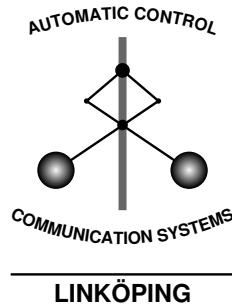


A Review of Time-Delay Estimation Techniques

Svante Björklund and Lennart Ljung

Control & Communication
Department of Electrical Engineering
Linköpings universitet, SE-581 83 Linköping, Sweden
WWW: <http://www.control.isy.liu.se>
E-mail: svabj@isy.liu.se

30th December 2003



Report no.: LiTH-ISY-R-2554

Technical reports from the Control & Communication group in Linköping are available at <http://www.control.isy.liu.se/publications>.



Avdelning, Institution
Division, department
Automatic Control
Department of Electrical Engineering

Datum
Date
2004-01-01

Språk
Language

Svenska/Swedish
 Engelska/English

Rapporttyp
Report: category

Licentiatavhandling
 Examensarbete
 C-uppsats
 D-uppsats
 Övrig rapport

ISBN

ISRN

Serietitel och serienummer **ISSN**
Title of series, numbering 1400-3902

LITH-ISY-R- 2554

URL för elektronisk version

<http://www.control.isy.liu.se>

Titel A Review of Time-Delay Estimation Techniques
Title

Författare Svante Björklund and Lennart Ljung
Author

Sammanfattning
Abstract

This paper reviews and evaluates suggested methods for estimating the time-delay of linear systems in automatic control applications. A classification of the methods according to the underlying principles is suggested. The evaluation, done by analyzing the estimates of the methods from extensive simulated data in open loop, shows that different classes of methods have different properties and are suitable in different cases. Some method are clearly inferior to others. Recommendations are given on how to choose estimation method and input signal.

Nyckelord
Keywords
time-delay, dead-time, estimation, system identification, linear systems, Laguerre, simulations, open loop, process industry

A Review of Time-Delay Estimation Techniques

Svante Björklund and Lennart Ljung

Control & Communication
Department of Electrical Engineering
Linköpings universitet, SE-581 83 Linköping, Sweden
WWW: <http://www.control.isy.liu.se>
E-mail: svabj@isy.liu.se, ljung@isy.liu.se

Abstract

This paper reviews and evaluates suggested methods for estimating the time-delay of linear systems in automatic control applications. A classification of the methods according to the underlying principles is suggested. The evaluation, done by analyzing the estimates of the methods from extensive simulated data in open loop, shows that different classes of methods have different properties and are suitable in different cases. Some methods are clearly inferior to others. Recommendations are given on how to choose estimation method and input signal.

1 Introduction

In this paper we will study the time-delay estimation (TDE) problem:

$$y(t) = G(p)u(t) + n(t) = G_r(p)u(t - \Delta t) + n(t),$$

where the system $G_r(p)$ is a SISO (single-input single-output) time-invariant linear rational transfer function. Often in signal processing applications, the system is restricted to be a constant [4, 13], but here $G_r(p)$ will be a transfer function with essential dynamics, typical in process industry, see e.g. [18, 31]. This means that both the open-loop and closed-loop cases are of interest and that we study TDE for SNRs (signal-to-noise ratio), input signals and systems, that are common in such applications.

In time-delay estimation, the objective can be either:

1. The time-delay estimate that makes the model approximate the true system “best” is wanted. What is “best” depends on the intended use of the

model. In automatic control the time-delay estimate can be a means to achieve a good model in the frequency band relevant to the control [7, 23], e.g. around the cross-over frequency. In [31] the *apparent time-delay* (the delay resulting from identifying a first order model with time-delay from the data) is used for control performance monitoring of PID control loops.

2. The *true time-delay* is desired. This is the case in “pure time-delay” estimation, diagnosis, radar range estimation [20, 30], direction of arrival estimation with array antennas [6, 15], signal averaging [11], etc.

In this paper we will evaluate the time delay estimates according to the second objective, since we have not determined any particular use of the estimate. One should keep in mind that the best model approximation in a restricted class of models, does not necessarily use the “true” time delay.

We consider it as an advantage if a method can estimate time-delays that also consists of fractions of the sampling interval. However, some methods can only estimate time-delays that are a multiple of the sampling interval. Sometimes such methods can be used to initialize other more “free” methods.

