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PARALLEL QR IMPLEMENTATION OF SUBSPACE IDENTIFICATION WITH PARSIMONIOUS MODELS

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Abstract:

In this paper we reveal that the typical subspace identification algorithms use non-parsimonious model formulations, with extra terms in the model that appear to be non-causal. These terms are the causes for inflated variance in the estimates and partially responsible for the loss of closed-loop identifiability. We then propose a parallel parsimonious formulation of a new subspace identification algorithm and demonstrate the effectiveness of the proposed algorithm via simulation.

Keywords: subspace identification; parsimonious formulation; parallel projections

1. INTRODUCTION

Subspace identification methods (SIM) are attractive not only because of their numerical simplicity and stability, but also for their state space form that is very convenient for estimation, filtering, prediction, and control. A few drawbacks, however, have been experienced with SIMs:

- (1) The estimation accuracy is in general not as good as the prediction error methods (PEM), represented by large variance.
- (2) The application of SIMs to closed-loop data is still a challenge, even though the data satisfy identifiability conditions for traditional methods such as PEMs.
- (3) The estimation of B and D is more problematic than that of A and C , which is reflected in the poor estimation of zeros and steady state gains.

In this paper, we are concerned with the reasons why subspace identification approaches exhibit these drawbacks and propose parsimonious SIMs

for open-loop applications. First of all, we start with the analysis of existing subspace formulation using the linear regression formulation (Jansson and Wahlberg, 1998; Knudsen, 2001). From this analysis we reveal that the typical SIM algorithms actually use non-parsimonious model formulation, with extra terms in the model that appear to be non-causal. These terms, although conveniently included for performing subspace projections, are the causes for inflated variance in the estimates and partially responsible for the loss of closed-loop identifiability.

2. ANALYSIS OF SUBSPACE MODEL FORMULATION

Parsimoniousness is a general rule in regression analysis and system identification. The typical subspace identification models, however, are not parsimonious and even non-causal as will be revealed in this section. We begin with an innovation model formulation,

$$x_{k+1} = Ax_k + Bu_k + Ke_k \quad (1a)$$

$$y_k = Cx_k + Du_k + e_k \quad (1b)$$

where $y_k \in R^{n_y}$, $x_k \in R^n$, $u_k \in R^{n_u}$, and $e_k \in R^{n_y}$ are the system output, state, input, and innovation, respectively. A, B, C, D and K are system matrices with appropriate dimensions.

An extended state space model can be formulated as

$$Y_f = \Gamma_f X_k + H_f U_f + G_f E_f \quad (2a)$$

$$Y_p = \Gamma_p X_{k-p} + H_p U_p + G_p E_p \quad (2b)$$

where the extended observability matrix

$$\Gamma_f = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{f-1} \end{bmatrix} \quad (3)$$

and the Toeplitz matrices are

$$H_f = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}B & CA^{f-3}B & \cdots & D \end{bmatrix} \quad (4a)$$

$$G_f = \begin{bmatrix} I & 0 & \cdots & 0 \\ CK & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}K & CA^{f-3}K & \cdots & I \end{bmatrix} \quad (4b)$$

The input and output data are arranged in the following Hankel form:

$$U_f = \begin{bmatrix} u_k & u_{k+1} & \cdots & u_{k+N-1} \\ u_{k+1} & u_{k+2} & \cdots & u_{k+N} \\ \vdots & \vdots & \ddots & \vdots \\ u_{k+f-1} & u_{k+f} & \cdots & u_{k+f+N-2} \end{bmatrix} \quad (5a)$$

$$\triangleq [u_f(k) \cdots u_f(k+N-1)] \quad (5b)$$

$$U_p = \begin{bmatrix} u_{k-p} & u_{k-p+1} & \cdots & u_{k-p+N-1} \\ u_{k-p+1} & u_{k-p+2} & \cdots & u_{k-p+N} \\ \vdots & \vdots & \ddots & \vdots \\ u_{k-1} & u_k & \cdots & u_{k+N-2} \end{bmatrix} \quad (5c)$$

$$\triangleq [u_p(k-p) \cdots u_p(k-p+N-1)] \quad (5d)$$

Similar formulations are made for Y_f , Y_p , E_f , and E_p . Subspace identification methods minimize the following objective function (Overschee and Moor, 1996),

$$\begin{aligned} [\hat{L}^1 \ \hat{L}^2 \ \hat{L}^3] &= \arg \min \{ \|Y_f - L^1 Y_p - L^2 U_p - L^3 U_f\|_F^2 \} \quad (6) \\ &= \arg \min \left\{ \sum_{j=0}^{N-1} \left\| y_f(k+j) - \begin{bmatrix} L^1 & L^2 & L^3 \end{bmatrix} \begin{bmatrix} y_p(k-p+j) \\ u_p(k-p+j) \\ u_f(k+j) \end{bmatrix} \right\|^2 \right\} \end{aligned}$$

