Iterative Learning Control of a Flexible Robot Arm Using Accelerometers

Svante Gunnarsson, Mikael Norrlöf
Enes Rahic, Markus Özbek

Division of Automatic Control
Department of Electrical Engineering
Linköpings universitet, SE-581 83 Linköping, Sweden
WWW: <http://www.control.isy.liu.se>
E-mail: svante@isy.liu.se, mino@isy.liu.se

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Abstract

Iterative learning control (ILC) is applied to a laboratory scale robot arm with joint flexibility. The ILC algorithm is based on an estimate of the arm angle, where the estimate is formed using measurements of the motor angle and the arm angular acceleration. The design of the ILC algorithm is based on a model obtained from system identification. The ILC algorithm is evaluated experimentally on the robot arm with good results.

Keywords: Iterative learning control, joint flexibility, accelerometer

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Department of Electrical Engineering
Linköping University, SE-58183 Linköping, Sweden
e-mail: svante@isy.liu.se, mino@isy.liu.se

Abstract

Iterative learning control (ILC) is applied to a laboratory scale robot arm with joint flexibility. The ILC algorithm is based on an estimate of the arm angle, where the estimate is formed using measurements of the motor angle and the arm angular acceleration. The design of the ILC algorithm is based on a model obtained from system identification. The ILC algorithm is evaluated experimentally on the robot arm with good results.

1 Introduction

Iterative Learning Control (ILC) has proven to be a competitive control method in many applications, and the most well known is probably the robotics domain. Some examples are given in [1, 6, 7]. The standard situation in robot control using ILC is that it is assumed that the robot is rigid, such that all relevant signals can be calculated from the actuator signals (motor angles). In reality there are always mechanical flexibilities in both joints and links. The aim of this paper is to present some experiences from the use of accelerometers in ILC applied to flexible mechanical systems. Different iterative learning control ideas applied to flexible mechanical systems have been studied previously in e.g. [12], [14] and [7]. In these papers it is assumed that the position (angle) of the object to be controlled can be measured. Here, instead, it is assumed that only the acceleration of the object to be controlled can be measured, in addition to the actuator angle. In

[11], which also deals with a flexible system, an acceleration signal is used in an ILC algorithm, but in a completely different way.

The viewpoint in this paper is that ILC is used as a complement to an existing control system consisting of, possibly, both a feed-back and feed-forward controller. The situation is depicted in Figure 1, where G represents the system to be controlled and F and F_f represent the feed-back and feed-forward controllers. The signals $r(t)$, $y_k(t)$ and $u_k(t)$ denote the reference signal, the output signal and the ILC input signal respectively, and the subscript k denotes the iteration number. Finally, the signals $d_k(t)$ and $n_k(t)$ denote load and measurement disturbance respectively. When dealing with commercial industrial robots the control system can not be affected by the user and it has to be considered as given. The control system will have some performance properties and the idea is to use ILC to improve the performance by generating a correction signal that is added to the signals generated in the existing control system.

Figure 1: Control system.

2 System description

The process that is studied in this paper is a Flexible Joint process from Quanser [13], and it consists of a SRV02 Rotary Servo Plant and a ROTOFLEX Rotary Flexible Joint. The process can, somewhat simplified, be described by Figure 2.

Figure 2: Simplified description of the Flexible Joint Process.

Using the state variables $x_1 = \theta_a, x_2 = \dot{\theta}_a, x_3 = \theta_m$ and $x_4 = \dot{\theta}_m$ torque balances for the arm and the motor give the state space model

$$\dot{x}(t) = Ax(t) + Bu(t) \quad y(t) = Cx(t) \quad (1)$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_a} & -\frac{b}{J_a} & \frac{k}{J_a r} & \frac{b}{J_a r} \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_m r} & \frac{b}{J_m r} & -\frac{k}{J_m r^2} & -\frac{f}{J_m} - \frac{b}{J_m r^2} \end{pmatrix} \quad (2)$$

$$B = \begin{pmatrix} 0 & 0 & 0 & \frac{k_\tau}{J_m} \end{pmatrix}^T \quad (3)$$

$$C = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} \quad (4)$$

In these matrices J_a and J_m denote the moment of inertia of the arm and the motor respectively. The parameters k and b denote stiffness and damping respectively of the spring, and f is the viscous friction coefficient of the motor. The variable θ_k denotes the angle after the gear box, which means that $\theta_k = \frac{1}{r}\theta_m$, where r is the gear ratio. Finally, the parameter k_τ denote the torque constant and represent the relationship between input voltage and generated torque. This means that the dynamics of the electrical actuator have been neglected

in the model. In industrial robots the normal situation is that the motor angle, θ_m can be measured, but not the arm angle θ_a . In the laboratory process also the arm angle can be measured but this signal will only be used for evaluation purposes and not in the control system.

3 Identification and Controller Design

From an initial experiment estimating the resonance frequency of the arm the sampling interval was set to $T = 0.016$ sec. The Nyquist frequency is then ten times the resonance frequency of the arm. The input signal for identification was chosen as a PRBS signal where the change probability was selected in order to get a suitable frequency content. Since the process is designed for educational purposes it is well behaved in the sense that the level of non-linear effects and measurement disturbance is relatively small. Data were collected during 100 sec., and after subtraction of the mean values the data set is split into an estimation set and a validation set. This implies that each data set contains approximately $N = 3000$ measurements.