TDE is a much studied problem, with very many references in the literature. Yet, it cannot be said that there is a clear solution to the problem: A general agreement on which method is “best”. It is the purpose of this contribution to review and evaluate a number of suggested approaches. The review will be done by grouping different methods together into classes according to underlying principles. The evaluation will be done by analyzing the quality of the estimates from simulated data in open loop. Since the space in this article is limited, see the reports [2] for details about implementation, simulation setup, analysis and results. See [14, 17, 18, 31] for time-delay estimation in closed loop.

2 Time-delay estimation methods

2.1 Classification of methods

Most methods that have been suggested for time-delay estimation (both in control and signal processing) can be put into one of the following classes:

1. *Time-delay approximation model methods.* The input and output signals are represented in a certain basis and the time-delay is estimated from an approximation of the relation (a model) between the signals in this basis. The time-delay is not an explicit parameter in the model. Depending on the basis there are several subclasses:

- (a) *Time domain approximation methods.* The time-delay is the delay for the impulse response to start [2, 3, 19, 21]. Finding the peak of the cross correlation between input and output, which is a common method [13], is in principle the same thing.
- (b) *Frequency domain approximation methods.* The time-delay is estimated from the phase of the time-delay $e^{-i\omega T_d}$ [6, 11, 15, 18, 19].
- (c) *Laguerre domain approximation methods.* The time-delay is estimated from a relation between the input and output signals expressed in Laguerre functions [9, 10]. Also other bases for the signals are possible, e.g. Kautz functions.

There are two independent steps in these methods: 1) Estimate the approximation model. 2) Estimate the time-delay from the model.

2. *Explicit time-delay parameter methods.* The time-delay is an explicit parameter in the model.

- (a) *One-step explicit methods.* The time-delay and the other model parameters are estimated simultaneously [23, 26]. Estimating several models, e.g. ARX models, with different time-delays and choosing the best is also of this subclass [2, 31].
- (b) *Two-step explicit methods* [5, 29]. Alternating between estimating the time-delay and the other parameters.
- (c) *Sampling methods.* Utilizing the sampling process to derive an expression for the time-delay. For example, zero order hold (zoh) sampling of a system with subsample time-delays creates an extra zero [7].

3. *Area and moment methods* [1, 16]. These methods utilize relations between the time-delay and certain areas over or below the step response $s(t)$ and certain moments of the impulse response $h(t)$ (integrals of the type $\int t^n h(t) dt$). There are two independent steps: 1) Estimate the step or impulse response. 2) Estimate the time-delay from these responses.

4. Higher-order statistics (HOS) methods. Their main advantage is that noise with a symmetric probability distribution function, e.g. Gaussian, theoretically can be removed completely by

HOS [27]. In [28], bispectra and 3rd order moments are used and methods in the 2D time and frequency domains, similar to subclasses 1a and 1b, are presented. They assume $G_r = 1$.

2.2 Compared methods

2.2.1 Time domain approximation methods : The methods IDT and SDT use thresholds $h(t) = h_{\text{std}} \cdot \hat{y}_{\text{std}}(t)$, where h_{std} is a user selected constant and $\hat{y}_{\text{std}}(t)$ is the estimated standard deviation of the impulse or step response, respectively. Since h_{std} is difficult to chose manually to suit all cases it has been chosen by a simulation study to $h_{\text{std}} = 5$ for both IDT and SDT. For low SNR the estimated impulse and step responses are very noisy [2]. (See also [21].) In an attempt to mitigate this, the methods ICT and SCT uses CUSUM (cumulative sum) thresholding, which is a nonlinear averaging operation [12]. The user-selected parameters in CUSUM (relative drift ν_{std} and threshold h_{std}) are also difficult to select manually. They have been chosen (also by a simulation study) to $\nu_{\text{std}} = 1$ & $h_{\text{std}} = 3$ for ICT and to $\nu_{\text{std}} = 6$ & $h_{\text{std}} = 1$ for SCT. The used drift and threshold are then $\nu = \nu_{\text{std}} \cdot \hat{y}_{\text{std}}(0)$ and $h = h_{\text{std}} \cdot \hat{y}_{\text{std}}(0)$. It is easy to realize that the methods IDT, SDT, ICT and SCT have positive bias. Another approach to the thresholding is employed in [21] and its implementation is here called KURZ.

