On the Use of a Multivariable Frequency Response Identification Method in the Presence of Periodic Disturbances

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Abstract

A method for estimating the Multivariable Frequency Response Function (MFRF) is evaluated with respect to different disturbance properties and excitation signals. For the robot application, disturbances are mainly of a periodic nature, and Gaussian disturbance descriptions, which are often used in identification literature, do not give the same result. It is shown that the averaging technique over multiple periods does not work as well for periodic disturbances as for Gaussian disturbances. The chirp excitation signal might be a better choice than multisine to reduce the influence of periodic disturbances. The main reason for this is a varying operating point.

Keywords: identification, robotics, periodic disturbances, excitation signals

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I. Introduction

Robot manufacturers are today faced with increasing demands on the controller design. Two main reasons are the need for higher performance (due to new applications) and price reduction (gives poor mechanical properties). For the design of an advanced robot controller, accurate dynamic robot models are crucial.

To obtain an accurate robot model, it is insufficient to use nominal parameter values from manufacturers, and direct measurement of physical parameters is unrealistic due to the complexity of a robot. The solution is to use experimental robot identification, where the robot model is estimated from the response measured during a robot experiment.

The robot application gives many challenging problems, such as a multivariable non-linear system, oscillatory behavior, closed loop identification (to stay centered around the working point), disturbances, and non-linearities such as e.g. static friction. The identification quality could be ruined if these problems are not considered.

The aim here is to analyze the use of a multivariable frequency response function (MFRF) estimation method (referred to as MFRFE in the sequel), described in [3], for robot identification. In particular, the method is evaluated with respect to disturbances and excitation signals.

The MFRFE method is mainly used with periodic excitation signals. The main benefits from periodic excitation signals for the MFRFE method rely on the fact that the disturbances are random variables, independent over the periods (see for example Assumption 5 in [8]). For the robot application, this is not a realistic disturbance description. In reality, the disturbances are periodic and can be modeled as a sum of sinusoids where the frequencies are multiples of the motor velocity. This will be further described in Section IV. We will here make a comparative study of the MFRFE method with realistic periodic disturbances versus Gaussian disturbances, which, as far as the authors know, is what is mainly considered in the literature [7, 9]. As will be shown, the result is drastically different in the two cases.

The choice of input signal is crucial in all types of identification for the quality of the identified model. See for example [12] where parameter identification of rigid robots is considered. Here we will compare the use of multisine (sum of sinusoids) and chirp (frequency swept sinusoid) excitation signals. It is shown that the periodic disturbances gives further restrictions for the design of excitation signals.

The work presented is carried out only on simulation data. The robot simulation model corresponds to an experimental robot with a load capacity of 250 kg. The model is a linearized version of a non-linear state-space model with 20 states, describing the dynamics of axes 1, 2 and 3 (see Fig. 1) from applied motor torque to achieved motor position. The non-linear robot model is linearized at zero position and velocity, corresponding to the position in Fig. 1, and a Bode diagram from motor torque to motor speed can be seen in Fig. 2.

The paper is organized as follows. In Section II, the identification method is described. In Section III, the selection of excitation signals is treated, and Section IV describes the disturbance properties. The data collection is described in Section V, and the results are presented in Section VI. Finally, Section VII contains some conclusions.

II. The Identification Method

The identification method that will be used is described in [3] and it is an estimation method for a multivariable frequency response function (MFRF).

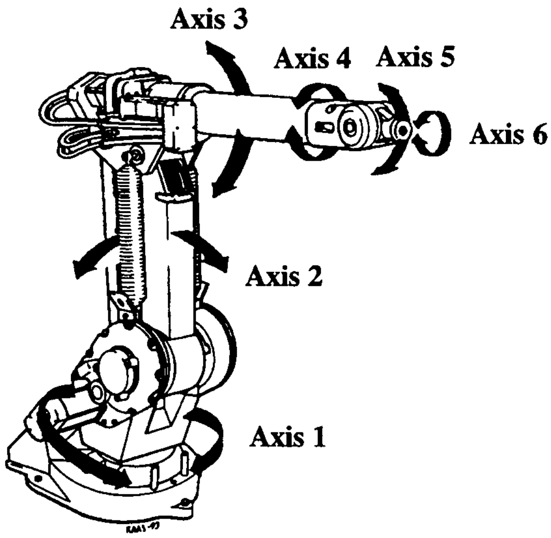


Fig. 1. The ABB manipulator IRB 1400.

