Identification of Wiener systems with hard and discontinuous nonlinearities

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Abstract

The problem of identification of Wiener systems with special types of hard and discontinuous nonlinearities in the presence of process and measurement noises in observations to be processed has been considered. It is shown, that the original problem could be reduced to the problem of determination of the subsystem from the auxiliary network of subsystems, equivalent to the true linear system (linear part of the Wiener system). A technique based on the ordinary least squares, to be used in a case of missing data, and on the expectation maximization algorithm is proposed here. The results of numerical simulation of the discrete-time Wiener system with various hard and discontinuous nonlinearities by computer are given.

Keywords: System identification, parameter estimation, nonlinear systems, Wiener systems

1 Introduction

The main problem, arising in the estimation of the Wiener systems is the determination of an intermediate signal, acting between linear and nonlinear blocks. To facilitate this problem often requirements, such as, the nonlinearity to be analytic, usually a linear in unknown parameters polynomial are insisted. In order to get an estimate of an intermediate signal it is often assumed that the nonlinearity is invertible or is linear in a small region around the origin. In such a case the system is not nonlinear for sufficiently small inputs [1]. However such assumptions are not satisfied for hard and discontinuous nonlinearities, because they can not be described by polynomials and are noninvertible in general. On
the other hand, the considered nonlinearities are common in computer controlled systems [2].

In spite of problems, arising in the performance of such systems, there exist only several reports on identification of Wiener systems with hard nonlinearities and, especially, with the discontinuous ones [3]. It is, because of the difficulty of parametric identification of the Wiener system with such types of nonlinearities.

In practice, one has to work out special techniques, efficient in a case of missing data, that are cut off by some of hard nonlinearities, i.e., the saturation or dead-zone. On the other hand, such nonlinearities as the preload or hysteresis, do not cut off observations of an intermediate signal, when it is passing through. In such a case, nevertheless, there arises the problem to restore an intermediate signal, having input–output observations, too.

In the following we introduce the concept of auxiliary network of subsystems working in parallel to the true linear Wiener part. Based on it in Section 2 we prove uniqueness of equivalent subsystem to the linear part of the Wiener system. The reduction of the problem of identification of the Wiener system to the determination of the equivalent subsystem is analytically investigated in Section 3. Section 4 presents the parameters estimation technique, based on the expectation maximization (EM) algorithm and used in a case of missing data. In Section 5 the simulation results, obtained for various nonlinearities are presented. In Section 6 a Monte Carlo simulation results for periodical and for random signals are given.

2 The linear part of the Wiener system and its equivalent

The linear part of the Wiener system is dynamic, time invariant, causal, and stable. It can be represented by the time invariant dynamic system (LTI) with the transfer function $G(q, \Theta)$ as a rational function of the form

$$G(q, \Theta) = \frac{b_1 q^{-1} + \ldots + b_m q^{-m}}{1 + a_1 q^{-1} + \ldots + a_m q^{-m}} = \frac{B(q, b)}{1 + A(q, a)}$$

with the finite number of parameters

$$\Theta^T = (b_1, \ldots, b_m, a_1, \ldots, a_m), \quad b^T = (b_1, \ldots, b_m), \quad a^T = (a_1, \ldots, a_m),$$

that are determined from the area $\Omega$ of the permissible parameter values. The area $\Omega$ is restricted by the conditions of the stability of the respective difference equation and normalized, requiring the static gain of the linear model to be 1.

Assume that in parallel to the true linear part of the Wiener system there is acting the auxiliary network of $L$ subsystems, described by the same difference equation with the same initial conditions but with different parameters $\Theta_i = \Theta + \Delta\Theta_i, \forall i = 1, L$. Thus, the transfer function of some $l$–th auxiliary subsystem from the above mentioned network can be described by the following expression

$$G_l(q, \Theta_i) = \frac{(b_1 + \Delta b_1) q^{-1} + \ldots + (b_m + \Delta b_m) q^{-m}}{1 + (a_1 + \Delta a_1) q^{-1} + \ldots + (a_m + \Delta a_m) q^{-m}} = \frac{B_l(q, b + \Delta b)}{1 + A_l(q, a + \Delta a)},$$

with
where
\[
\Delta b_i^T = (\Delta b_1, \ldots, \Delta b_m)_i, \Delta a_i^T = (\Delta a_1, \ldots, \Delta a_m)_i.
\] (4)

The true linear part (1) and linear auxiliary subsystems are excited by the same persistently exciting input \(u(t)\). The true system (1) responds producing the output \(x(t)\), that is the intermediate signal of the Wiener system, while the auxiliary subsystem by generating the \(i\)th output \(\hat{x}_i(t) \forall i = 1, L\), that is the estimate of the intermediate signal. In order to determine the deviation of \(\hat{x}_i(t) \forall i = 1, L\) from \(x(t)\) the intermediate signal error
\[
\Delta x_i = \sqrt{\frac{1}{N} \sum_{k=1}^{N} [x(k) - \hat{x}_i(k)]^2} \forall k = 1, L,
\] (5)
is calculated, assuming that the true intermediate signal \(x(k) \forall k = 1, N\) is known beforehand. Here \(N\) is the number of observations to be processed.