2.2.2 Frequency domain approximation methods: In the method LAGC a discrete-time Laguerre model, with pole $\alpha = 0.8$ and $N_l = 10$ Laguerre coefficients, of a continuous-time system is identified. The zeros of the model are translated to continuous-time. By comparing the dead-time with a Padé approximation, the dead-time may be estimated from the continuous-time non-minimum phase zeros [19]. This method can deliver complex valued estimates if the zeros of the model happens to be negative due to the noise. In an improved method, described in [14, 18], the discrete-time non-minimum phase zeros of the Laguerre model form the allpass part, which directly represents the dead-time. The dead-time is estimated by studying the slope at low frequencies of the phase of the allpass part. This method can give very incorrect estimates if the non-minimum phase zeros of the model are displaced due to the noise [2]. This method with a protection against displaced zeros [2] and $\alpha = 0.8$ and $N_l = 10$ we call LAGD. By exchanging the Laguerre model with a FIR (15 taps), ARX ($n_a = 4$, $n_b = 15$) or output error ($n_f = 2$, $n_b = 15$) model [22] we get the methods FIRD, ARXD and OED.

2.2.3 Laguerre domain approximation methods: Some methods in this subclass are described in [9, 10]. Several parameters must be selected

by the user, most importantly the Laguerre pole α and the number N_l of Laguerre coordinates to use. These must be selected to suit the input and output signals and the available execution time [2]. Not all input signal types can be used. Although the methods in [9, 10] assume $G(s) = 1 \cdot e^{-sT_d}$, we have applied them to more general systems. The method FIS uses $\alpha = 0.955$ and $N_l = 150$.

2.2.4 One-step explicit methods: The methods IPC1 and IPC2 use the continuous-time models $G(s) = \frac{K}{1+sT}e^{-sL}$ and $G(s) = \frac{K}{(1+sT_1)(1+sT_2)}e^{-sL}$, respectively. The time-delay is a continuous parameter, and all parameters are estimated by a prediction error/maximum likelihood method, using iterative search [23]. It is well known that the objective function may have many local minima in this case, [8, 26, 29], so the initialization of the parameters must be done with great care. Here we use an initialization method implemented in [24], based on ARXS (see below), followed by a global search for best time delay and model zero for fixed poles, followed by local Gauss-Newton search for all free parameters. Other ideas how to handle the problem of local minima are described in [8].

The methods OES, ARXS and PFAS employ the discrete-time model structures OE, ARX and ARX, respectively. The model orders [22], $(n_f = 2, n_b = 1)$, $(n_a = 10, n_b = 5)$ and $(n_a = 10, n_b = 1)$, were chosen by a simulation study. Several models of each model structure with different time-delays are estimated by the prediction error method (PEM) [22] and the best is chosen. PFAS uses prefiltering of the data to resemble the OE model structure [2].

2.2.5 Two-step explicit methods : ELNA is a recursive discrete-time two-step method [5].

2.2.6 Area and moment methods: In [1] some area and moment methods that use measured step and impulse responses are described. Two of these methods are implemented in AREA and MOM but with estimated step and impulse responses.

3 Simulations

3.1 Simulation setup

A factorial experiment (several factors varied simultaneously) with simulated signals in open loop was performed in MATLAB. The signal-to-noise ratio (SNR), measured at the system output, was either 1 or 100. For each factor level combination, 1024 trials or repetitions were conducted. The noise $n(t)$ was white and Gaussian. The sampling interval was $T_s = 1$. The used input signals had a length of 500 samples and were:

- White (RBS 0-100%) or narrowband (RBS 10-30%, most energy between 10% and 30% of the Nyquist frequency) random binary input signals. These input signals are common in system identification if the input signal can be chosen freely.
- Step input signals in the form [`zeros(50,1); ones(150,1); -ones(150,1); zeros(150,1)`] (MATLAB code). Steps are common when we cannot choose the input signal, e.g. when identifying during normal operation, but are restricted to utilize set-point changes.

