
Svante Björklund

Control & Communication
Department of Electrical Engineering
Linköpings universitet, SE-581 83 Linköping, Sweden
WWW: http://www.control.isy.liu.se
E-mail: svabj@isy.liu.se

15th July 2003

Report no.: LiTH-ISY-R-2526

Technical reports from the Control & Communication group in Linköping are available at http://www.control.isy.liu.se/publications.

In this report we study estimation of time-delays in linear dynamical systems with additive noise. Estimating time-delays is a common engineering problem, e.g. in automatic control, system identification and signal processing.

The purpose with this work is to test and evaluate a certain class of methods for time-delay estimation, especially with automatic control applications in mind. The class of methods consists of estimating the time-delay as a continuous parameter with a prediction error method in some simple model structures which are often used in process industry. The methods are evaluated experimentally with the aid of simulations and plots of RMS error, bias, standard deviation and confidence intervals for different cases.

The results are: It is best not to prewhite the input signal. There should be at most one real pole in the model structure. In some cases the simplest model structure, a first order system with time-delay “idproc6”, is clearly the best. It is not clearly worse than the best in any case. The RMS error varies much with the system, the input signal type and the SNR. For idproc6 it varies between 0.3 and 12.1 sampling intervals in the performed simulations with a mean of 4.7.
# Contents

1 Introduction ................................................. 1

2 The time-delay estimation methods .................. 2

3 Simulation setup ........................................... 4

4 Analysis ..................................................... 7

5 Discussion and conclusions ............................... 16
   5.1 Discussion ........................................... 16
   5.2 Conclusions ......................................... 16
   5.3 Recommendations .................................. 16
   5.4 Future work ......................................... 17

References ..................................................... 1

A Validation of ANOVA and confidence intervals .... 1
1 Introduction

The problem we address in this report is estimating time-delays in linear dynamical systems with additive noise. A synonym for time delay is dead-time. Estimating time-delays is a common engineering problem, e.g. in control performance monitoring of industrial processes [Hor00, Swa99], in design and tuning of controllers, in range estimation in radar [KQ92] and in direction estimation by time-delay of arrival in signal intelligence [HR97, Wik02]. Dead-time estimation is also a necessary part in all system identification [Lju99].

The purpose with this work is to test and evaluate a certain class of methods for time-delay estimation, especially with automatic control applications in mind. Particularly interesting is it to determine the best method. Is one method best in all situations or should different methods be used for different situations? The class of methods consists of estimating the time-delay as a continuous parameter with a prediction error method in some simple model structures which are often used in process industry [Lju02]. The methods are tested and evaluated experimentally with the aid of simulations.

In the next chapter, the time-delay estimation methods are briefly described. Then, Chapter 3 is about the simulation setup. After that, in Chapter 4 the analysis of the simulations is conducted. Following, Chapter 5 contains discussion, conclusions and suggestions for further work. Appendix A contains validation of required prerequisites for the analysis.
2 The time-delay estimation methods

The class of methods which are studied in this report consists of estimating the time-delay as a continuous parameter with a prediction error method in some simple model structures which are often used in process industry [Lju02]. The process models are:

- **idproc1.** A first order system with time-delay:
  \[ G(s) = \frac{K}{1 + sT} e^{-sL} \]  
  \(2.1\)

- **idproc2.** A second order system with real poles and time-delay:
  \[ G(s) = \frac{K}{(1 + sT_1)(1 + sT_2)} e^{-sL} \]  
  \(2.2\)

- **idproc3.** A second order system with complex poles and time-delay:
  \[ G(s) = \frac{K}{1 + 2\zeta \omega_n s + (\omega_n)^2} e^{-sL} \]  
  \(2.3\)

- **idproc4.** A third order system with real poles and time-delay:
  \[ G(s) = \frac{K}{(1 + sT_1)(1 + sT_2)(1 + sT_3)} e^{-sL} \]  
  \(2.4\)