**Proposition 1.** The error (5) is equal to zero if and only if
\[
\frac{B_i(q, b + \Delta b_i)}{1 + A_i(q, a + \Delta a_i)} = \frac{B(q, b)}{1 + A(q, a)}.
\] (6)

**Proof.** The intermediate signal error could be rewritten in such a form
\[
\Delta x_i = \sqrt{\frac{1}{N} \sum_{k=1}^{N} [x(k) - \hat{x}_i(k)]^2}
\] (7)
\[
= \sqrt{\frac{1}{N} \sum_{k=1}^{N} \left[ \frac{B(q, b)}{1 + A(q, a)} u(k) - \frac{B_i(q, b + \Delta b_i)}{1 + A_i(q, a + \Delta a_i)} u(k) \right]^2}
\] (8)
\[
= \sqrt{\frac{1}{N} \sum_{k=1}^{N} \left[ \left( \frac{B(q, b)}{1 + A(q, a)} - \frac{B_i(q, b + \Delta b_i)}{1 + A_i(q, a + \Delta a_i)} \right) u(k) \right]^2}.
\]
The expression in brackets (difference of fractions) is equal to zero for such \(i\)th linear subsystem from the network of auxiliary subsystems, that assures
\[
B_i(q, b + \Delta b_i) = B(q, b) \quad \text{and} \quad A_i(q, a + \Delta a_i) = A(q, a).
\]
For the same order polynomials to be equal follows that
\[
\Delta b_1 = \ldots = \Delta b_m = \Delta a_1 = \ldots = \Delta a_m = 0.
\] (8)

**Remark 1.** Under considered conditions each linear subsystem is unique in the meaning of parameters and output.

**Remark 2.** If two subsystems, described by the same difference equations with the same initial conditions are excited by the same input signal and generate the same output signal, then both subsystems have the same values of parameters and are equivalent.
Conclusion 1. If in parallel to the true linear part of the Wiener system (1) is acting the auxiliary network of subsystems, that are described by the linear difference equations of the same order with the same initial conditions, and all of them are excited by the same input, then the output of the true linear part will be equal to the output of that auxiliary subsystem, that is equivalent to the true one.

3 Reduction of the problem of identification of the Wiener system

In practice, the intermediate signal $x(t)$ is unknown. Thus, the criterion (5) cannot be used directly to find the subsystem, equivalent to the true linear part (1). Therefore there arises a problem to change or to modify the criterion (5).

The unknown intermediate signal $x(t)$, generated by the linear part of the Wiener system as response to the input $u(t)$, is acting on the static nonlinear part $f(\cdot, \eta)$, which responds by generating the signal $y(t)$. At first, let us assume, that the structure of output nonlinearity is known beforehand. We consider here the common examples of, so called, hard nonlinearities, such as, the saturation, preload, dead-zone and hysteresis (see Fig. 1) and some of discontinuous nonlinearities (see Fig. 2) [3], [4]. For simplicity, the threshold $\eta = a$ for each of nonlinearities is chosen the same and known, too.

![Figure 1: Examples of hard nonlinearities: saturation (a), dead-zone (b), preload (c), hysteresis (d).](image-url)
Suppose now, that output signals $\hat{x}_1(t), \hat{x}_2(t), \ldots, \hat{x}_L(t)$ of the linear auxiliary subsystems $G_1(q, \Theta_1), G_2(q, \Theta_2), \ldots, G_L(q, \Theta_L)$ are passed through the same nonlinear blocks, acting in parallel. The nonlinear blocks are equivalent to the nonlinear part of the true Wiener system. Then on the output of a nonlinearities $f_i(\cdot, a) \forall i = 1, L$ signals $\hat{y}_1(t), \hat{y}_2(t), \ldots, \hat{y}_L(t)$ are observed.

In order to determine the deviation of any observed $\hat{y}_i(t) \forall i = 1, L$ from the output $y(t)$ of the true Wiener system one can calculate the output error

$$\Delta y_i = \sqrt{\frac{1}{N} \sum_{k=1}^{N} [y(k) - \hat{y}_i(k)]^2} \quad \forall \quad i = 1, L. \quad (9)$$

**Proposition 2.** The output error (9) is equal to zero if the true linear part of the Wiener system and the $l$th $\forall l \in 1, L$ linear auxiliary subsystem are equivalent, i.e., $G_i(q, \Theta) = G(q, \Theta)$.

**Proof (Case of the saturation).** Suppose, that the nonlinear block of the true Wiener system is the saturation. Then, the signal $y(k)$ in the formula (9) is equal to $x(k)$ if $|x(k)| < a$, assuming that for the saturation

$$y(k) \equiv y^s(k) = \frac{1 + sgn(a - |x(k)|)}{2} x(k) + \frac{1 + sgn(|x(k)| - a)}{2} a sgn(x(k)). \quad (10)$$

Here $sgn$ is the standard sign function; $a > 0$ is the threshold.

In the opposite case $y^s(k)$ is equal to $a$ if $x(k) > 0$ or is equal to $-a$ if $x(k) < 0$. In the first case $\Delta y_i$ will be equal to zero only for the subsystem (6), that is equivalent to the true linear part of the Wiener system. In the opposite case $\Delta y_i$ is equal to zero for all subsystems outputs, including the output $\hat{x}_1(k)$ of the subsystem (6), if absolute values of outputs $\hat{x}_1(k), \hat{x}_2(k), \ldots, \hat{x}_L(k)$ of $G_1(q, \Theta_1), G_2(q, \Theta_2), \ldots, G_L(q, \Theta_L)$ are larger than $a$.

**Remark 3.** For the saturation the output error (9) could be reduced to

$$\Delta y_i = \Delta y^s_i = \sqrt{\frac{1}{N_1} \sum_{k=1}^{N_1} [y(k) - \hat{y}_i(k)]^2} \equiv \Delta x^s_i = \sqrt{\frac{1}{N_1} \sum_{k=1}^{N_1} [x(k) - \hat{x}_i(k)]^2}, \quad (11)$$

if the subsystem (6) is applied. Here $N_1 < N$ is large enough number of observations, that are left after rejecting the observations $y(k) \equiv y^s(k) = a$ and the same observations $\hat{y}_i(k)$ of the output of the subsystem (6).

**Proof (Case of the dead-zone).** Suppose now, that the nonlinear block of the true Wiener system is the dead-zone. Then, the signal $y(k)$ in the formula (9) is equal to 0 if $|x(k)| < a$, assuming that for the dead-zone

$$y(k) \equiv y^{dz}(k) = x(k) - a sgn(x(k)) - \frac{1 + sgn(a - |x(k)|)}{2} (x(k) - a sgn(x(k))). \quad (12)$$

In the opposite case $y^{dz}(k)$ is equal to $x(k) - a sgn(x(k))$. In the first case $\Delta y_i$ is equal to zero for all outputs of subsystems, including the output $\hat{x}_i(k)$ of the subsystem (6), if absolute values of auxiliary subsystems $G_1(q, \Theta_1), G_2(q, \Theta_2), \ldots, G_L(q, \Theta_L)$ are larger than $a$.