All systems were of the form $G_j(s) = e^{-9s} \cdot \bar{G}_j(s)$. \bar{G}_1 had poles -0.1 & -1 , no zeros and DC gain 1 (a slow second order system). \bar{G}_2 had poles -1 & -10 , no zeros and DC gain 1 (a fast second order system). \bar{G}_5 had poles $-0.1, -0.3, -0.6$ & -1 , zeros -0.4 & -0.9 and DC gain 1 (a fourth order system with real poles). \bar{G}_6 had poles $-0.1(1 \pm i)/\sqrt{2}$ & $-(1 \pm i)/\sqrt{2}$, zeros -0.4 & -0.9 and DC gain 1 (a fourth order system with complex poles). For all the systems the time delay was 10 after the (zero order hold) sampling.

3.2 Analysis methods

In order to draw conclusions we have studied the time-delay estimates themselves and their RMS error, bias and variance. We have sometimes also used ANOVA and confidence intervals for pair-wise comparisons [25]. We will in this paper only present graphs of the RMS error (in number of sampling intervals).

Many methods sometimes fail and return estimates with negative, large positive, complex or NaN values. To handle this, complex and NaN values and values ≤ 0 or ≥ 20 are replaced with the value 20. This will give a large error for these estimates as the true time delay is 10. The maximum RMS error will be 10.

Then, the 90%, 95% or 100% best estimates are retained. The motivation for removing the worst estimates is that a good implementation should achieve the resulting performance, e.g. by detecting failures in the optimization and restarting it with a different start value. Then the RMS error is computed. Shown in the graphs is the RMS error averaged over different factor level combinations. Keep in mind that the presented RMS values only are estimates of the “true” ones.

3.3 Simulation results

Figure 1 shows the average (over all factors) RMS estimation error for all tested methods when the 90%, 95% and 100% best estimates are retained. The methods OES and PFAS are the best methods when 100% are used. For 90%-95%, they are challenged by IPC2. OES and PFAS are better than ARXS. IPC2 is better than IPC1. IPC2 mostly gives very accurate estimates

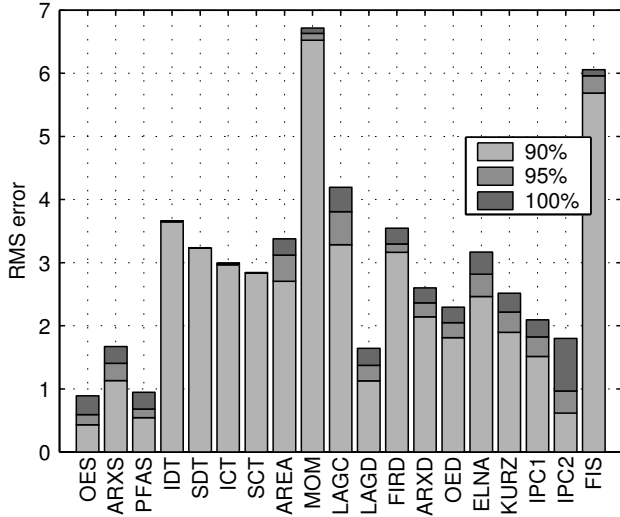


Figure 1: RMS error (in sampling intervals) for different methods when the 90%, 95% or 100% best estimates are retained. Average RMS error over SNRs, input signal types and systems. Maximum RMS error is 10 according to Section 3.2. See Table 1 for where the method names OES, ARXS, etc. are described.

but sometimes fails. It is favored by only retaining the 90%-95% best estimates. LAGD also belongs to the best methods. The methods LAGC, IDT, SDT, ICT, SCT, AREA, MOM and FIS often fail.

Testing the methods on 10^{th} order random systems gives similar results [2]. The RMS error is for most methods somewhat higher than for the fixed systems used in Figure 1. The RMS error of ARXS, FIRD, ELNA and IPC1 seems to be nearly unchanged. The CUSUM and area methods have a much higher RMS error, probably because these implementations are not adapted to all types of systems. Also LAGD has a much higher RMS error and is on the same level as FIRD, ARXD and OED.

