$

where $\mathbf{Y}_k^0$ is the output from real system leads to

$$\mathbf{U}_{k+1} = \mathbf{Q}(\mathbf{U}_k + \mathbf{L}\mathbf{E}_k^0) \quad (23)$$

where $\mathbf{Q}$ and $\mathbf{L}$ are given by (17) and (18). The convergence properties of equation (23) are determined by how well the nominal model describes the true system. Assuming that

$$\mathbf{Y}_k^0 = \mathbf{T}_u^0 \mathbf{U}_k \quad (24)$$

it is shown in [5] that the algorithm converges provided that

$$\bar{\sigma}(\mathbf{T}_u - \mathbf{T}_u^0) < \frac{2\rho}{\sqrt{\rho + \lambda}} \quad (25)$$

where here $\mathbf{W}_u = \rho \mathbf{I}$ and $\bar{\sigma}$ denotes the largest singular value.

4 Frequency Domain Interpretation

The starting point when deriving a frequency domain interpretation of the updating equation (23) is to use equation (8)

$$\mathbf{Y} = \mathbf{T}_u \mathbf{U} \quad (26)$$

i.e. to express the filtering of a data vector as a matrix multiplication. Consider a discrete time system with pulse response t_l , $l = 0, \dots, \infty$ and transfer operator $T_u(q)$ defined by

$$T_u(q) = \sum_{l=0}^{\infty} t_l q^{-l} \quad (27)$$

where q denotes the shift operator. Applying an input signal $u(t)$ which is zero for $t < 0$ the output can be written

$$y(t) = \sum_{l=0}^t t_l u(t-l) \quad (28)$$

i.e.

$$y(t) = T_u(q)u(t) \quad (29)$$

Now, provided that the inverse exists the relationship

$$\mathbf{Y} = \mathbf{T}_u^{-1} \mathbf{U} \quad (30)$$

corresponds to the filtering

$$y(t) = \frac{1}{T_u(q)} u(t) \quad (31)$$

Since $\mathbf{T}_u$ is triangular the inverse exists if and only if $t_0 > 0$, i.e. when the system has relative degree zero. Even though $\mathbf{T}_u^{-1}$ exists it has undesirable behavior when $T_u(q)$ has zeros outside the unit circle, since then the elements of $\mathbf{T}_u^{-1}$ become very large. Furthermore the operation

$$\mathbf{Y} = \mathbf{T}_u^T \mathbf{U} \quad (32)$$

corresponds to a non-causal filtering

$$\begin{aligned} y(t) &= t_0 u(t) + \dots + t_{N-t} u(N) = \\ &= \sum_{l=0}^{N-t} t_l u(t+l) = \sum_{l=0}^{N-t} t_l \left(\frac{1}{q}\right)^{-l} u(t) \approx T_u\left(\frac{1}{q}\right) u(t) \end{aligned} \quad (33)$$

Hence the operation

$$\mathbf{Y} = \mathbf{T}_u^T \mathbf{T}_u \mathbf{U} \quad (34)$$

can be interpreted as

$$y(t) = T_u(q) T_u\left(\frac{1}{q}\right) u(t) \quad (35)$$

or, in the frequency domain,

$$Y(e^{i\omega T}) = |T_u(e^{i\omega T})|^2 U(e^{i\omega T}) \quad (36)$$

i.e. a filtering operation that gives zero phase shift. One implementation of this operation is provided by the Matlab command `filtfilt`.

Now consider equation (23) with $\mathbf{Q}$ and $\mathbf{L}$ given by equations (17) and (18) with $\mathbf{W}_u = \rho \cdot \mathbf{I}$ and $\mathbf{W}_e = \mathbf{I}$. In the frequency domain this consequently corresponds to the filtering operation

$$U_{k+1}(z) = Q(z)(U_k(z) + L(z)E_k^0(z)) \quad (37)$$

where

$$Q(z) = \frac{\lambda + T_u(z)T_u(z^{-1})}{\rho + \lambda + T_u(z)T_u(z^{-1})} \quad (38)$$

and

$$L(z) = \frac{T_u(z^{-1})}{\lambda + T_u(z)T_u(z^{-1})} \quad (39)$$

It should be observed that all boundary effects in the beginning and the end of the data sequences are neglected in the derivation of the transfer function formulation. If the length of each movement N is large the frequency domain expressions however give a useful interpretation of the optimization based ILC algorithm.

Assuming that the nominal model equals the true system, i.e. $\mathbf{T}_u = \mathbf{T}_u^0$ the well known convergence criterion

$$\|Q(e^{i\omega})\| \|1 - L(e^{i\omega})T_u(e^{i\omega})\| < 1 \quad (40)$$

becomes

$$\frac{\lambda}{\lambda + \rho + |T_u(e^{i\omega})|^2} < 1 \quad (41)$$

This condition is satisfied for all ω provided that $\rho > 0$. Since the gain of the system tends to zero for high frequencies the left hand side of the inequality tends to $\lambda/(\lambda + \rho)$ for high frequencies. It is also of interest to compute the asymptotic error. Still assuming $\mathbf{T}_u = \mathbf{T}_u^0$, insertion of $U_{k+1}(z) = U_k(z) = U_\infty(z)$ and $E_k(z) = E_\infty(z)$ in equation (37) gives $\rho U_\infty(z) = T_u(z^{-1})E_\infty(z)$ which together with $E_\infty(z) = R(z) - T_u(z)U_\infty(z)$ yield

$$E_\infty(z) = \frac{\rho}{\rho + T_u(z)T_u(z^{-1})}R(z) \quad (42)$$

The value of ρ hence influences the magnitude of the error after the iterations have converged.