Denoting

$$L^1 = \begin{bmatrix} L_{11}^1 & L_{12}^1 & \cdots & L_{1p}^1 \\ L_{21}^1 & L_{22}^1 & \cdots & L_{2p}^1 \\ \vdots & & \ddots & \\ L_{f1}^1 & L_{f1}^1 & & L_{fp}^1 \end{bmatrix} \triangleq \begin{bmatrix} L_1^1 \\ L_2^1 \\ \vdots \\ L_f^1 \end{bmatrix} \quad (7a)$$

$$L^2 = \begin{bmatrix} L_{11}^2 & L_{12}^2 & \cdots & L_{1p}^2 \\ L_{21}^2 & L_{22}^2 & \cdots & L_{2p}^2 \\ \vdots & & \ddots & \\ L_{f1}^2 & L_{f1}^2 & & L_{fp}^2 \end{bmatrix} \triangleq \begin{bmatrix} L_1^2 \\ L_2^2 \\ \vdots \\ L_f^2 \end{bmatrix} \quad (7b)$$

$$L^3 = \begin{bmatrix} L_{11}^3 & L_{12}^3 & \cdots & L_{1f}^3 \\ L_{21}^3 & L_{22}^3 & \cdots & L_{2f}^3 \\ \vdots & & \ddots & \\ L_{f1}^3 & L_{f1}^3 & & L_{ff}^3 \end{bmatrix} \triangleq \begin{bmatrix} L_1^3 \\ L_2^3 \\ \vdots \\ L_f^3 \end{bmatrix} \quad (7c)$$

the above problem is equivalent to f separate sub-problems:

$$\begin{aligned} [\hat{L}_i^1 \ \hat{L}_i^2 \ \hat{L}_i^3] &= \\ \arg \min \left\{ \sum_{j=0}^{N-1} \left\| y(k+j+i-1) - \begin{bmatrix} L_i^1 & L_i^2 & L_i^3 \end{bmatrix} \begin{bmatrix} y_p(k-p+j) \\ u_p(k-p+j) \\ u_f(k+j) \end{bmatrix} \right\|^2 \right\} \quad (8) \\ \text{for } i &= 1, 2, \dots, f \end{aligned}$$

For the moment consider the first subproblem, that is, $i = 1$. In this case the problem implies that the following model is specified:

$$\begin{aligned} y(k) &= \begin{bmatrix} L_1^1 & L_1^2 & L_1^3 \end{bmatrix} \begin{bmatrix} y_p(k-p) \\ u_p(k-p) \\ u_f(k) \end{bmatrix} + v(k) \\ &= \begin{bmatrix} L_1^1 & L_1^2 \end{bmatrix} \begin{bmatrix} y_p(k-p) \\ u_p(k-p) \end{bmatrix} \\ &\quad + L_{11}^3 u(k) + \sum_{j=2}^f L_{1j}^3 u(k+j-1) + v(k) \quad (9) \end{aligned}$$

Note that the third term on the RHS of the above equation is non-causal and unnecessary. Therefore, we can make the following statements about the typical SIM formulation in general.

- (1) The model format used in SIM during the projection step is non-causal. This would result in non-causal models in the projection step. Although the non-causal terms are ignored at the step to estimate B , D , all the model parameters estimate have inflated variance due to the fact that extra and unnecessary terms are included in the model, making the model nonparsimonious.
- (2) Because of the extra terms that turn out to be 'future' inputs, SIMs in general have

problems with closed-loop data using direct identification methods. Most SIMs usually project out U_f as follows:

$$Y_f/\Pi_{U_f}^\perp = \Gamma_f X_k/\Pi_{U_f}^\perp + G_f E_f/\Pi_{U_f}^\perp \quad (10)$$

where $\Pi_{U_f}^\perp = I - U_f^T(U_f U_f^T)^{-1}U_f$. Because of the non-causal terms in the model, $\frac{1}{N}E_f U_f^T \neq 0$ as $N \rightarrow \infty$ for closed-loop data. As a consequence, many SIMs fail to work on closed loop data, except for a few SIM algorithms that avoid this projection (Chou and Verhaegen, 1997; Wang and Qin, 2001; Wang and Qin, 2002).