- **idproc5.** A third order system with two complex poles, one real pole and time-delay:
  \[ G(s) = \frac{K}{(1 + 2\zeta \omega_n s + (\omega_n)^2)(1 + sT_3)} e^{-sL} \]  
  \(2.5\)

In the prediction error method (PEM) [Lju99]:

\[ \hat{\theta}_N = \arg \min_{\theta} V_N(\theta) = \arg \min_{\theta} \frac{1}{N} \sum_{t=1}^{N} \frac{1}{2} \varepsilon^2(t, \theta), \]

where \(\varepsilon(t|\theta) = y(t) - \hat{y}(t|\theta)\) is the prediction error of the model.

The steps of the implemented algorithm are:

1. Find an initial estimate of the time delay by choosing the estimated ARX model [Lju99] (with \(n_a\) and \(n_b\) depending on the process model (\(2.1\))-(\(2.5\)) ) whose time delay \(n_k\) gives the lowest criterion function \(V_N(\theta)\). This estimate will be an multiple of the sampling interval.

2. Depending on the selected process model (\(2.1\))-(\(2.5\)) choose the orders \(n_b\) and \(n_f\) of an OE model [Lju99] suitably. The time delay \(n_k\) is taken from step 1.

3. Convert the estimated discrete-time poles of the OE model into continuous-time and use these as the poles of the resulting process model (\(2.1\))-(\(2.5\)). Now only it remains to find the final time-delay estimate.
4. Perform a line search over several sample intervals around the previously estimated \( n_k \) from the ARX model. The reason is to avoid local minima. The other parameters of the process model are held fix.

5. Conduct a Gauss-Newton search [Lju99] for the final time-delay estimate. The other parameters of the process model are held fix.

In this chapter we also want to mention a special name that we give to one model structure together with the absence of prewhitening that is used later in this report (see Chapter 4):

- idproc6: This is a synonym for idproc1*nopw. nopw means without prewhitening the input signal (see [Bjo03b]).
3 Simulation setup

The setup for the simulations is the same as in [Bjo03b] with the following exceptions:

- The number of trials was 128. The trials were split into four groups of 32 trials each and each group was used to compute an estimate of the RMS error (our response variable) of the time-delay estimate. This gave 4 estimates of the RMS error that was used in the calculation of the confidence intervals.
- The methods were different. See Chapter 2.
- The true time-delay was set to 9 sampling intervals instead of 10 because we want to estimate the true continuous-time time-delay which was 9 sampling intervals. (This will be 10 sampling intervals after zero-order hold sampling.)

Three environment factors were varied during the simulations: The system, the input signal type and the SNR [Bjo03b]. The signal-to-noise ratio (SNR) was either 1 or 100. See [Bjo03b] for the definition of the SNR. The impulse responses of the four used systems are depicted in Figure 3.1. Note that for all the systems the time delay will be 10 after the sampling. More information about the systems can be found in [Bjo03b].

![Impulse response G1](image1)
![Impulse response G2](image2)
![Impulse response G5](image3)
![Impulse response G6](image4)

Figure 3.1: Impulse response of system $G_1$-$G_2$ and $G_5$-$G_6$. True time-delay after sampling $T_d = 10$. 
Figures 3.2-3.4 show the used input signals in the time and frequency domains. Figure 3.2 depicts the signal RBS 10-30% which is a bandpass random signal with frequency contents between 10% and 30% of the Nyquist frequency. Figure 3.2 depicts the signal RBS 0-100% which is a wideband random signal with frequency contents between 0% and 100% of the Nyquist frequency. It is actually white noise. Figure 3.4 depicts the signal Steps which is a signal with three steps. It has a frequency contents between 0% and about 5% of the Nyquist frequency. More information about the input signals can be found in [Bjö03b].

Figure 3.2: Time signal (left) and frequency spectrum (right) for a realization of the input signal type RBS 10-30%.