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$G_L(q, \Theta_L)$ outputs $\hat{x}_1(k), \hat{x}_2(k), \ldots, \hat{x}_L(k)$ are less than $a$. In the opposite case $\Delta \hat{y}_i$ will be equal to zero only for the subsystem (6), that is equivalent to the true subsystem.

Remark 4. For the dead-zone the output error (9) could be reduced to

$$\Delta y_i = \Delta \hat{y}_i^{dz} = \sqrt{\frac{1}{N_1} \sum_{k=1}^{N_1} [y(k) - \hat{y}(k)]^2} \equiv \Delta x_i^{dz} = \sqrt{\frac{1}{N_1} \sum_{k=1}^{N_1} [x(k) - \hat{x}(k)]^2},$$

if the subsystem (6) is applied. Here $N_1 < N$ is enough large number of observations, that are left after rejecting the observations $y(k) \equiv y^{dz}(k) = 0$ and the same observations $\hat{y}(k)$ of the output of the subsystem (6); $\hat{x}_i(k) = \hat{y}_i(k) + \text{sgn}(\hat{y}_i(k))$.

Proof (Case of the preload and hysteresis). The signals $y(k)$ for the preload and hysteresis are determined according to

$$y(k) \equiv y^h(k) = x(k) + a \text{sgn}(x(k))$$

and

$$y(k) \equiv y^h(k) = \begin{cases} x(k) - a & \text{if } x(k) - x(k-1) > 0, \\ x(k) + a & \text{if } x(k) - x(k-1) < 0, \\ y(k-1) & \text{if } x(k) = x(k-1), \end{cases}$$

Figure 2: Examples of discontinuous nonlinearities with the different leaning over of linear segments: $m_1 \neq m_2$ (a), $m_1 = m_2$ (b), $m_2 = 0$ (c), $m_1 = 0$ (d).
Remark 5. For the preload the output error (9) could be reduced to

$$\Delta x_i = \Delta x_i^p = \frac{1}{N} \sqrt{\sum_{k=1}^{N} [x(k) - \hat{x}_i(k)]^2},$$

(16)

if the subsystem (6) is applied. Here \( \hat{x}_1(k) = \hat{y}_1(k) - a \text{sgn}(\hat{y}_1(k)) \).

Remark 6. For the hysteresis the output error (9) could be reduced to

$$\Delta x_i \equiv \Delta x_i^h = \sqrt{\frac{1}{N_1} \sum_{k=1}^{N_1} e_1^2 + \frac{1}{N_2} \sum_{k=1}^{N_2} e_2^2 + \frac{1}{N_3} \sum_{k=1}^{N_3} e_3^2},$$

(17)

if the subsystem (6) is applied. Here

- \( e_1(k) = x(k) - \hat{x}_i^{(1)}(k) \) for \( \hat{x}_i^{(1)}(k) = \hat{y}_i(k) + a, \) when \( \hat{x}_i^{(1)}(k) - \hat{x}_i^{(1)}(k-1) > 0 \),
- \( e_2(k) = x(k) - \hat{x}_i^{(2)}(k) \) for \( \hat{x}_i^{(2)}(k) = \hat{y}_i(k) - a, \) when \( \hat{x}_i^{(2)}(k) - \hat{x}_i^{(2)}(k-1) < 0 \),
- \( e_3(j) = x(k) - \hat{x}_i^{(3)}(k-1), \) when \( \hat{x}_i^{(3)}(k) = \hat{x}_i^{(3)}(k-1), N_1 + N_2 + N_3 = N \).

Proof (Case of the discontinuous nonlinearity). For the discontinuous nonlinearity the output signal \( y(k) \) is calculated according to

$$y(k) \equiv y^{dn}(k) = \begin{cases} \hat{y}(k) = K(k) \{ x(k) - D \text{sgn}[x(k)] \} + b \text{sgn}[x(k)] & \text{if } |x(k)| > D, \\ 0 & \text{if } |x(k)| < D, \end{cases}$$

(18)

$$K(k) = m_1 + (m_2 - m_1)h(k),$$

(19)

where \( |m_1| < \infty, |m_2| < \infty \) are respective coefficients, determining leaning over of corresponding linear segments, \( 0 \leq D < \infty \) is the dead zone and \( |b| < \infty \) is the preload constant.

The switching function \( h(k) \) has the form

$$h(k) \equiv h[x(k)] = \begin{cases} 0 & \text{if } x(k) > 0, \\ 1 & \text{if } x(k) < 0. \end{cases}$$

(20)

Then, according to (18) the signal \( y(k) \) is equal to \( \hat{y}(k) \) if \( |x(k)| > D \) or is equal to 0 in the opposite case.