- (3) Because U_f contains extra rows due to the extra terms, the projection in Eq. 10 tends to reduce the information content unnecessarily even for open-loop data, leading to inefficient use of the data.
- (4) These non-causal terms will have negligible coefficients only when the number of data is very large and process is very well excited. For limited number of samples or non-white input signals, SIM algorithms tend to have large estimation errors.

To avoid these problems the SIM model must not include these non-causal terms. We propose a parallel QR implementation of a parsimonious subspace identification method (PARSIM) which removes these non-causal terms by enforcing triangular structure of the Toeplitz matrix H_f at every step of the SIM procedure. The parallel PARSIM (PARSIM-P) method involves a bank of least squares problems in parallel. This idea was presented in (Qin *et al.*, 2002). In this paper, the bank of least squares problems is implemented via QR factorization. Optimal weighting is derived for this PARSIM-P method. An optimal estimate of the B, D matrices is given using the Kalman filter structure. Numerical simulation is used to demonstrate the effectiveness of the proposed method.

3. PARALLEL PARSIM METHOD

The key idea in the proposed method is to exclude those non-causal terms of U_f . To accomplish this we partition the extended state space model row-wise as follows:

$$Y_f = \begin{bmatrix} Y_{f1} \\ Y_{f2} \\ \vdots \\ Y_{ff} \end{bmatrix}; Y_i \triangleq \begin{bmatrix} Y_{f1} \\ Y_{f2} \\ \vdots \\ Y_{fi} \end{bmatrix}; i = 1, 2, \dots, f \quad (11)$$

Partition U_f and E_f in a similar way to define U_{fi} , U_i , E_{fi} , and E_i , respectively, for $i = 1, 2, \dots, f$. Denote further

$$\Gamma_f = \begin{bmatrix} \Gamma_{f1} \\ \Gamma_{f2} \\ \vdots \\ \Gamma_{ff} \end{bmatrix} \quad (12a)$$

$$H_{fi} \triangleq [CA^{i-2}B \ \dots \ CB \ D] \quad (12b)$$

$$\triangleq [H_{i-1} \ \dots \ H_1 \ H_0] \quad (12c)$$

$$G_{fi} \triangleq [CA^{i-2}K \ \dots \ CK \ I] \quad (12d)$$

$$\triangleq [G_{i-1} \ \dots \ G_1 \ G_0] \quad (12e)$$

$$\forall i = 1, 2, \dots, f$$

where H_i and G_i are the Markov parameters for the deterministic input and innovation sequence, respectively. We have the following partitioned equations:

$$Y_{fi} = \Gamma_{fi} X_k + H_{fi} U_i + G_{fi} E_i \quad (13)$$

$$\forall i = 1, 2, \dots, f$$

Note that each of the above equation is guaranteed causal.

3.1 Parallel Estimation of Γ_{fi} and H_{fi}

By eliminating $e(k)$ in the innovation model through iteration, it is straightforward to derive the following relation (Knudsen, 2001),

$$X_k = L_z Z_p + A_K^p X_{k-p} \quad (14)$$

where

$$L_z \triangleq [\Delta_p(A_K, K) \ \Delta_p(A_K, B_K)] \quad (15a)$$

$$\Delta_p(A, B) \triangleq [A^{p-1}B \ \dots \ AB \ B] \quad (15b)$$

$$A_K \triangleq A - KC \quad (15c)$$

$$B_K \triangleq B - KD \quad (15d)$$

Substituting this equation into Eq. 13, we obtain

$$Y_{fi} = \Gamma_{fi} L_z Z_p + \Gamma_{fi} A_K^p X_{k-p} + H_{fi} U_i + G_{fi} E_i \quad (16)$$

$$\forall i = 1, 2, \dots, f$$

Since the second term in the RHS of Eq. 16 tends to zero as p tends to infinity, we have the following least squares estimates:

$$[\hat{\Gamma}_{fi} L_z \ \hat{H}_{fi}] = Y_{fi} \begin{bmatrix} Z_p \\ U_i \end{bmatrix}^+ \quad (17)$$

$$\forall i = 1, 2, \dots, f$$

Augmenting all estimates $\hat{\Gamma}_{fi} L_z, \forall i = 1, 2, \dots, f$, we have

$$\begin{bmatrix} \hat{\Gamma}_{f1} L_z \\ \hat{\Gamma}_{f2} L_z \\ \vdots \\ \hat{\Gamma}_{ff} L_z \end{bmatrix} = \hat{\Gamma}_f L_z \quad (18)$$

Now we have the following parallel PARSIM algorithm to estimate Γ_{fi} and H_{fi} , for $i = 1, 2, \dots, f$.