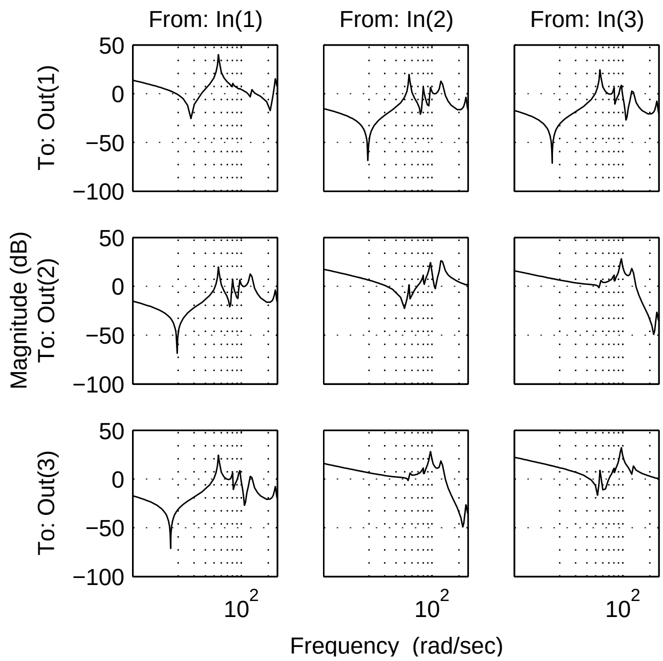


Fig. 2. Bode diagram of the transfer function from motor torque to motor speed.

Consider a discrete time multivariable system $G(\omega_k)$ with m inputs and p outputs. The input and output signals, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$, are sampled at time instants $t_n = nT_s,\ n = 0, 1, \ldots, N-1$, with T_s the sample time and $T_0 = NT_s$ the signal period. Let

$$U_N(\omega_k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} u(nT_s) e^{i\omega_k nT_s},$$

$$Y_N(\omega_k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(nT_s) e^{i\omega_k nT_s},$$

be the DFT of the sampled signals, where

$$\omega_k = k\frac{2\pi}{NT_s}\ , \quad k = 0, 1, \ldots, N-1.$$

If the sampled signals are periodic, the following linear mapping will hold exactly

$$Y_N(\omega_k) = G(\omega_k) U_N(\omega_k), \tag{1}$$

where $G(\omega_k) \in \mathbb{C}^{p\times m}$ is the MFRF.

Note that if the sampled signals are not periodic, the DFT will introduce leakage errors due to the limited time window and (1) will no longer hold exactly (see Section 2.2.2. in [9] for details). This is the main reason why only periodic excitation is considered.

To be able to extract $G(\omega_k)$ from data, at least m different (independent) experiments are needed. The relation between the input and output can then be written as

$$\mathbf{Y}_N(\omega_k) = G(\omega_k)\mathbf{U}_N(\omega_k),$$

where the m columns of $\mathbf{U}_N(\omega_k) \in \mathbb{C}^{m\times m}$ and $\mathbf{Y}_N(\omega_k) \in \mathbb{C}^{p\times m}$ correspond to the m different experiments. If $\mathbf{U}_N(\omega_k)$ has full rank, an estimate of $G(\omega_k)$ can be formed as

$$\hat{G}_N(\omega_k) = \mathbf{Y}_N(\omega_k)\mathbf{U}_N^{-1}(\omega_k).$$

If more than m experiments are carried out, $\mathbf{U}_N(\omega_k)^{-1}$ can be replaced by the pseudo-inverse.