In the first case \( \Delta y_i \) will be equal to zero only for the subsystem (6), that is equivalent to the true linear part of the Wiener system. In such a case the output error (9) will be equal to zero only if \( \hat{x}_i(k) = \hat{x}_i(k) \) for \( i = 1, L \). In the opposite case \( \Delta y_i \) is equal to zero for all outputs of subsystems, including the output \( \hat{x}_1(k) \) of the subsystem (6), if absolute values of outputs \( \hat{x}_1(k), \hat{x}_2(k), \ldots, \hat{x}_L(k) \) of auxiliary subsystems \( G_1(q, \Theta_1), G_2(q, \Theta_2), \ldots, G_L(q, \Theta_L) \) are less than \( D \). \( \square \)
Remark 7. For the discontinuous nonlinearity the output error (9) could be reduced to

\[ \Delta x_l \equiv \Delta x_l^{dn} = \sqrt{\frac{1}{N_1} \sum_{k=1}^{N_1} e_1^2 + \frac{1}{N_2} \sum_{k=1}^{N_2} e_2^2}, \]  

if the subsystem (6) is applied. Here

\[ e_1(k) = x(k) - \hat{x}_i^{(1)}(k) \text{ for } \hat{x}_i^{(1)}(k) = m_1^{-1} \hat{y}_i(k) + D \text{sgn}(\hat{y}_i(k)) - b m_1^{-1} \text{sgn}(\hat{y}_i(k)), \]

when \( \hat{x}_i^{(1)}(k) > 0 \),

\[ e_2(k) = x(k) - \hat{x}_i^{(2)}(k) \text{ for } \hat{x}_i^{(2)}(k) = m_2^{-1} \hat{y}_i(k) + D \text{sgn}(\hat{y}_i(k)) - b m_2^{-1} \text{sgn}(\hat{y}_i(k)), \]

when \( \hat{x}_i^{(2)}(k) < 0 \), \( N_1 + N_2 < N \).

Statement 1. If in the auxiliary network of subsystems \( G_1(q, \Theta_1), G_2(q, \Theta_2), \ldots, G_L(q, \Theta_L) \) there exists the \( l \)th \( (8 \leq l \leq L) \) subsystem, ensuring that the output error (9) is equal to zero or is minimal as compared with output errors, calculated for others subsystems, then such \( l \)th subsystem will be equivalent to the linear part of the Wiener system.

The Statement 1 follows from Proposition 1 and Proposition 2.

Conclusion 2. For all above mentioned nonlinearities, including the discontinuous nonlinearity, \( \Delta y_i \) of the form (9) will be zero only for the auxiliary subsystem (6), that is equivalent to the true linear part of the Wiener system. Under considered conditions the problem of identification of the Wiener system with hard nonlinearities (the saturation, dead-zone, preload and hysteresis) or with the discontinuous nonlinearity (18)–(20) could be reduced to the problem of a determination of the linear auxiliary subsystem (6).

4 Estimation of parameters, using data sets with missing observations

There are at least two ways to solve the problem of a determination of the linear subsystem, equivalent to the true linear part of the Wiener system.

First way is based on the simulation of a network of auxiliary subsystems and on the generation of their outputs \( \hat{y}_1(t), \hat{y}_2(t), \ldots, \hat{y}_{L-1}(t), \hat{y}_L(t) \), choosing for each output the denominator coefficients \( a_1, \ldots, a_m \) of \( G_i(q, \Theta_i) \forall i \in \overline{1,L} \) from the area \( \Omega \), restricted by the conditions of the stability of the respective difference equation. In turn, the coefficients of the numerator \( b_1, \ldots, b_m \), are determined by the constraint, requiring the static gain of the linear model to be 1. Afterwards, the subsystem (6) could be obtained by calculating for each subsystem from the network the output error (9) and solving its minimization problem.

Second way is more attractive and is based on linear system parameters estimation techniques, but used in a case of cut off or missing data. Here we propose the new Wiener system parameters identification technique, that fall in the framework of the EM algorithm.

It was mentioned above, that for the saturation the output \( y(t) \) of the Wiener system partly coincides with an intermediate signal \( x(t) \), assuming that there
exist some parts of $x(t)$, when $|x(t)| < a$, that passed through the nonlinearity $f(\cdot, a)$ without any special processing. Then the different parts of $y(t)$, coincides with those respective parts of $x(t)$. However, in fact, the most observations of an intermediate signal $x(t)$ are cut off by the nonlinear block—saturation. Therefore there arrises the problem to identify the parameters of the linear block of the Wiener system by $u(t)$ and $y(t)$ with cut-off observations or, the same, by $u(t)$ and $x(t)$ with missing observations. In such a case the LS estimate

$$\hat{\Theta} = \left( X^T X \right)^{-1} X^T Y$$  \hspace{1cm} (22)$$

will fail because of the presence of such observations not only in the vector $Y$ but also in the matrix $X$. Here

$$\hat{\Theta}^T = \begin{pmatrix} \hat{b}, \hat{a} \end{pmatrix}^T, \quad \hat{b}^T = (\hat{b}_1, \ldots, \hat{b}_m), \quad \hat{a}^T = (\hat{a}_1, \ldots, \hat{a}_m)$$  \hspace{1cm} (23)$$

are $2m \times 1, m \times 1, m \times 1$ vectors of estimates of parameters, respectively,

$$X = \begin{bmatrix} u(m) & \ldots & u(1) & -y(m) & \ldots & -y(1) \\ u(m+1) & \ldots & u(2) & -y(m+1) & \ldots & -y(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u(N-1) & \ldots & u(N-m) & -y(N-1) & \ldots & -y(N-m) \end{bmatrix}$$  \hspace{1cm} (24)$$

is the $(N - m) \times 2m$ matrix, consisting of input-output observations,

$$Y = (y(m+1), y(m+2), \ldots, y(N))^T$$  \hspace{1cm} (25)$$

is the $(N - m) \times 1$ vector of output’s observations.

Usually during the estimation of parameters (23) the regression matrix $X$ contains observations of the output corrupted by the process and measurements noises, as well as the observations, that are cut off by the nonlinear part of the Wiener system. Therefore all proofs based on a deterministic approach of the LS cannot be applied here anymore. In such a case the ordinary LS loses its optimality and, as a result, the expression (22) applied for $\hat{\Theta}$ calculation appeared to be inefficient.

The problem could be solved by substituting instead of cut off observations in (24) and (25) the respective observations of the auxiliary signal $\hat{x}_l(t) \forall l = 1, \ldots, L$. That for enough large $N$ would be approximately equal to $x(k) \forall k = 1, \ldots, N$. In such a case the intermediate signal error (9) acquires a minimum. It could be noted, that in a noisy environment of data to be processed preferably to change all $y(t)$ observations in spite of they are cut off or non-cut off.