Figure 2 displays the RMS error for different combinations of input signal and SNR (*cases*) when the 95% best estimates are used. Here we can see which method to choose when the input signal and/or the SNR is given. We can also choose the best input signal. We see that RBS signals are better than step signals. For high SNR there are more methods among the best than for low SNR. The discrete-time one-step methods OES and PFAS belong to the best methods in all cases. The time domain approximation methods are very inaccurate for step inputs but can be really accurate for RBS and high SNR. The area and moment methods and LAGC are not good in any case. The other frequency domain approximation methods (LAGD, FIRD, ARXD and OED) and ELNA are neither among the best nor the worst methods for any case except for LAGD, which is among

Methods	Section
OES, ARXS, PFAS	2.2.4
IDT, SDT, ICT, SCT	2.2.1
AREA, MOM	2.2.6
LAGC, LAGD, FIRD, ARXD, OED	2.2.2
ELNA	2.2.5
KURZ	2.2.1
IPC1, IPC2	2.2.4
FIS	2.2.3

Table 1: Method names and where they are described.

the best for steps with high SNR. The method IPC2 is among the best methods for steps and with low SNR for RBS signals. For high SNR and RBS the method IPC2 is beaten by several methods. The method IPC1 is not as good as IPC2 for any case. The method FIS is very inaccurate for RBS signals but among the best for step input signals. It is especially good for low SNR.

4 Discussion

Since OES and PFAS have the correct model structure (output error) they are better than ARXS. The methods IDT, SDT, ICT and SCT often miss to detect, especially for low SNR, because of noisy and uncertain impulse and step response estimates [2].

The method LAGD performs well in most cases (but not best) and seldom really bad. Two reasons for LAGD being better than FIRD, ARXD and OED are probably that typical impulse responses can be described well by Laguerre functions and that the model orders of the latter methods are not optimal. The results in [14] for LAGD is in agreement with our results. LAGC often fails, probably, because it has no protection against noise-corrupted zeros. The results in [14] for LAGC is better than our results. Perhaps has the implementation in [14] some protection against bad zeros.

IPC2 is better than IPC1 as it has a more suitable model structure for the used systems. Note however that both these methods work on a lower order model than some of the tested systems, and therefore have some inherent bias.

AREA and MOM often give very inaccurate estimates due to poor estimates of step and impulse responses [2, 16]. These methods would probably perform better with measured step and impulse responses as in [1]. Another improvement is described in [16].

The method FIS often give very poor estimates. This is probably due to its inability to describe certain signals