5 Numerical Example

In this section a numerical example will be given where the properties of the filters $L(z)$ and $Q(z)$ are investigated. The example will be based on the first order

discrete-time system (sampling interval $T = 0.004$ sec)

$$T_u(z) = \frac{0.07z^{-1}}{1 - 0.93z^{-1}} \quad (43)$$

presented in [9] and obtained by identification of an ABB IRB 1400. This model will also be used in the experiments in Section 6. The data used for the identification were collected while the robot was controlled by the conventional control system. This means that $T_u(z)$ in equation (43) is a model of the closed loop system from the ILC input signal $u_k(t)$ to the output signal $y_k(t)$. See also Figure 1.

The filter $L(z)$ is determined by the nominal model $T_u(z)$ and the parameter λ and in equation (39) it is seen that when λ decreases the filter tends towards the inverse of the nominal model. In Figure 2 and Figure 3 respectively the gain and phase of $L(z)$ are shown for different values of λ . For small values of λ the gain of $L(z)$ is close to the gain of $T_u^{-1}(z)$ while it for larger values obtains more low-pass character. The phase of $L(z)$ is however not affected by λ and it is equal to the phase of $T_u^{-1}(z)$.

The properties of $Q(z)$ are determined by both λ and ρ and the shape of $Q(z)$ depends on both their values and their relative size. Provided that the gain of $T_u(z)$ decreases as ω increases the gain of $Q(z)$ tends to $\lambda/(\lambda + \rho)$. This means that the value of ρ has to be of the same magnitude as λ in order to give a significant low pass behavior of $Q(z)$. When ρ is essentially smaller than λ the high frequency gain of $Q(z)$ is close to unity. This can be seen in Figure 5. The low frequency gain of $Q(z)$ does also depend on the low frequency gain of the nominal model $T_u(z)$. Some examples are given in Figure 4 and Figure 5 respectively.

6 Experiments

The experiments are carried out on the ABB IRB 1400 installed at the Division of Automatic Control, Linköping University. In the experiment a desired motion of axis one is specified, and the desired trajectory is given in Figure 6. It should be noticed that all signals correspond to the motor side, i.e. before the gear-box, since no measurements on the arm side are available.

The nominal model required in order to design the ILC updating algorithm is obtained by using system identification and this results in the first order model, equation (43), presented in the previous section. Further details concerning this step is given in [9]. Using this model the matrix $\mathbf{T}_u$ containing the impulse response coefficients is formed and then the matrices $\mathbf{Q}$ and $\mathbf{L}$ are computed. The design variables were chosen as $\lambda = 10^{-3}$ and $\rho = 10^{-2}$. Since the updating of the ILC

input signal in the experiments is carried out in Matlab a straightforward implementation of the updating formula (23) is feasible. In the experiments an interface between the robot control system S4C and Matlab is used. See also [9].

Four iterations are carried out and the maximum value of the absolute value of the error in each iteration is shown in Figure 7. The first value in the plot hence corresponds to the error obtained without any ILC input applied. Already after one iteration the error is almost completely eliminated, and after two iterations the level of the error corresponds to the resolution in the measurement device. Figure 8 shows the error signal at iterations 0, 1 and 2 respectively. In Figures 7 and 8 the errors are normalized.

7 Conclusions

An optimization approach to Iterative Learning Control has been studied. By minimizing a quadratic criterion including the control error and the input signal subject to a constraint on the change of the input signal an algorithm for updating the input signal is obtained. The updating has an appealing interpretation in the frequency domain where the filter properties coincide well with previous experience. The optimization approach has also been tested in experiment on an ABB IRB 1400 with very good results.

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Figure 2: Magnitude of $T_u^{-1}(z)$ (solid) and $L(z)$. Dashed: $\lambda = 10^{-3}$, dotted: $\lambda = 10^{-2}$, dash-dotted: $\lambda = 10^{-1}$

Figure 3: Phase of $T_u^{-1}(z)$ and $L(z)$.

Figure 6: Reference trajectory on the motor side

Figure 4: Gain of $Q(z)$ for $\lambda = 10^{-3}$. Solid: $\rho = 10^{-3}$, dashed: $\rho = 10^{-2}$, dash-dotted: $\rho = 10^{-1}$

Figure 7: Normalized maximum absolute value of the error in each iteration.

Figure 5: Gain of $Q(z)$ for $\lambda = 10^{-1}$. Solid: $\rho = 10^{-3}$, dashed: $\rho = 10^{-2}$, dash-dotted: $\rho = 10^{-1}$

Figure 8: Normalized error signal. Solid: $k = 0$ (without ILC input), Dashed: $k = 1$, Dash-dotted: $k = 2$.