**Statement 2.** If there exist such $\hat{x}_l(t) \forall l \in \overline{1, L}$ of the $l$th subsystem, that for enough large $N$ partly coincides with the $y(t)$ of the true Wiener system and coinciding parts are, namely, the parts of the intermediate signal $x(t)$, then the criterion of the form (11) will be minimal with respect of others $\Delta x_i \forall i \in \overline{1, L}$.

The **Statement 2** follows from the **Proposition 2**.

**Remark 8.** An auxiliary signal $\hat{x}_l(t) \forall l \in \overline{1, L}$ is the estimate of the output of the auxiliary subsystem, equivalent to the true linear part of the Wiener system.
The signal \( \hat{x}_l(t) \) \( \forall l \in \mathbb{Z} \) can be determined using the technique, based on the EM algorithm, consisting of the expectation step (E-step) in which the conditional expectation of the sufficient statistics (in our case the auxiliary signal) is calculated given available data and the current estimates of the parameters, and the maximization step (M-step) in which the estimated sufficient statistics obtained in the E-step are applied to recalculate parameter estimates. The technique to be used could be explained by the following steps:

(a) cut off observations of \( y(t) \) could be rejected beforehand;
(b) the LS problem, consisting of the identification of parameters of a LTI system with a finite impulse response (FIR), could be solved;
(c) the auxiliary signal \( \hat{x}_l(t) \) \( \forall l \in \mathbb{Z} \) (the estimate of \( x(t) \), including its missing values) could be calculated, using estimates of parameters of FIR filter, that was determined on (b);
(d) return to (b) and estimate the parameters using both observed data and missing values estimates.

Iteration of above mentioned calculations are continued until convergence is assured.

Thus, by rejecting some portions of incorrect observations one will deal with missing data, that further will be replaced by the missing value estimates, determined at a E-step of EM.

In such a case a dependence of some regressors from the process output will be facilitated and the assumption of the ordinary LS, that the regressors depend only on input signal, will be satisfied. However, before the estimation of parameters of FIR filter one has to sort out correct samples from the initial set of observations \( y(t) \) or to delete some incorrect data portions. Anyway one has to deal the regression problem with missing data. It is known, that in the linear regression model an observation could be deleted without affecting the consecutive ones. On the other hand, it is obvious, that the elimination of observations could have some influence on the accuracy of estimates of parameters to be calculated. In ours work it will be investigated by the comprehensive simulation.

For the estimation of parameters of FIR filter

\[
y(t) = f_0u(t) + f_1u(t-1) + \ldots + f_\nu u(t-\nu) \quad (26)
\]

one can use the expression

\[
\hat{\beta} = \left( \mathbf{A}^T \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{\hat{Y}}. \quad (27)
\]

Here

\[
\hat{\beta}^T = (\hat{f}_0, \hat{f}_1, \ldots, \hat{f}_\nu) \quad (28)
\]

is a \((\nu + 1) \times 1\) vector of estimates of parameters \( \beta^T = (f_0, f_1, \ldots, f_\nu) \),

\[
\mathbf{A} = \begin{bmatrix}
u(\nu + 1) & u(\nu) & \ldots & u(1) \\
u(\nu + 2) & u(\nu + 1) & \ldots & u(2) \\
\vdots & \vdots & \ddots & \vdots \\
u(N-K) & u(N-K-1) & \ldots & u(N-K-\nu)
\end{bmatrix} \quad (29)
\]
is the \((N - K - \nu) \times \nu\) regression matrix, consisting only of observations of an input \(u(t)\) of the true Wiener system,

\[
\hat{Y} = (y(\nu + 1), y(\nu + 2), \ldots, y(N - K))^T
\]

(30)
is the \((N - K - \nu) \times 1\) vector of sorted out observations of the output \(y(t)\) of the true Wiener system, \(K\) is the number of cut off observations, that are rejected beforehand.

Then the auxiliary signal \(\hat{x}_l(t) \forall l = \overline{1, L}\) is calculated, according to the expression

\[
\hat{x}_l(j) = f_0 u(j) + f_1 u(j - 1) + \ldots + f_\nu u(j - \nu) \quad \forall \quad j = \nu + 1, N - K,
\]

(31)
and the estimation of parameters \(\beta^T = (f_0, f_1, \ldots, f_\nu)\) is repeated, substituting instead of missing observations in (30) respective values of the auxiliary signal \(\hat{x}_l(t) \forall l = \overline{1, L}\). Afterwards, the samples of \(\hat{x}_l(t) \forall l \in \overline{1, L}\) are substituted instead of \(y(t)\) observations into (24) and (25) and estimate (23) is calculated by (22).

It could be mentioned, that the deletion of any observation or some of observations is equivalent to the deletion of an respective equation or some equations, respectively, in

\[
\hat{Y} = \Lambda \beta.
\]

(32)

For the comparison of estimates the estimation technique (27), based on the FIR filter, as well as the ordinary LS of the form (22) will be used here.

5 Simulation results

5.1 Nonlinear block—saturation

The true intermediate signal of the Wiener system is given by

\[
x(t) = G(q, \Theta)u(t),
\]

(33)
and the true output signal by

\[
y^*(t) = \frac{1 + \text{sgn}(a - |x(t)|)}{2} x(t) + \frac{1 + \text{sgn}(|x(t)| - a)}{2} a \text{sgn} x(t)
\]

(34)
with the sum of sinusoids (see Fig. 3)

\[
u(t) = \frac{1}{20} \sum_{k=1}^{20} \sin(k \pi t / 10 + \phi_k)
\]

(35)
as an input to the linear block

\[
G(q, \Theta) = \frac{b_1 q^{-1}}{(1 + c_1 q^{-1})^2}.
\]

(36)
Here \(b_1 = 1, c_1 = -0.7, a = 7.5\); in (35) the stochastic variables \(\phi_k\) with uniform distribution on \([0, 2\pi]\) were chosen.
The process noise
\[ v(t) = \frac{1}{(1 + c_1 q^{-1})^2} \xi(t), \]  
(37)
and the measurement noise
\[ e(t) = \zeta(t), \]  
(38)
are added to the signals \( x(t) \) and \( y(t) \), respectively. Here \( \zeta(t) \) and \( \xi(t) \) are noncorrelated between each other sequences of independent Gaussian variables with zero means and variances \( \sigma^2_\zeta, \sigma^2_\xi \).