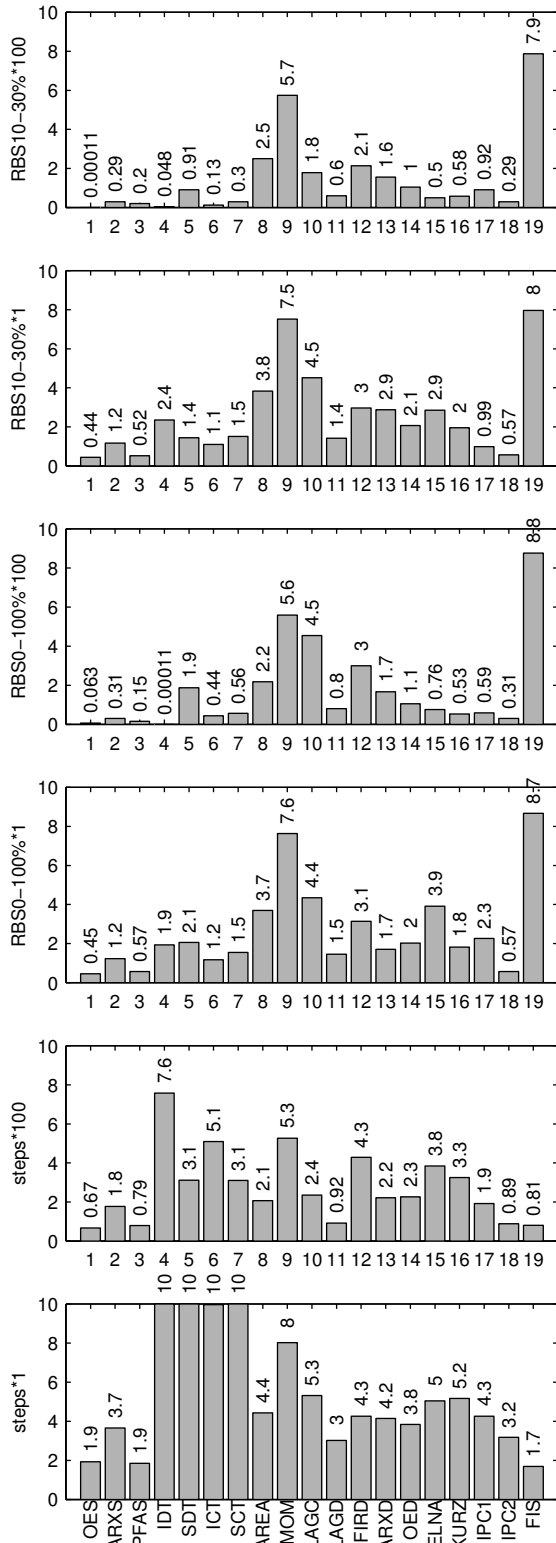


Figure 2: RMS error for different methods when the 95% best estimates are retained. Six bar plots for six combinations of input signal and SNR. Average RMS error over the systems. Maximum RMS error is 10 according to Section 3.2. See Table 1 for where the method names OES, ARXS, etc. are described.

in the Laguerre domain [2]. On the other side, FIS performs astonishingly well for steps, especially at low SNR. Perhaps it would be even better with more Laguerre coefficients. The used number, 150, was on the limit to be too computer intensive. This method apparently works for dynamic systems despite it is derived for $G_1 = 1$.

5 Conclusions

We have made a classification of existing time-delay estimation methods according to underlying principles.

Different classes have different properties and are suitable in different cases. Some methods are, however, clearly inferior to others. The winner in average with respect to estimation quality is OES. If sub-sample time-delay estimates are needed, the best method is IPC2 but keep in mind that the model structure should be the same as the true system.

Recommendations for the choice of input signal and estimation methods for the best estimation quality are:

- If you cannot use other input signals than steps: For high SNR use OES, PFAS, LAGD, IPC2 or FIS. If the SNR is low use only OES, PFAS or FIS. For subsample time-delay estimates use IPC2.
- If you can choose the input signal, use RBS (Random Binary) signals. For high SNR the best methods are OES and IDT. There are however more good methods. If subsample time-delay estimates are desired, use IPC2. For low SNR use OES, PFAS or IPC2. If the SNR is unknown, use only OES (or IPC2 for sub-sample time-delays).

Finally, we may note that our simulation study is somewhat unfair to IPC2 since the true time delay was a multiple of the sampling interval, and since IPC2 actually estimates a low order approximation of the true system (cf objective 1 in the Introduction) and does not primarily deal with the time delay.

References

[1] K. Åström and T. Hägglund. *PID Controllers: Theory, Design and Tuning*. Instrument Society of America, 2nd edition, 1995. ISBN 1-55617-516-7.

[2] S. Björklund. A collection of technical reports on time-delay estimation. Technical Report LiTH-ISY-R-2467, LiTH-ISY-R-2513, LiTH-ISY-R-2525, LiTH-ISY-R-2536, LiTH-ISY-R-2537, LiTH-ISY-R-2538, Department of Electrical Engineering, Linköping University, SE-581 83 Linköping, Sweden, 2003.