Usually, input/output measurements are corrupted by disturbances. To reduce the variance of the estimate, the periodic excitation can be repeated M times, which gives [3]

$$\hat{G}_{N,M}(\omega_k) = \hat{\mathbf{Y}}_{N,M}(\omega_k)\hat{\mathbf{U}}_{N,M}^{-1}(\omega_k) = \\ = \left(\frac{1}{M}\sum_{i=1}^{M} \mathbf{Y}_N^{[i]}(\omega_k)\right)\left(\frac{1}{M}\sum_{i=1}^{M} \mathbf{U}_N^{[i]}(\omega_k)\right)^{-1}, \tag{2}$$

where $\mathbf{U}_N^{[i]}(\omega_k)$ and $\mathbf{Y}_N^{[i]}(\omega_k)$, $i = 1, 2, \ldots, M$ are the DFT of the M synchronized input/output records $\mathbf{u}(t_i + nT_s)$ and $\mathbf{y}(t_i + nT_s)$, $n = 0, 1, \ldots, N-1$, with $i = 1, 2, \ldots, M$, $t_{i+1} = t_i + T_0$.

In order to evaluate the quality of the estimated models, some quality measures are needed. Bias and variance expressions in the SISO case can be found in [8]. Here we will mainly look at the *relative error* of the estimate, defined as

$$\Delta G_{N,M}(\omega_k) = \frac{\hat{G}_{N,M}(\omega_k)}{G(\omega_k)} - 1, \tag{3}$$

and the *relative absolute error*, defined as

$$\Delta|G|_{N,M}(\omega_k) = \left|\frac{|\hat{G}_{N,M}(\omega_k)|}{|G(\omega_k)|} - 1\right|. \tag{4}$$

For MIMO systems, which is the case here, the relative error is calculated element-wise.

III. Excitation Signals

The selection of excitation signals is an important step in the design of good experiments. For a detailed treatment of different excitation signals, see e.g. Chapter 4 in [9].

What is an optimal excitation signal depends on the goal of the experiment. Here we will focus on the non-parametric approach and the energy should then be distributed to achieve a predefined accuracy in the frequency band of interest. The signals will have a specified maximum peak value, which makes it convenient to use the, so called, *crest factor* [9]. The crest factor of a signal is given by the ratio of the peak value of the signal to its effective root mean square (rms) value, where *effective* means that only the signal power in the frequency band of interest is used for the rms calculation. The crest factor gives an idea of the compactness of the signal. Signals with an impulsive behavior have a large crest factor and will, for a given peak value, inject much less power into the system than a signal with a small crest factor.

For the MFRF estimation method to work as well as possible, one should use periodic excitations [9]. Here we will make a comparison between chirp (also called swept sine) and multisine signals.

A signal of length T_0 is designed, and a periodic signal of length $T = MT_0$ is achieved by periodic continuation like

$$u(t + kT_0) = u_0(t), \quad 0 \le t < T_0, \quad k = 0, 1, \dots, M-1. \tag{5}$$

A *chirp signal* is a sinusoid with a frequency that changes continuously over a certain frequency band $f_1 \le f \le f_2$ like

$$u_0(t) = A\sin(2\pi f_1 t + \pi/T_0(f_2 - f_1)t^2 + \phi)\ , \quad 0 \le t \le T_0, \tag{6}$$

with amplitude A, and phase ϕ. In order to get a periodic signal, f_1 and f_2 must be a multiple of $f_0 = 1/T_0$ [9]. To get a continuous derivative also for $t = kT_0$, $(f_1 + f_2)/2$ must be a multiple of f_0. The chirp signal will have the same crest factor as a sinusoid, i.e. $\sqrt{2}A$.

The *multisine signal* can be written as

$$u_0(t) = \sum_{k=1}^{F} A_k \sin(2\pi f_k t + \phi_k), \tag{7}$$

with amplitudes A_k, phases ϕ_k, and frequencies $f_k = l_k f_0 = l_k/T_0$ with $l_k \in \mathbb{N}$. The phases ϕ_k are chosen to get a low crest factor. Here, we will use the Schroeder phases [11] in combination with an iterative optimization procedure described in [2], which is referred as the clipping algorithm in the literature.