\( N = 100 \) data points were generated, without additive process and measurement noises (see Fig. 3), and with ones (see Fig. 4) according to
\[ x(t) = x(t) + \lambda_1 v(t) \]  
(39)
and
\[ y^s(t) = y^s(t) + \lambda_2 e(t), \]  
(40)
respectively. Here \( \lambda_1 = 0.185, \lambda_2 = 0.704 \) were chosen such that variances \( \sigma^2_v = \sigma^2_e = 0.5 \). Then, signal-to-noise ratios (SNR—the square root of the ratio of signal and noise variances) are equal to 20 for the process noise SNR\( v \) and to 9 for the measurement one SNR\( e \).

The LS estimates (22) of parameters of the transfer function \( G(q, \Theta) \) were calculated, using observations of \( u(t) \) and \( y(t) \). Afterwards, the estimate of the intermediate signal \( x(t) \) could be found as follows (see expressions (33), (36))
\[ \hat{x}_1(t) = b_1 u(t) - a_1 \hat{x}_1(t - 1) - a_2 \hat{x}_1(t - 2) \quad \forall \quad t = \nu + 1, N \]  
(41)
by substituting instead of true values of parameters $b_1 = 1$, $a_1 = -1.4$, $a_2 = 0.49$ their estimates: $\hat{b}_1 = 0.5034$, $\hat{a}_1 = -1.2660$, $\hat{a}_2 = 0.4117$, respectively, in the unnoisy case, and $\tilde{b}_1 = 0.5681$, $\tilde{a}_1 = -1.1312$, $\tilde{a}_2 = 0.2781$ in the opposite one. In (41) $\hat{x}_1(2) = \hat{x}_1(1) = 0$, $\nu = 2$ were chosen.

The intermediate signal $x(t)$ (curve 1) and its reconstructed counterpart $\hat{x}_1(t)$ (curve 2) are shown in Fig. 5.

The LS problem (27), consisting of the identification of parameters of FIR filter

$$y(t) = f_0 u(t) + f_1 u(t - 1) + \ldots + f_\nu u(t - \nu) \quad \forall \quad t = \nu + 1, N.$$  \hspace{1cm} (42)

was solved, at first, without rejecting of incorrect observations. The whole number of parameters $\nu = 14$ was chosen. Afterwards, the intermediate signal $x(t)$ was reconstructed

$$\hat{x}_2(t) = f_0 u(t) + f_1 u(t - 1) + \ldots + f_\nu u(t - \nu),$$ \hspace{1cm} (43)

by substituting instead of unknown true values of parameters their estimates, calculated before.

The FIR filter parameters estimation problem was repeated once more. In such a case the 60 cut off observations from 100 initial ones were rejected beforehand and estimates of FIR filter parameters were calculated, using only 40 observations $y(t)$. The auxiliary signal $\hat{x}_3(t)$ was determined according to the expression (43), by substituting instead of respective estimates, calculated without rejection of cut off observations, the estimates obtained with rejection ones. In Fig. 5 curves 3,4 correspond to the signals $\hat{x}_2$ and $\hat{x}_3$, respectively.
Observations

Figure 5: The intermediate signal \( x(t) \) and its reconstructed versions for the saturation. Curves: \( x(t)|_1, ^\hat{x}_1(t)|_2, ^\hat{x}_2(t)|_3, ^\hat{x}_3(t)|_4 \). Unnoisy case (a), the noisy one with \( \sigma_u^2 = \sigma_\epsilon^2 = 0.5 \) (b).

The final estimate of parameters \( \Theta \) of the transfer function \( G(q, \Theta) \) was calculated according to

\[
\hat{\Theta} = (\hat{X}^T \hat{X})^{-1} \hat{X}^T \hat{Y},
\]

using the observations of the auxiliary signal \( \hat{x}_3(t) \). Here

\[
\Theta^T = (\hat{b}, \hat{a})^T, \quad \hat{b}^T = (b_1, \ldots, b_m), \quad \hat{a}^T = (a_1, \ldots, a_m)
\]

is a \( 2m \times 1, m \times 1, m \times 1 \) vectors of estimates of parameters, respectively,

\[
X = \begin{bmatrix}
  u(m) & \cdots & u(1) & -\hat{x}_3(m) & \cdots & -\hat{x}_3(1) \\
  u(m+1) & \cdots & u(2) & -\hat{x}_3(m+1) & \cdots & -\hat{x}_3(2) \\
  \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
  u(N-1) & \cdots & u(N-m) & -\hat{x}_3(N-1) & \cdots & -\hat{x}_3(N-m)
\end{bmatrix}
\]

is the \((N-m) \times 2m\) matrix, consisting of observations of input \( u(t) \) and the auxiliary signal \( \hat{x}_3(t) \),

\[
\hat{Y} = (\hat{x}_3(m+1), \hat{x}_3(m+2), \ldots, \hat{x}_3(N))^T
\]

is the \((N-m) \times 1\) vector of reconstructed output’s \( \hat{x}_3(t) \) observations.