Figure 3.3: Time signal (left) and frequency spectrum (right) for a realization of the input signal type RBS 0-100%.
Figure 3.4: Time signal (left) and frequency spectrum (right) for a realization of the input signal type Steps.
4 Analysis

In this chapter we compare the tested methods and see in which cases they could be used. We will perform a statistical analysis from which it is possible to draw conclusions. Bear in mind that:

- All results are in average over all the factors that are not explicitly studied. Certain special cases can give a different result.
- Even if there is a statistically significant difference (seen in the confidence interval plots), it is not sure that the difference (see the RMS error bar plots) has any practical importance.

The statistical analysis is conducted in the same way as in Chapter 7 in [Bjo03b]. The program t155b2.m generated the ANOVA table and the graphs in this section. The estimated time-delay was not allowed to be larger than 30. A positive transformation: \( x^{(0.435939)} \) was used. This means that "the lower value the better" in the confidence interval plots.

We see from Table 4.1 and Figures 4.1-4.9 that:

- High SNR is better in average over all methods and all other factors (Figure 4.6).
- Without prewhitening is significantly better in average over all methods and all other factors (Figure 4.6). Therefore we concentrate on without prewhitening in the following.
- The method idproc1*nopw has the lowest estimated RMS error (=4.71) in our simulation (Figure 4.1) but still we cannot say that it or any other method is significantly best in average (Figure 4.5). We give the name idproc6 to the method idproc*nopw.
- Without prewhitening there is no significant difference between idproc1,2,3,5 in average (Figure 4.5).
- Without prewhitening idproc4 is significantly worse in average (Figure 4.5).
- The method idproc6 has the lowest estimated RMS error due to its low standard deviation. Its bias is not the lowest. See Figure 4.2.
- The bias of idproc6 is probably due to the fact that the model structure cannot describe the true system. Not even with a high SNR, the method idproc6 cannot give a bias near zero. Some methods with models of higher order, e.g. idproc2*nopw and idproc3*nopw, have a bias near zero for high SNR. See Figure 4.3. They have, however, a high bias for low SNR.
- The method idproc6 is significantly better (Figure 4.9) than the other methods for the fast low order system \( G_2 \). No method is significantly worst for \( G_2 \). For the other systems there is no method that is significantly best or worst. The method idproc6 is not significantly worse than any of the other methods in average. See Figure 4.9.
- The input signal type RBS 0-100% is significantly easier in average than the other signal types. The signal type Steps is significantly harder in average than the other. See Figure 4.7.
The fast low order system $G_2$ is significantly easier in average than the other systems. The high order system with complex poles $G_6$ is significantly harder in average than the other. See Figure 4.7.

For the input signal types RBS 10-30% and RBS 0-100% no method is significantly best. For the input signal type Steps there is a group of methods that are significantly better than the other. No method in this group employs prewhitening. The method idproc6, which is in this group, is not significantly worse than any of the other methods in this group in average. See Figure 4.8.

We observe in Table 4.1 that all main factors and all factor interactions (except Prewhite* SNR, Prewhite* Sys, Method* Prewhite* SNR and Method* InType* Prewhite* SNR* Sys) on the 95% confidence level have significant effect since their p-values (the column Prob>F) are lower than 0.05. It is another question if these effects have practical importance.

In Figure 4.5 we see that for the third order systems more than one real pole significantly decreases the performance. For the second order systems we cannot say whether real or complex poles are best.

One of the best choices of method is idproc1*nopw. It has the lowest estimated RMS error in average (=4.7). It has RMS error 1.7 for high SNR and 5.6 for low SNR. It has RMS error 4.7-4.8 for all input signal types. It has RMS error 1.1 for the fast system and 5.9 in average over the other systems. All RMS error numbers are in average over the other factors.

The RMS error varies much with the system, the input signal type and the SNR. For idproc6 it varies between 0.3 and 12.1 sampling intervals in the performed simulations with a mean of 4.7.