[Algorithm 1] Parallel PARSIM (PARSIM-P)

- (1) Perform the following least squares estimates,

$$\begin{bmatrix} \hat{\Gamma}_{fi} L_z & \hat{H}_{fi} \end{bmatrix} = Y_{fi} \begin{bmatrix} Z_p \\ U_i \end{bmatrix}^+ \quad (19)$$

$$\forall i = 1, 2, \dots, f$$

- (2) Perform SVD for the following weighted matrix

$$W_1(\hat{\Gamma}_f L_z)W_2 = USV^T \quad (20)$$

where W_1 is nonsingular and $L_z W_2$ does not lose rank. We choose

$$\hat{\Gamma}_f = W_1^{-1} U S^{1/2} \quad (21)$$

which will yield the estimate for A and C (Verhaegen, 1994).

- (3) The estimates for H_{i-1} , $\forall i = 1, 2, \dots, f$ can be calculated by averaging repeated estimates

$$\hat{H}_{i-1} = \frac{1}{f-i+1} \sum_{j=i}^f \hat{H}_{fj}(:, (j-i)lf+1 : (j-i+1)lf)$$

$$\forall i = 1, 2, \dots, f \quad (22)$$

which can be used to estimate B and D .

[Theorem 1]

Algorithm 1 gives *consistent* estimates for Γ_f and H_{i-1} , $\forall i = 1, 2, \dots, f$ under the following conditions:

- (1) The past horizon $p \rightarrow \infty$.
- (2) The input $u(k)$ and innovation sequence $e(k)$ are uncorrelated, i.e., $\frac{1}{N} E_i U_i^T \rightarrow 0$ as $N \rightarrow \infty$.
- (3) $\{C, A\}$ is observable and $\{A, [B \ K]\}$ reachable.

[Proof] See Appendix A.

[Remark 1] For finite past horizon p the algorithm is biased, but the bias decays to zero exponentially with p . If p is too large in practice, however, large variance is expected for the estimates. Therefore, it is necessary in practice to use a finite p for the best trade-off. Cross-validation can be used to select an optimal p .

[Remark 2] Because of Condition 2 in Theorem 1 where the $E_i U_i^T$ term requires no correlation between future u_k and past e_k , this PARSIM-P algorithm is biased for direct closed loop identification.

3.2 Optimal Weighting and QR Implementation

3.2.1. Optimal Weighting In the conventional SIM formulation under open-loop conditions,

$$E_f \Pi_{U_f}^\perp \rightarrow E_f \text{ as } N \rightarrow \infty \quad (23)$$

since E_f is uncorrelated with U_f . Therefore Eq. 10 becomes:

$$Y_f \Pi_{U_f}^\perp = \Gamma_f X_k \Pi_{U_f}^\perp + G_f E_f \quad (24)$$

Post-multiplying $\frac{1}{N} Z_p^T$ to eliminate the noise term,

$$Y_f \Pi_{U_f}^\perp Z_p^T = \Gamma_f X_k \Pi_{U_f}^\perp Z_p^T \quad (25)$$

Van Overshee and De Moor (1995) show that all SIM methods do SVD on the following weighted matrix:

$$W_r Y_f \Pi_{U_f}^\perp Z_p^T W_c = W_r \Gamma_f X_k \Pi_{U_f}^\perp Z_p^T W_c \quad (26)$$

where W_r and W_c are the row and column weighting matrices, respectively. In CVA $W_r = (Y_f \Pi_{U_f}^\perp Y_f^T)^{-1/2}$ which basically normalizes the output variables to achieve a maximum likelihood type of weighting. Gustafsson (Gustafsson, 2002) shows that an approximately optimal weighting for W_c is