As was mentioned previously, (1) will not hold exactly if the sampled signals are not periodic. Using a periodic excitation signal (for closed loop as reference value) will give periodic signals in steady state. However, for a poorly damped system like a robot, transients will make the required waiting time, T_w, long (see [10] for estimations of the required waiting time).

To reduce the transient effects, the total excitation signal

$$u_{tot}(t) = \begin{cases} u_w(t) = \tilde{u}_w(t + \Delta T) & \text{if } 0 \le t < T_w \\ u(t - T_w) & \text{if } T_w \le t < T + T_w \end{cases},$$

will be used, where $\tilde{u}_w(t)$ is defined similar to (5) with $M_w = \lceil T_w/T_0 \rceil$ periods of $u_0(t)$ and $\Delta T = M_w T_0 - T_w$.

IV. Disturbance Properties

Industrial robots usually have AC permanent magnet motors as actuators, which are extremely fast, compact, and robust. A drawback, however, is that the generated torque changes periodically with the rotor position. The resulting torque ripple is caused by distortion of the stator flux linkage distribution, variable magnetic reluctance at the stator slots, and secondary phenomena such as, e.g., the feeding power converter [6]. The ripple caused by the variable magnetic reluctance is proportional to the current, which we here approximate with the commanded torque, τ_c, from the robot controller (neglecting the fast power controller). Since the torque ripple is periodic in the rotor position φ, it can be modeled as a sum of sinusoids like [4]

$$\begin{aligned} v_\tau(t) = &\sum_{n \in \mathbb{N}_a} a_n \sin(n\varphi(t) + \phi_{a,n}) + \\ &+ \tau_c(t) \sum_{n \in \mathbb{N}_b} b_n \sin(n\varphi(t) + \phi_{b,n}), \end{aligned} \tag{8}$$

where the number of components in $\mathbb{N}_a$ and $\mathbb{N}_b$ depends on the specific motor type and the level of approximation. The applied torque τ can therefore be seen as a sum of the commanded torque τ_c and the input disturbance v_τ (see Fig. 3).

Measurements of the rotor position is normally obtained by using Tracking Resolver-to-Digital Converters [5]. It is shown in [5] that the position error, due to non-ideal resolver characteristics, can be be described as a sum of sinusoids like in

$$v_\varphi(t) = \sum_{n \in \mathbb{N}_c} c_n \sin(n\varphi(t) + \phi_{c,n}) + e_\varphi(t), \tag{9}$$

where $e_\varphi(t)$ is Gaussian noise, added to take into account measurement noise. v_φ is hereafter denoted output disturbance.

Numerical values which are considered to be relevant for the robot application (see e.g. [13]) can be seen in Table I. $e_\varphi(t)$ is chosen as Gaussian white noise with zero mean and $1.5 \cdot 10^{-10}$ variance.

V. Data Collection

A. Simulation Model

The comparison will be carried out only on simulation data. The robot simulation model was briefly described in

TABLE I

NUMERICAL VALUES FOR THE DISTURBANCES.

n	a_n (Nm)	$\phi_{a,n}$ (rad)
3	0.06	1.45
36	0.11	0.9
72	0.08	0.2
n	b_n (-)	$\phi_{b,n}$ (rad)
18	0.04	0
n	c_n (mrad)	$\phi_{c,n}$ (rad)
2	0.40	-1.84
10	0.20	0.81
12	0.22	2.94
48	0.18	2.94

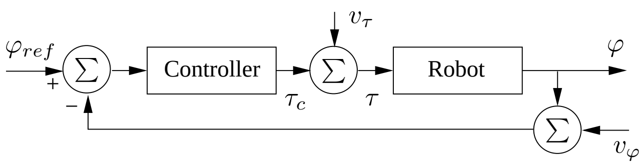


Fig. 3. Block diagram.