Such estimates of parameters of equation (41) were determined: \( \hat{b}_1 = 0.9467, \hat{a}_1 = -1.4650, \hat{a}_2 = 0.4947 \) for the unnoisy case, and \( \hat{b}_1 = 0.9951, \hat{a}_1 = -1.3999, \hat{a}_2 = 0.4867 \) for the opposite one.
The ordinary LS estimates of parameters of $G(q, \Theta)$, calculated by (22), using observations $u(t)$ and even unnoisy $y(t)$, are biased. Influence of the noisy $y(t)$ on the accuracy of estimates is significant. Therefore such LS estimates (see Fig. 5, curve 2), as well as the estimates of parameters of FIR filter (43), obtained without rejection of cut off observations (see Fig. 5, curve 3) can not be applied to reconstruct the intermediate signal $x(t)$. It can be noticed, that both curves (2 and 3) nearly coincide, approaching to each other, but not to curve 1, that represents the true $x(t)$. On the other hand, the same FIR filter could be applied if some set of incorrect observations is rejected beforehand. In such a case the reconstructed signal (see Fig. 5, curve 4) approaches to the true signal $x(t)$ (curve 1) even for the intensive noise. The estimates of parameters, determined by expression (44), are close to the respective true parameter values of $G(q, \Theta)$ even in a noisy case.

5.2 Nonlinear block—dead-zone

The true output signal $y(t)$ is calculated, using the expression

$$y^{dz}(t) = x(t) - a \operatorname{sgn}(x(t)) - \frac{1 + \operatorname{sgn}(a - |x(t)|)}{2}(x(t) - a \operatorname{sgn}(x(t))).$$ (48)

$N = 100$ data points were generated, without additive process’s and measurement noises and with ones (see Fig. 6) according to (39) and

$$y^{dz}(t) = y^{dz}(t) + \lambda_2 e(t),$$ (49)

respectively. Here $\lambda_1 = 0.185, \lambda_2 = 0.704$ were chosen such, that variances $\sigma_v^2 = \sigma_e^2 = 0.5$. Then, signal-to-noise ratios are $\text{SNR}_v=20$ and $\text{SNR}_e=12$. The Wiener system’s intermediate and output signals without added noises and in the opposite case are shown in Fig. 6a, c, respectively. The signal $\hat{x}_1(t)$ is calculated according to the expression (41) by substituting instead of true values of parameters their LS estimates (22): $\hat{b}_1 = 0.5314, \hat{a}_1 = -1.3617, \hat{a}_2 = 0.4712$ in an unnoisy case, and $\hat{b}_1 = 0.6401, \hat{a}_1 = -1.2307, \hat{a}_2 = 0.3416$ in an opposite one.

Then, the LS problem (27), consisting of the identification of parameters of FIR filter (42) was solved, at first, without rejecting of incorrect observations. Here the whole number of parameters $\nu = 14$ was chosen. Afterwards, the intermediate signal $x(t)$ was reconstructed according to (43) and the signal $\hat{x}_2(t)$ was determined. The FIR filter parameters estimation problem was repeated once more. In such a case, similar to the case of the saturation, the 60 cut off observations from 100 initial ones were rejected beforehand. Then, estimates of FIR filter parameters were calculated, using only 40 observations

$$\hat{x}(t) = y(t) + a \operatorname{sgn}(y(t)),$$ (50)

Afterwards, they were substituted instead of respective values of $y(\cdot)$ in (30). Then, the auxiliary signal $\hat{x}_3$ was calculated according to the expression (43).

The intermediate signal $x(t)$ and three its reconstructed versions $\hat{x}_1(t), \hat{x}_2(t), \hat{x}_3(t)$ are shown in Fig. 6 b, d as curves 3—5, respectively. The values of the signal $\hat{x}_3(t)$ were substituted into the matrix (46) and vector (47), and estimates of parameters were calculated by (44). For the unnoisy case $\hat{b}_1 = 0.9688, \hat{a}_1 = \ldots$
Figure 6: The intermediate signal $x(t)$, output signal $y^{\pm}(t)$ and reconstructed versions of $x(t)$ for the dead-zone. Unnoisy environment (a, b), noisy one(c, d). Curves: $x(t)|1$, $y^{(d-1)}(t)|2$, $\hat{x}_1(t)|3$, $\hat{x}_2(t)|4$, $\hat{x}_3(t)|5$.

$-1.3762$, $\hat{a}_2 = 0.4690$ and for the noisy one $\hat{b}_1 = 0.8731$, $\hat{a}_1 = -1.3819$, $\hat{a}_2 = 0.4777$

The signal $\hat{x}_1(t)$ (curve 3), based on the estimates of the ordinary LS (22), as compared with the signals $\hat{x}_2(t)$, $\hat{x}_3(t)$ is the worst estimate of $x(t)$. Signal $\hat{x}_2(t)$ is less accurate, comparing with $\hat{x}_3(t)$ (curves 4 and 5, respectively). The signal $\hat{x}_3(t)$ (curve 5) deviates from $x(t)$ (curve 1) negligible, when a transition process of parameters estimation is over.

5.3 Nonlinear block—preload

The true output signal $y(t)$ is calculated, using the expression

$$ y^p(t) = x(t) + a \text{sgn}(x(t)). $$

$N = 100$ data points were generated, without additive process and measurement noises and with ones (see Fig. 7) according to (39) and

$$ y^p(t) = y^p(t) + \lambda_2 e(t), $$

respectively. Here $\lambda_1 = 0.185, \lambda_2 = 0.704$ were chosen such that variances $\sigma^2_{\epsilon} = \sigma^2_{\xi} = 0.5$. Then, signal-to-noise ratios are $SNR^p = 20$ and $SNR^e = 29$. The Wiener system’s intermediate and output signals without added noises and in the opposite case are shown in Fig. 7 a, c, respectively. The signal $\hat{x}_1(t)$ is calculated according to the expression (41) by substituting instead of true values of parameters their LS estimates (22): $\hat{b}_1 = 1.9149$, $\hat{a}_1 = -1.0404$, $\hat{a}_2 = 0.1443$
Figure 7: The intermediate signal $x(t)$, output signal $y^p(t)$ and the reconstructed versions of $x(t)$ for the preload. Unnoisy environment (a, b), noisy one (c, d). Curves: $x(t)$—1, $y^p(t)$—2, $\hat{x}_1(t)$—3, $\hat{x}_2(t)$—4.