The identification is carried out using measurements of the motor angle θ_m , but for the design of the ILC algorithm it is necessary to have a model from input signal to arm angle θ_a . Therefore it is logical to identify a physically parameterized state space model, from which the model from u to θ_a easily can be obtained. The state space model given by equation (4) contains seven physical parameters. Two of these J_m and r respectively are specified in the data sheet of the process and they have the values $J_m = 0.0021$ and $r = 5$ respectively. The remaining parameters to be estimated are therefore J_a, k, b, f , and k_τ respectively. The starting point for the identification, see [9], is the state space model structure on innovations form

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) + K(\theta)e(t) \quad (5)$$

$$y(t) = Cx(t) + e(t) \quad (6)$$

where θ is a vector containing the parameters to be estimated. The signal $e(t)$ is a zero mean

white disturbance and $K(\theta)$ is the Kalman filter gain. The unknown parameters in the vector θ are estimated using the System Identification Toolbox, see [10], and the model structure given by the problem is specified as a Matlab m-file. The identification results in the parameters presented in Table 1.

J_a	0.0991
k	35.1
b	0.0924
f	0.0713
k_τ	0.122

Table 1: Identified values of physical parameters.

The model is validated by simulating the model using the validation data set, and Figure 3 shows the measured and simulated output resulting from the validation. As can be seen the agreement is high. The Bode diagram of the identified model is shown in Figure 4.

Figure 3: Measured and simulated output from the physically parameterized models.

Before ILC can be applied the system has to be stabilized by some appropriate feedback. Since the aim is that the ILC algorithm shall give the desired servo performance the requirements for the feedback can be chosen fairly moderate. The regulator can be of PID or some more sophisticated type. In this paper it has been chosen to use an LQG controller since a state space model is available from the identification of the physically parameterized model. In the LQG controller only

Figure 4: Bode diagram of the estimated model.

the motor angle $\theta_m(t)$ is used for the feedback.

The reference signal is chosen such that the reference acceleration first is constant and positive during a first time interval and then negative and constant during a second interval. This means that the reference velocity will first grow linealy and then decrease linearly. The resulting reference angle is given by Figure 5.

Figure 5: Reference angle.

4 Arm angle estimation

The main idea in the paper is to use an estimate of the arm angle in the ILC algorithm. The estimate is formed as a combination of arm acceleration and motor angle by using a state estimator based on the nominal relationship between the motor and arm angles. Consider the torque bal-

ance of the arm

$$J_a \ddot{\theta}_a + f_a \dot{\theta}_a - k(\theta_m - \theta_a) = 0 \quad (7)$$

where for simplicity the damping coefficient d is put to zero. By considering θ_a and $\dot{\theta}_a$ as state variables, the arm acceleration $\ddot{\theta}_a$ as measured output signal, and the motor angle as input signal equation (7) can be expressed in state space form. Forming a Kalman filter for this system, the estimated arm angle can then be expressed using transfer functions as

$$\hat{\Theta}_a(s) = F_{\ddot{y}}(s)\bar{Y}(s) + F_{\theta_m}(s)\Theta_m(s) \quad (8)$$

where $\bar{Y}(s)$ denotes the Laplace transform of the arm angular acceleration. When designing the Kalman filter the variance R_2 of the arm acceleration signal can be used as a design variable that affects the balance between the measured arm acceleration and the measured motor angle. Choosing a high value of R_2 implies that the arm acceleration is almost neglected. The resulting filter from θ_m to $\hat{\theta}_a$ is then almost exactly the transfer function obtained from equation (7). The arm angle estimate is then obtained by feeding the motor angle through the nominal transfer function from motor angle to arm angle. In the nominal case this gives a good estimate of the arm angle but the estimate is very sensitive to model errors. On the other hand, choosing a low value of R_2 means that the arm acceleration plays a large role in the estimation of arm angle. The gain of the filters $F_{\ddot{y}}$ and F_{θ_m} for this frequency range is almost constant, see [5]. Considering equation (7) this behavior is logical since a slight reformulation of equation (7) gives

$$\theta_a(t) = \theta_m(t) - \frac{J_a}{k} \ddot{\theta}_a(t) - \frac{f_a}{k} \dot{\theta}_a(t) \quad (9)$$

With good measurements of $\ddot{\theta}_a(t)$ available an approximation of the arm angle is given by

$$\hat{\theta}_a(t) = \theta_m(t) - \frac{\hat{J}_a}{\hat{k}} \ddot{\theta}_a(t) \quad (10)$$

where the effect of the viscous friction has been neglected. The parameters $\hat{k}$ and $\hat{J}_a$ denote estimates of the spring constant and moment of inertia.