![Figure 4.1: RMS error for different methods. (t155b2.m)](image-url)
Figure 4.2: Bias (left) and standard deviation (right) of the estimates for different methods with or without prewhitening. (t155b2.m)

Figure 4.3: Bias (left) and standard deviation (right) of the estimates for different SNRs and methods. (t155b2.m)
Figure 4.4: RMS error for different methods and some environment factors: Input signal type to the left and system to the right. (t155b2.m)

Figure 4.5: Confidence intervals (95% simultaneous confidence level) for combinations of model structure and prewhitening (= method). Since the prerequisites are not completely fulfilled (Appendix A), we cannot say that any method is significantly best. (t155b2.m)
<table>
<thead>
<tr>
<th>Source</th>
<th>Sum Sq.</th>
<th>d.f</th>
<th>Mean Sq.</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>31.3133</td>
<td>4</td>
<td>7.8283</td>
<td>168.1455</td>
<td>0</td>
</tr>
<tr>
<td>InType</td>
<td>15.3686</td>
<td>2</td>
<td>7.6843</td>
<td>165.0516</td>
<td>0</td>
</tr>
<tr>
<td>Prewhite</td>
<td>27.6304</td>
<td>1</td>
<td>27.6304</td>
<td>593.4761</td>
<td>0</td>
</tr>
<tr>
<td>SNR</td>
<td>254.4791</td>
<td>1</td>
<td>254.4791</td>
<td>5465.9847</td>
<td>0</td>
</tr>
<tr>
<td>Sys</td>
<td>36.5241</td>
<td>3</td>
<td>12.1747</td>
<td>261.5019</td>
<td>0</td>
</tr>
<tr>
<td>Method*InType</td>
<td>2.8977</td>
<td>8</td>
<td>0.36222</td>
<td>7.7801</td>
<td>4.7824e-10</td>
</tr>
<tr>
<td>Method*Prewhite</td>
<td>3.2521</td>
<td>4</td>
<td>0.81303</td>
<td>17.4631</td>
<td>1.0958e-13</td>
</tr>
<tr>
<td>Method*SNR</td>
<td>4.3107</td>
<td>4</td>
<td>1.0777</td>
<td>23.1473</td>
<td>0</td>
</tr>
<tr>
<td>Method*Sys</td>
<td>27.6585</td>
<td>12</td>
<td>2.3049</td>
<td>49.5066</td>
<td>0</td>
</tr>
<tr>
<td>InType*Prewhite</td>
<td>44.388</td>
<td>2</td>
<td>22.194</td>
<td>476.7075</td>
<td>0</td>
</tr>
<tr>
<td>InType*SNR</td>
<td>7.9038</td>
<td>2</td>
<td>3.9519</td>
<td>84.8833</td>
<td>0</td>
</tr>
<tr>
<td>InType*Sys</td>
<td>5.4145</td>
<td>6</td>
<td>0.90242</td>
<td>19.3831</td>
<td>0</td>
</tr>
<tr>
<td>Prewhite*SNR</td>
<td>0.00325</td>
<td>1</td>
<td>0.00325</td>
<td>0.069806</td>
<td>0.79169</td>
</tr>
<tr>
<td>Prewhite*Sys</td>
<td>0.35104</td>
<td>3</td>
<td>0.11701</td>
<td>2.5134</td>
<td>0.057402</td>
</tr>
<tr>
<td>SNR*Sys</td>
<td>31.3915</td>
<td>3</td>
<td>10.4638</td>
<td>224.7536</td>
<td>0</td>
</tr>
<tr>
<td>Method<em>InType</em>Prewhite</td>
<td>4.7158</td>
<td>8</td>
<td>0.58948</td>
<td>12.6615</td>
<td>0</td>