Section I. Some simplifying conditions are that the simulation model does not include non-linearities such as, e.g., backlash in the gear-box, static friction, and non-linear stiffness in the springs. These non-linearities would affect the estimation quality, which has been studied in ,e.g., [1] for black-box identification of an industrial robot.

The data collection will be carried out in closed loop, which is schematically depicted in Fig. 3. The periodic disturbances described in Section IV will be used according to Fig. 3. For comparison, Gaussian disturbances will be used as well, as will be further described in Section V-C.

Limitations according to real life experiments are imposed on the experiments, such as limitations in amplitude and bandwidth for the motor torque, speed, and acceleration.

B. Closed loop identification

An important question in closed loop identification is how the model quality, and in particular the bias, is affected by the feedback. In [8] there are bias expressions for the FRF estimate in the SISO case in the presence of correlated input/output errors. However it needs to be further investigated how the multivariable FRF estimate is affected by correlated periodic disturbances.

According to [7] there are three different approaches to identification in closed loop. Here we will use the direct approach, which means that the input/output data from the experiment are used for the identification, ignoring that the data are collected in closed loop and not using the reference signal. To be noted is that closed loop identification puts further restrictions on the the controller and excitation signal in order to get data that are informative enough, see [7] for details.

C. Experiment Design

A comparison between multisine and chirp excitation signals will be made, both for periodic disturbances (described in Section IV) and Gaussian disturbances. The chirp signal will be applied with two different phase angles (ϕ in (6)). Additionally, input and output disturbances are treated separately. This gives us a total of 12 different settings. To be noted is that only one realization will be used for each setting. The aim of this study is to show some principal characteristics, but more extensive studies, such as for example Monte Carlo simulations, must take place in order to quantify the differences.

The motor speed will be used as output in the identification, and the excitation signals are therefore applied as reference speed for the controller, which can be seen as a PI-controller for speed. Hence, φ_{ref} in Fig. 3 is the integral of the excitation signals (7) and (6).

Both reference speed signals have a peak value of 8 rad/s and their frequency contents in the interval $[1,\ 40]$ Hz. The multisine signal is generated using Schroeder phases and crest factor optimization with 100 iterations, giving a crest factor of 1.47. The chirp signal has a linearly swept frequency from 1 to 40 Hz, with a crest factor of 1.42. Each excitation signal has a signal period $T_0 = 10$ s, a waiting time $T_w = 20$ s, and $M = 10$ periods. The resulting signal amplitudes correspond to realistic real life experiments on a robot with comparable size.

For each setting, three experiments are carried out, where the same reference signal is applied to axes 1, 2, and 3, respectively (with zero reference for the other axes). This is not the optimal input design (see [3] for a discussion), but is sufficient for our purposes. The variance of the periodic disturbances is calculated for each of the three experiments and used as the variance for the corresponding experiments with Gaussian disturbances.