$$W_c = (Z_p Z_p^T - Z_p U_f^T (U_f U_f^T)^{-1} U_f Z_p^T)^{-1/2}$$

$$= (Z_p \Pi_{U_f}^\perp Z_p^T)^{-1/2} \quad (27)$$

which is used in CVA and MOESP. Substituting Eq. 27 into Eq. 26, and replacing X_k with $L_z Z_p$ as an instrumental variable, we obtain,

$$W_r Y_f \Pi_{U_f}^\perp Z_p^T W_c = W_r \Gamma_f L_z Z_p \Pi_{U_f}^\perp Z_p^T (Z_p \Pi_{U_f}^\perp Z_p^T)^{-1/2}$$

$$= W_r \Gamma_f L_z (Z_p \Pi_{U_f}^\perp Z_p^T)^{1/2} \quad (28)$$

Comparing Eq. 28 with Eq. 20, the equivalent weightings for the PARSIM-P algorithm is

$$W_1 = W_r = (Y_f \Pi_{U_f}^\perp Y_f^T)^{-1/2} \quad (29a)$$

$$W_2 = (Z_p \Pi_{U_f}^\perp Z_p^T)^{1/2} \quad (29b)$$

Jansson and Wahlberg (1996) show that the row-weighting W_1 (or W_r) has no influence on the asymptotic accuracy of the estimate.

3.2.2. QR Implementation for Γ_f To implement the PARSIM-P algorithm efficiently, we perform the following QR decomposition

$$\begin{bmatrix} Z_p \\ U_i \\ Y_{fi} \end{bmatrix} = \begin{bmatrix} R_{11,i} & \\ R_{21,i} & R_{22,i} \end{bmatrix} \begin{bmatrix} Q_{1,i} \\ Q_{2,i} \end{bmatrix} \quad (30)$$

Using this relation in Eq. 19, we obtain

$$[\hat{\Gamma}_{fi} L_z \hat{H}_{fi}] = R_{21,i} R_{11,i}^+ \quad (31)$$

Notice that the QR decomposition in Eq. 30 can be implemented recursively. Given that the i^{th} QR decomposition is performed, the next QR decomposition can be performed on

$$\begin{bmatrix} Z_p \\ U_{i+1} \\ Y_{f,i+1} \end{bmatrix} = \begin{bmatrix} Z_p \\ U_i \\ U_{f,i+1} \\ Y_{f,i+1} \end{bmatrix} = \begin{bmatrix} R_{11,i} Q_{1,i} \\ U_{f,i+1} \\ Y_{f,i+1} \end{bmatrix} \quad (32)$$

which is in QR decomposition form except for the last two rows.

3.2.3. QR Implementation for K Once $\hat{\Gamma}_f$ is known in Step 2 of Algorithm 1, the Kalman filter gain K can be estimated similar to Wang and Qin (Wang and Qin, 2002).

With large p , substituting Eq. 14 into Eq. 2 leads to:

$$Y_f = \Gamma_f L_z Z_p + H_f U_f + G_f E_f \quad (33)$$

Therefore,

$$Y_f \Pi_{\begin{bmatrix} Z_p \\ U_f \end{bmatrix}}^\perp = G_f E_f \Pi_{\begin{bmatrix} Z_p \\ U_f \end{bmatrix}}^\perp = G_f E_f \quad (34)$$

since E_f is not correlated with Z_p and U_f in open-loop.

Performing QR decomposition,

$$\begin{bmatrix} Z_p \\ U_f \\ Y_f \end{bmatrix} = \begin{bmatrix} R_{11} & & \\ R_{21} & R_{22} & \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \quad (35)$$

then

$$R_{33} Q_3 = G_f E_f \quad (36)$$

Denoting $e_k = F e_k^*$ such that $\text{cov}(e_k^*) = I$,

$$G_f E_f = (G_f \otimes F) E_f^* \triangleq G_f^* E_f^* \quad (37)$$

where $\otimes$ denotes Kronecker product and

$$G_f^* = \begin{bmatrix} F & 0 & \cdots & 0 \\ CKF & F & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}KF & CA^{f-3}KF & \cdots & F \end{bmatrix} \in \mathbb{R}^{m \cdot f \times m \cdot f} \quad (38)$$

From Eqs. 36 and 37 and using the fact that Q_3 is an orthonormal matrix, we choose

$$\hat{E}_f^* = Q_3 \quad (39a)$$

$$\hat{G}_f^* = R_{33} \quad (39b)$$

Therefore,

$$\hat{F} = R_{33}(1:m, 1:m) \quad (40)$$

and K can be calculated from G_f^* using Γ_f .