</tr>
<tr>
<td>Method<em>InType</em>SNR</td>
<td>7.5517</td>
<td>8</td>
<td>0.94396</td>
<td>20.2754</td>
<td>0</td>
</tr>
<tr>
<td>Method<em>InType</em>Sys</td>
<td>4.3384</td>
<td>24</td>
<td>0.18077</td>
<td>3.8827</td>
<td>2.1494e-09</td>
</tr>
<tr>
<td>Method<em>Prewhite</em>SNR</td>
<td>0.2098</td>
<td>4</td>
<td>0.052451</td>
<td>1.1266</td>
<td>0.34272</td>
</tr>
<tr>
<td>Method<em>Prewhite</em>Sys</td>
<td>1.0951</td>
<td>12</td>
<td>0.091258</td>
<td>1.9601</td>
<td>0.02524</td>
</tr>
<tr>
<td>Method<em>SNR</em>Sys</td>
<td>2.1137</td>
<td>12</td>
<td>0.17615</td>
<td>3.7834</td>
<td>1.341e-05</td>
</tr>
<tr>
<td>InType<em>Prewhite</em>SNR</td>
<td>0.53357</td>
<td>2</td>
<td>0.26678</td>
<td>5.7303</td>
<td>0.003396</td>
</tr>
<tr>
<td>InType<em>Prewhite</em>Sys</td>
<td>0.9937</td>
<td>6</td>
<td>0.16562</td>
<td>3.5573</td>
<td>0.0017662</td>
</tr>
<tr>
<td>InType<em>SNR</em>Sys</td>
<td>6.759</td>
<td>6</td>
<td>1.1265</td>
<td>24.1962</td>
<td>0</td>
</tr>
<tr>
<td>Prewhite<em>SNR</em>Sys</td>
<td>5.726</td>
<td>3</td>
<td>1.9087</td>
<td>40.9962</td>
<td>0</td>
</tr>
<tr>
<td>Method<em>InType</em>Prewhite*SNR</td>
<td>3.4172</td>
<td>8</td>
<td>0.42714</td>
<td>9.1747</td>
<td>4.5227e-12</td>
</tr>
<tr>
<td>Method<em>InType</em>Prewhite*Sys</td>
<td>2.4918</td>
<td>24</td>
<td>0.10383</td>
<td>2.2301</td>
<td>0.00069197</td>
</tr>
<tr>
<td>Method<em>InType</em>SNR*Sys</td>
<td>3.9783</td>
<td>24</td>
<td>0.16576</td>
<td>3.5605</td>
<td>2.9474e-08</td>
</tr>
<tr>
<td>Method<em>Prewhite</em>SNR*Sys</td>
<td>2.0705</td>
<td>12</td>
<td>0.17254</td>
<td>3.706</td>
<td>1.8938e-05</td>
</tr>
<tr>
<td>InType<em>Prewhite</em>SNR*Sys</td>
<td>2.5402</td>
<td>6</td>
<td>0.42337</td>
<td>9.0936</td>
<td>1.3247e-09</td>
</tr>
<tr>
<td>Method<em>InType</em>Prewhite<em>SNR</em>Sys</td>
<td>1.3271</td>
<td>24</td>
<td>0.055295</td>
<td>1.1877</td>
<td>0.24437</td>
</tr>
<tr>
<td>Error</td>
<td>33.5209</td>
<td>720</td>
<td>0.046557</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>576.2693</td>
<td>959</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Analysis of Variance table for all methods. Constrained (Type III) sums of squares. Positive transformation: (RMS error)\(^{(0.435939)}\). (t155b2.m)
The population marginal means of groups 1 and 2 are significantly different.

Figure 4.6: Confidence intervals (95% simultaneous confidence level) for prewhitening (left) and SNR (right). (t155b2.m)

The population marginal means of groups 1 and 2 are significantly different.