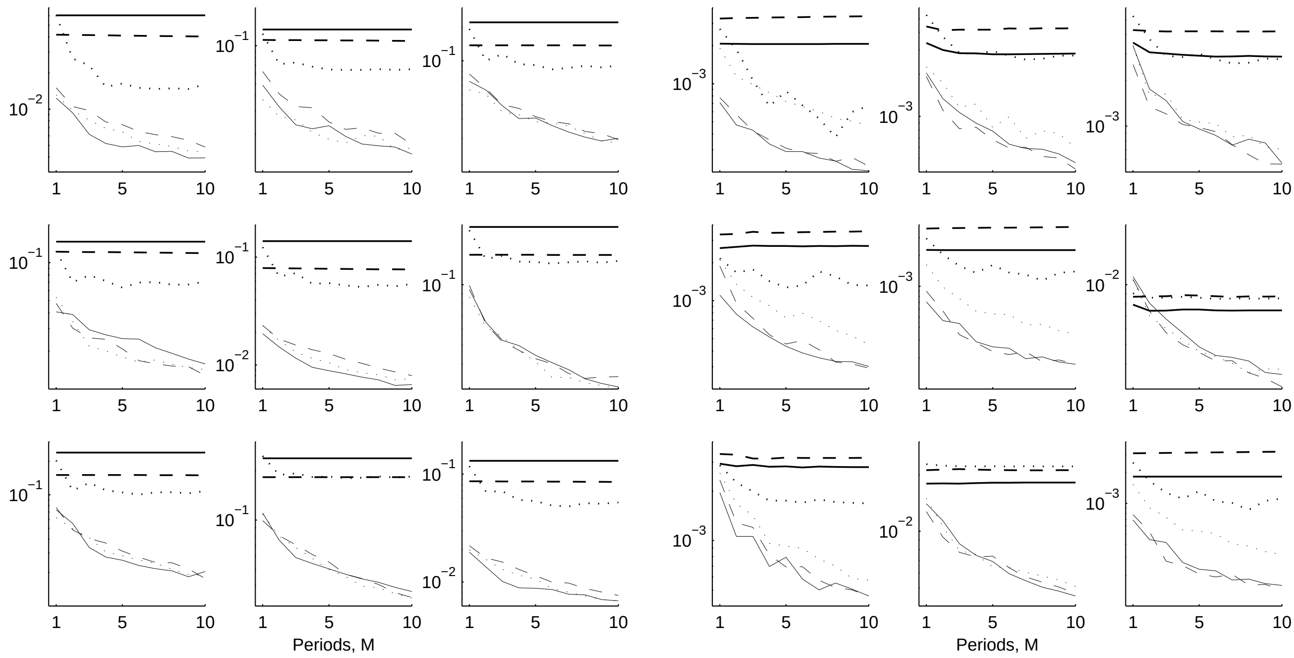
VI. RESULTS

The main point in this paper is to show that the identification method works quite differently in the presence of more realistic (periodic) disturbances. Input and output disturbances are treated separately to see their different influences more clearly. The result for input-output disturbances is similar to the input disturbance case.

To compare the influence of different excitation signals and disturbance descriptions, the relative absolute error, (4), is averaged over the excited frequency interval, Ω, like

$$\overline{\Delta|G|}_M = \frac{1}{N_\Omega} \sum_{\omega_k \in \Omega} \Delta|G|_M(\omega_k),$$

where N_Ω is the number of excited frequencies (N in (4) is omitted for easier notation). $\overline{\Delta|G|}_M$ is plotted for $1 \leq M \leq 10$ in Fig. 4 for input disturbance (left) and output disturbance (right). From these two plots, a number of conclusions can be made.



(a) Input disturbance

(b) Output disturbance

Fig. 4. Mean relative absolute error for different number of periods, M. A comparison between periodic (thick lines) and Gaussian (thin lines) disturbances using three different reference signals; multisine (solid), chirp with phase $\phi = 0$ (dotted), and chirp with phase $\phi = 1.89$ rad (dashed).

First, lets consider periodic disturbances (thick lines). One can clearly see that $\overline{\Delta|G|}_M$ depends on the excitation signal. For the input disturbance case, no improvement for increasing M can be seen using multisine and chirp with $\phi = 1.89$ rad. Using chirp with $\phi = 0$, $\overline{\Delta|G|}_M$ is actually decreasing with increasing M. The main difference between the results is due to a varying operating point for the chirp signal with $\phi = 0$, which will be explained in the next paragraph. For the output disturbance case, a slight improvement can be seen even for the multisine and chirp with $\phi = 1.89$ rad, which can be explained by the Gaussian noise $e_\varphi(t)$ in (9).