- [3] C. Carlemalm, S. Halvarsson, T. Wigren, and B. Wahlberg. Algorithms for time delay estimation using a low complexity exhaustive search. *IEEE Trans. Automatic Control*, 44(5):1031–1037, May 1999.
- [4] G. C. Carter et al. Special issue on time delay estimation. *IEEE Transactions on Acoustics, Speech and Signal Processing*, 29(3), June 1981. pt 2.
- [5] A. Elnaggar, G. A. Dumont, and A.-L. Elshafei. Recursive estimation for system of unknown delay. In *Proceedings of the 28th Conference on Decision and Control*, Tampa, Florida, USA, December 1989.
- [6] J. Falk, P. Händel, and M. Jansson. Direction finding for electronic warfare systems using the phase of the cross spectral density. In *RadioVetenskap och Kommunikation (RVK)*, pages 264–268, June 2002.
- [7] G. Ferretti, C. Maffezzoni, and R. Scattolini. Recursive estimation of time delay in sampled systems. *Automatica*, 27(4):653–661, 1991.
- [8] G. Ferretti, C. Maffezzoni, and R. Scattolini. On the identifiability of the time delay with least-squares methods. *Automatica*, 32(3):449–453, 1996.
- [9] B. R. Fischer. *System Identification in Alternative Shift Operators with Applications and Some Other Topics*. Phd thesis LTU-DT-99/18-SE, Department of Computer Science and Electrical Engineering, Luleå University of Technology, Luleå, Sweden, August 1999.
- [10] B. R. Fischer and A. Medvedev. Laguerre domain estimation of time delays in narrowband ultrasonic echoes. In *14th Triennial IFAC World Congress*, pages 361–366, Beijing, China, July 1999.
- [11] A. Grennberg and M. Sandell. Estimation of sub-sample time delay differences in narrowband ultrasonic echoes using the hilbert transform correlation. *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, 41(5):588–595, September 1994.
- [12] F. Gustafsson. *Adaptive Filtering and Change Detection*. Wiley, 2000. ISBN 0-471-49287-6.
- [13] A. Hero, H. Messer, J. Goldberg, et al. Highlights of statistical signal and array processing. *IEEE Signal Processing Magazine*, pages 21–64, September 1998.
- [14] A. Horch. *Condition Monitoring of Control Loops*. Phd thesis TRITA-S3-REG-0002, Dep. of Signals, Sensors and Systems, Royal Institute of Technology, Stockholm, Sweden, 2000.
- [15] A. W. Houghton and C. D. Reeve. Direction finding on spread spectrum signals using the time-domain filtered cross spectral density. *IEE Proceedings - Radar, Sonar and Navigation*, 144(6):315–320, December 1997.
- [16] A. Ingimundarson. *Dead-Time Compensation and Performance Monitoring in Process Control*. Phd thesis, Dep. of Automatic Control, Lund Institute of Technology, Lund, Sweden, 2003.
- [17] A. J. Isaksson, A. Horch, and G. A. Dumont. Event-triggered deadtime estimation – comparison of methods. In *Control Systems 2000*, pages 209–215, Victoria, Canada, 5 2000.
- [18] A. J. Isaksson, A. Horch, and G. A. Dumont. Event-triggered deadtime estimation from closed-loop data. In *Proceedings of American Control Conference*, Arlington, VA, USA, June 2001.
- [19] M. Isaksson. A comparison of some approaches to time-delay estimation. Master’s thesis ISRN LUTFD2/TFRT-5580-SE, Dep. Automatic Control, Lund Institute of Technology, Sweden, 1997.
- [20] S. Kingsley and S. Quegan. *Understanding Radar Systems*. McGraw-Hill, 1992. ISBN 0-07-707426-2.
- [21] H. Kurz and W. Goedecke. Digital parameter-adaptive control of processes with unknown dead time. *Automatica*, 17:245–252, January 1981.
- [22] L. Ljung. *System Identification: Theory for the User*. Prentice-Hall, 2nd edition, 1999.
- [23] L. Ljung. Identification for control: Simple process models. In *Proceedings of the 41st IEEE Conference on Decision and Control*, pages 4652–4657, Las Vegas, Nevada, USA, December 2002.
- [24] L. Ljung. *System Identification Toolbox for use with MATLAB. Version 6*. The MathWorks, Inc, Natick, MA, 6th edition, 2003.
- [25] D. C. Montgomery. *Design and Analysis of Experiments*. Wiley, 1997. ISBN 0-471-15746-5.
- [26] P. Nagy and L. Ljung. Estimating time-delays via state-space identification methods. In *Preprints 9th IFAC Symposium on System Identification and System Parameter Estimation*, pages 1141–1144, Budapest, Hungary, Jul 1991.
- [27] C. L. Nikias and J. M. Mendel. Signal processing with higher-order spectra. *IEEE Signal Processing Magazine*, pages 10–37, July 1993.
- [28] C. L. Nikias and R. Pan. Time delay estimation in unknown gaussian spatially correlated noise. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 36(11):1706–1714, November 1988.
- [29] R. Pupeikis. Recursive estimation of the parameters of linear systems with time delay. In *Proc. 7th IFAC/IFORS Symp. Identification and System Parameter Estimation*, pages 787–792, York, UK, July 1985.
- [30] A. Sume. Kursmaterial i radarteori. Technical Report FOA-D-95-00108-3.3-SE, Department of Sensor Technology, Swedish Defence Research Establishment, Box 1165, 581 11 Linköping, Sweden, April 1995. In Swedish.
- [31] A. P. Swanda. *PID Controller Performance Assessment Based on Closed-Loop Response Data*. Phd thesis, University of California, Santa Barbara, California, USA, June 1999.