Figure 4.7: Confidence intervals (95% simultaneous confidence level) for input signal type (left) and system (right). (t155b2.m)
Figure 4.8: Confidence intervals (95% simultaneous confidence level) for combinations of method and input signal type. (t155b2.m)
39 groups have population marginal means significantly different from 11:proc1*nopw*fast2

Figure 4.9: Confidence intervals (95% simultaneous confidence level) for combinations of method and input signal type. (t155b2.m)
Figure 4.10 displays all factor combinations, method parameters on the one axis and the environment factors on the other.

![Diagram](t155b2-030504 16:29 rms: rms, data(:,4;:,:,:,m,m,m,:))

Figure 4.10: RMS error for different methods and environment factors. (t155b2.m)
5 Discussion and conclusions

5.1 Discussion

- Not only the model structure matters. The initialization of the time-delay in the optimization is important.
- It is unclear why the RMS error for the methods in this report (Figure 4.1) is so much higher than arxstruc in [Bjo03a] (with RMS error ≈ 2.7)? Arxstruc is used here also, for initialization of the time-delay parameter (Chapter 2).

5.2 Conclusions

We draw the following conclusions from the work in this report. The model structures idproc1,2,3,4,5 are defined in Chapter 2.

- It is best without prewhitening (nopw) of the input data.
- For nopw we cannot say that there is any difference between the model structures idproc1,2,3,5 in average but idproc4 is worst in average.
- For nopw and the fast system, The model structure idproc1 is the best and idproc4 is the worst. For nopw and the other systems we can not say that any of the model structures is best.
- For nopw and different input signal types:
  - The bandpass random signal (RBS 10-30%): The model structure idproc1 is best and idproc4 is worst.
  - The wideband random signal (RBS 0-100%): We can not say that there is any difference between the model structures.
  - The signal with steps (Steps): The model structure idproc4 is worst. The other model structures are better but we cannot say that any of them is the best.
- The RMS error varies much with the system, the input signal type and the SNR. For idproc6 it varies between 0.3 and 12.1 sampling intervals in the performed simulations with a mean of 4.7.
- The model model structure should not have more than one real pole (Figure 4.5).

5.3 Recommendations

Use the model structure idproc1

\[ G(s) = \frac{K}{1 + sT}e^{-sL} \]  \hspace{1cm} (5.1)

without prewhitening the input.
5.4 Future work

Some suggestions of future work:

- Better initialization of the time-delay in the optimization.
- Investigate why these methods are worse than arxstruc type methods [Bjö03a].
References


A Validation of ANOVA and confidence intervals

This appendix comments on the use and applicability of the ANOVA and confidence intervals in Section 4.

The 4 RMS estimates (Chapter 3) have first gone through a transformation to make the variance more constant [Bjo03b]. The transformation became (RMS error)^(0.435939) (Figure A.1). Then ANOVA [Mon97] was executed and confidence intervals for pairwise comparisons [Mat01] plotted. The traditional ANOVA table is given in Table 4.1. Since all p-values (the column Prob>F) are very small, all factors and interactions have effect with a very high confidence level. The confidence intervals are plotted in Chapter 4. For the ANOVA and confidence intervals to be valid some prerequisites must be fulfilled [Mon97, Bjo03b]. These are usually tested by studying some validation graphs [Mon97, Bjo03b] (Figures A.2-A.3). We see in these graphs that the prerequisites are not completely fulfilled so we must be somewhat careful in the interpretation of the ANOVA and confidence interval. See [Mon97, Bjo03b] for more information on this.

Figure A.1: Plot (without transform) for choosing a variance-stabilizing transform [Mon97] for ANOVA of all methods. The transformation is chosen by fitting a straight line to the data points by the least squares method.
Figure A.2: Residual analysis for ANOVA and confidence intervals. Ideally the residuals should be Gaussian and the residuals vs. time and fitted value should be within a horizontal band and be structureless [Mon97, Bjo03b].
Figure A.3: Residual standard deviation versus factor levels for ANOVA and confidence intervals. Ideally the standard deviation for all factor levels should be equal[Mon97, Bjo03b].