To simplify the notation, a convention like

$$v_\tau^{[k]}(t) = v_\tau(t + (k-1)T_0),\ 0 \le t < T_0,\ 1 \le k \le M,$$

(and similarly for the other signals), will be used in this section. Since the excitation signal is used as reference speed, its integral will be the reference position like

$$\varphi_{ref}^{[k]}(t) = \varphi_{ref}^{[k]}(0) + \tilde{u}_0(t),$$

where $\tilde{u}_0(t) = \int_0^t u_0(s)ds$ is introduced for easier notation. From this expression, we get the relation

$$\varphi_{ref}^{[k]}(0) = \varphi_{ref}(0) + (k-1)\tilde{u}_0(T_0).$$

Depending on the value of $\tilde{u}_0(T_0)$, each period will have different starting points. Lets consider the input disturbance case. The output disturbance case gives a similar discussion, except that the effects of $e_\varphi(t)$ in (9) will be reduced by the averaging technique. Using $\varphi(t) \approx \varphi_{ref}(t)$ in (8) gives an approximate expression for the input disturbance in period k like

$$v_\tau^{[k]}(t) \approx \sum_{n\in\mathbb{N}_a} a_n \sin\left(n\tilde{u}_0(t) + n(k-1)\tilde{u}_0(T_0) + \phi_{a,n}\right) + \\ +\tau_c(t) \sum_{n\in\mathbb{N}_b} b_n \sin\left(n\tilde{u}_0(t) + n(k-1)\tilde{u}_0(T_0) + \phi_{b,n}\right).$$

$n(k-1)\tilde{u}_0(T_0)$ can be seen as an additional phase shift for period k. The averaging in (2) gives that the disturbance part will be averaged like $\bar{v}_\tau(t) = \frac{1}{M}\sum_{k=1}^M v_\tau^{[k]}(t)$. If $\tilde{u}_0(T_0) = 0$, this averaging will have no effect. For multisine excitation, this is always the case since the frequencies are multiples of $1/T_0$. For the chirp excitation, $\tilde{u}_0(T_0)$ depends on the value of ϕ in (6). For $\phi = 1.89$ rad, $\tilde{u}_0(T_0) = 0$ and for $\phi = 0$, $\tilde{u}_0(T_0) \neq 0$. This gives a hint of a way of reducing the influence of periodic disturbances that needs to be further investigated. A varying operating point might reduce the influence from periodic disturbances.

Another interesting result is that the chirp and multisine gives different results for input and output disturbances, respectively (Compare the solid and dashed thick lines in Fig. 4). For input disturbances, the chirp signal is better, and for output disturbances, the multisine is better. However, chirp with phase $\phi = 0$ is the best for both types of disturbances (see the thick dotted lines in Fig. 4).

If the elements in $\overline{\Delta|G|}_M$ in Fig. 4 are compared with the elements of the Bode diagram in Fig. 2, one can notice the strange behavior that even though the true multivariable system is symmetric, $\overline{\Delta|G|}_M$ is not. This phenomenon needs further investigation.

Next, lets consider the Gaussian disturbances in Fig. 4 (thin lines). To be noted is that the disturbance variance may differ between the excitation signals, since the variance is selected to be the same as for the periodic disturbance in each case. However, for all three excitation signals, one can notice that $\overline{\Delta|G|}_M$ is decreasing for increasing M, which goes well in hand with variance expressions for SISO systems, which gives a standard deviation proportional to $1/\sqrt{M}$ [8].

VII. Conclusions and Future Work

A method for estimating the Multivariable Frequency Response Function (MFRF) has been evaluated with respect to different disturbance properties and excitation signals. For the robot application, disturbances are mainly of a periodic nature, and Gaussian disturbance descriptions, which are often used in identification literature, do not give the same result. The averaging technique over multiple periods does not work as well for periodic disturbances as for Gaussian disturbances. The chirp signal might be a better choice than the multisine to reduce the influence of periodic disturbances. The main reason for this is a varying operating point.

There are a number of aspects of the presented results that are subjects for future work. It would be interesting to verify the results in real life experiments. A varying operating point can also be further analyzed. Another possible solution to reduce the influence of periodic disturbances might be to estimate the disturbances according to [6], and compensate for the disturbances before applying the identification method. A further topic is to include non-linearities such as e.g. backlash in the gear-box, static friction, and non-linear stiffness in the springs. These non-linearities is present in real life experiments and would probably affect the quality of the estimated MFRF.

VIII. Acknowledgments

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