Since the actual arm angle $\theta_a(t)$ can be measured on the laboratory process it is straightforward to evaluate the idea. The process is controlled by the stabilizing controller, described in Section 3, which uses the motor angle $\theta_m(t)$ as feedback signal, and the reference signal described in Section 3 is applied. Figure 6 shows the measured and estimated arm angle during the last part of this experiment. The figure indicates that the simple estimation procedure gives a reasonably good estimate of the arm angle. One difference is the lower damping in the estimate, and this property can be explained by the neglected friction in the model used for estimation.

Figure 6: Measured (solid) and estimated (dashed) arm angle in the nominal case.

An important question is how sensitive the estimation procedure is for errors in the estimate of the factor J_a/k . This problem is dealt with in [5], and the main result is that the estimation procedure is rather insensitive to errors in this factor.

5 ILC algorithm

The problem can be described as in Figure 1 (without feed-forward controller) where the actual measurement of the controlled output is replaced by an estimate. Since the main goal is to achieve good servo properties the influence of the disturbances is not taken into account. There exist several approaches to the design of ILC algorithms, and in this paper an ILC algorithm based on optimization will be used. This approach has previously been considered in e.g [3], [8], [2], and

[4]. Introduce the vectors

$$\mathbf{Y}_k = (y_k(0), \dots, y_k(N))^T \quad (11)$$

$$\mathbf{U}_k = (u_k(0), \dots, u_k(N))^T \quad (12)$$

$$\mathbf{R} = (r(0), \dots, r(N))^T \quad (13)$$

$$\mathbf{E}_k = \mathbf{R} - \mathbf{Y}_k \quad (14)$$

where $u_k(t)$ and $y_k(t)$ denote the ILC input signal and the output signal respectively. Furthermore $r(t)$ denotes the desired output (reference) signal. The subscript k denotes iteration number. Using these notations the system can be described by the equation

$$\mathbf{Y}_k = \mathbf{T}_r \mathbf{R} + \mathbf{T}_u \mathbf{U}_k \quad (15)$$

where $\mathbf{T}_u$ is the lower triangular matrix formed by the impulse response coefficients of the transfer operator from ILC input signal $u_k(t)$ to output signal $y_k(t)$ and $\mathbf{T}_r$ is defined analogously. Equation (15) is a general formulation that covers systems working in open loop, where $u_k(t)$ represents the actual system input, as well as systems working in closed loop, where the signal $u_k(t)$ is added to the control signal generated by the conventional control system. The difference in conditions is reflected in different definitions of the matrices in equation (15). Consider now the criterion

$$\mathbf{J} = \mathbf{E}_{k+1}^T \mathbf{W}_e \mathbf{E}_{k+1} + \mathbf{U}_{k+1}^T \mathbf{W}_u \mathbf{U}_{k+1} \quad (16)$$

where $\mathbf{W}_e$ and $\mathbf{W}_u$ are weight matrices determining the trade off between performance and input energy. The criterion is minimized subject to the constraint

$$(\mathbf{U}_{k+1} - \mathbf{U}_k)^T (\mathbf{U}_{k+1} - \mathbf{U}_k) \leq \delta \quad (17)$$

The optimal input vector is updated according to

$$\mathbf{U}_{k+1} = \mathbf{Q}(\mathbf{U}_k + \mathbf{L}\mathbf{E}_k) \quad (18)$$

where

$$\mathbf{Q} = (\mathbf{W}_u + \lambda \cdot \mathbf{I} + \mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u)^{-1} \times (\lambda \cdot \mathbf{I} + \mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u) \quad (19)$$

and

$$\mathbf{L} = (\lambda \cdot \mathbf{I} + \mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u)^{-1} \mathbf{T}_u^T \mathbf{W}_e \quad (20)$$

The updating matrices $\mathbf{Q}$ and $\mathbf{L}$ hence depend on the nominal model $\mathbf{T}_u$ which is considered to be given and the weight matrices $\mathbf{W}_u$ and $\mathbf{W}_e$. The Lagrange multiplier λ is not computed explicitly but instead used as a design variable.

6 Experiments

Using the identified model and the feed-back controller it is straightforward to compute the discrete time transfer function from the ILC input to the arm angle, and hence also generate the matrix $\mathbf{T}_u$. Choosing appropriate design variables λ and ρ the matrices $\mathbf{Q}$ and $\mathbf{L}$ can be computed according to equations (19) and (20). A suitable choice of design variables, i.e. that gives a good trade-off between convergence rate and robustness, results in the convergence properties shown in figure 7. The algorithm has converged after five iterations and the max error is reduced to approximately 4% of the error obtained without ILC.

Figure 7: $\max_t |e_k(t)|$ as function of iteration number k .

Figure 8 shows the error signal $e_k(t)$ after 20 iterations, and the figure shows that the resulting error is of mainly high frequency character, which is a typical behavior of an ILC algorithm.

7 Conclusions

An ILC algorithm for a flexible mechanical system has been developed and evaluated using a laboratory scale robot arm with joint flexibility. The

Figure 8: The error signal after 20 iterations.

main components of the ILC algorithm is a dynamic model of the process obtained by using system identification and a procedure for estimating the arm angle by using the measurements of the arm acceleration. The algorithm performs well in experiments with rapid convergence and substantial reduction of the error.

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