3.2.4. Optimal Estimation for B and D With A and C estimates, Section 10.6 in (Ljung, 1999) gives an effective approach to estimate B and D with an output error formulation. Here we give a modified approach to estimating B , D optimally using A , C , K and F for the general innovation form.

From the innovation form of the system we have:

$$x_{k+1} = A_K x_k + B_K u_k + K y_k \quad (41)$$

The process output can be represented as

$$y_k = C(qI - A_K)^{-1} x_0 + [C(qI - A_K)^{-1} B_K + D] u_k + C(qI - A_K)^{-1} K y_k + e_k \quad (42)$$

or:

$$[I - C(qI - A_K)^{-1} K] y_k = C(qI - A_K)^{-1} x_0 + [C(qI - A_K)^{-1} B_K + D] u_k + e_k \quad (43)$$

using $e_k = F e_k^*$ where e_k^* has an identity covariance matrix, and defining

$$\tilde{y}_k = F^{-1} [I - C(qI - A_K)^{-1} K] y_k \quad (44a)$$

$$G(q) = F^{-1} C(qI - A_K)^{-1} \quad (44b)$$

$$D^* = F^{-1} D \quad (44c)$$

we obtain,

$$\begin{aligned} \tilde{y}_k &= G(q) B_K u_k + D^* u_k + G(q) x_0 \delta_k + e_k^* \\ &= G(q) \otimes u_k^T \text{vec}(B_K) + I_m \otimes u_k^T \text{vec}(D^*) \\ &\quad + G(q) x_0 \delta_k + e_k^* \end{aligned} \quad (45)$$

where $\text{vec}(B_K)$ and $\text{vec}(D^*)$ are vectorized B_K and D^* matrices along the rows. δ_k is the Kronecker delta function. Now $\text{vec}(B_K)$, $\text{vec}(D^*)$ and x_0 can be estimated using least squares from the above equation. The B , D matrices can be backed out as:

$$\hat{D} = F \hat{D}^* \quad (46a)$$

$$\hat{B} = \hat{B}_K + K \hat{D} \quad (46b)$$

3.2.5. PARSIM-P with QR Implementation We summarize the above procedure into following algorithm that implements PARSIM-P with QR decomposition.

[Algorithm 2] PARSIM-P with QR decomposition

- (1) Calculate $\Gamma_{f_i} L_z$ as in Eq. 31 via a bank of QR decompositions for $i = 1, 2, \dots, f$. Form $\Gamma_f L_z$ and calculate A and C estimates from SVD of $W_1 \hat{\Gamma}_f L_z W_2$ as in Algorithm 1.

- (2) Find the Kalman filter gain K and matrix F using A and C estimates.
- (3) Calculate B , D estimates from A , C , K and F estimates.

Using these three steps, it is straightforward to show that the estimates are consistent if the prior steps are consistent.

4. SIMULATION RESULTS

The counter example proposed in (Jansson and Wahlberg, 1998) is used here to test the effectiveness of the proposed parallel parsimonious method. The case of $K = [-0.21 \ -0.559]^T$ is used here. For comparison any standard SIM algorithms can be used, but we choose the MOESP algorithm in (Verhaegen, 1994). Figure 1 shows the poles and zero estimates using the PARSIM-P algorithm and MOESP. In this simulation, we choose $N=2000$, $f=3$, $p=5$. It can be seen from the results that the PARSIM-P gives better estimates of the poles than the MOESP algorithm, which knows the benefit of the parsimonious formulation. The zero estimates from the PARSIM-P is much better than the MOESP estimates.

Fig. 1. Poles and zero estimates for the counter example.

5. CONCLUSIONS

The conventional subspace identification models are non-parsimonious with extra non-causal input terms. The extra terms in the models are responsible for large variance in SIM estimates and partially responsible for the loss of closed-loop identifiability. The proposed parallel subspace identification algorithm overcomes these problems.

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APPENDIX A

Condition 1 implies the initial state has zero effect on the estimate since A_K is always stable. From Condition 2 we have $\frac{1}{N}E_i Z_p^T \rightarrow 0$ as $N \rightarrow \infty$, that is, the effect of noise is zero under condition 2. It is straightforward to show that Condition 3 is need for L_z to have full row rank and Γ_f full column rank.

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