in an unnoisy case, and $\hat{b}_1 = 1.8889, \hat{a}_1 = -1.0817, \hat{a}_2 = 0.1830$ in an opposite one. The observations

$$\hat{x}(t) = y(t) - \text{asgn}(y(t)),$$  \hspace{1cm} (53)

were substituted instead of respective values of $y(\cdot)$ in (30), and the LS problem (27), consisting of the identification of parameters of FIR filter (42) with $\nu = 14$, was solved. Afterwards, the intermediate signal $x(t)$ was reconstructed according to (43) and the signal $\hat{x}_2$ was determined. FIR filter parameters estimation problem was not repeated once more, because in the case of the preload no one observation of $y(t)$ was cut off. The intermediate signal $x(t)$, the true output $y^p(t)$ and two reconstructed versions of $x(t)$—$\hat{x}_1(t), \hat{x}_2(t)$, are shown in Fig. 7. In Fig. 7b, d curves 3, 4 correspond to the signals $\hat{x}_1$ and $\hat{x}_2$, respectively. The values of the signal $\hat{x}_2(t)$ were substituted into the matrix (46) and vector (47) and estimates of parameters were calculated by (44). For the unnoisy case $\hat{b}_1 = 1.2103, \hat{a}_1 = -1.2394, \hat{a}_2 = 0.3353$ and for the noisy one $\hat{b}_1 = 1.0948, \hat{a}_1 = -1.3827, \hat{a}_2 = 0.4713$. The signal $\hat{x}_1(t)$ (curve 3), based on the estimates of the ordinary LS (22) only at very small intervals coincides with the true signal $x(t)$ (curve 1). On the other hand, the signal $\hat{x}_2(t)$ (curve 4), in fact, is repeating the behaviour of the true signal $x(t)$, when the estimation transition process is finished.
Figure 8: The intermediate signal \( x(t) \), output signal \( y^h(t) \) and the reconstructed versions of \( x(t) \) for the hysteresis. Unnoisy environment (a, b), noisy one (c, d). Curves: \( x(t) \) — 1, \( y^h(t) \) — 2, \( \hat{x}_1(t) \) — 3, \( \hat{x}_2(t) \) — 4.

5.4 Nonlinear block—hysteresis

The true output signal \( y(t) \) is calculated, using the expression

\[
y^h(t) = \begin{cases} 
x(t) - a & \text{if } x(t) - x(t-1) > 0, \\
x(t) + a & \text{if } x(t) - x(t-1) < 0, \\
y(t-1) & \text{if } x(t) = x(t-1). 
\end{cases}
\]

respectively. Here \( \lambda_1 = 0.185, \lambda_2 = 0.704 \) were chosen such that variances \( \sigma_i^2 = \sigma_e^2 = 0.5. \) Then, signal-to-noise ratios are \( SNR_i = 20 \) and \( SNR_e = 20. \) The Wiener system’s intermediate and output signals without added noises and with ones (see Fig. 8) according to (39) and

\[
y^h(t) = y^{(h)}(t) + \lambda_2 e(t),
\]

\( N = 100 \) data points were generated, without additive process and measurement noises and with ones (see Fig. 8) according to (39) and

The observations

\[
\hat{x}(t) = \begin{cases} 
y(t) + a & \text{if } \hat{x}_1(t) - \hat{x}_1(t-1) > 0, \\
y(t) - a & \text{if } \hat{x}_1(t) - \hat{x}_1(t-1) < 0, \\
\hat{x}(t-1) & \text{if } \hat{x}_1(t) = \hat{x}_1(t-1), 
\end{cases}
\]

18
were substituted instead of respective values of \( y(\cdot) \) in (30), and the LS problem (27), consisting of the identification of parameters of FIR filter (42) with \( \nu = 14 \), was solved. Afterwards, the intermediate signal \( x(t) \) was reconstructed according to (43) and the signal \( \hat{x}_2(t) \) was determined. Here FIR filter parameters estimation problem was not repeated once more, too.

The intermediate signal \( x(t) \), the true output \( y_h(t) \) and two reconstructed versions of \( \hat{x}(t) \), \( \hat{x}_2(t) \), are shown in Fig. 8. In Fig. 8b, d curves 3, 4 correspond to the signals \( \hat{x}_1(t) \) and \( \hat{x}_2(t) \), respectively. The values of the signal \( \hat{x}_2(t) \) were substituted into the matrix (46) and vector (47) and estimates of parameters were calculated by (44). For the unnoisy case \( \hat{b}_1 = 0.8305, \hat{a}_1 = -1.2410, \hat{a}_2 = 0.3448 \) and for the noisy one \( \hat{b}_1 = 0.8367, \hat{a}_1 = -1.2172, \hat{a}_2 = 0.3098 \).

The signal \( \hat{x}_1(t) \) (curve 3), based on the estimates of the ordinary LS (22), does not approach to the true signal \( x(t) \) at all (curve 1). The signal \( \hat{x}_2(t) \) (curve 4) is more accurate version of the true signal \( x(t) \).

5.5 Nonlinear block—discontinuous nonlinearity

The true output signal \( y(t) \) is calculated according to

\[
y^{dn}(t) = \begin{cases} \bar{y}(t) & \text{if } |x(t)| > D, \\ 0 & \text{if } |x(t)| < D, \end{cases}
\]

where

\[
\bar{y}(t) = K(t) \{x(t) - D \text{sgn}[x(t)]\} + b \text{sgn}[x(t)],
\]

\[
K(t) = m_1 + (m_2 - m_1)h(t),
\]

\[
h(t) = h[x(t)] = \begin{cases} 0 & \text{if } x(t) > 0, \\ 1 & \text{if } x(t) < 0. \end{cases}
\]

\( N = 100 \) data points were generated, without additive process and measurement noises and with ones (see Fig. 9) according to (39) and

\[
y^{(p)}(t) = y^{(p)}(t) + \lambda_2 e(t),
\]

respectively. Here \( \lambda_1 = 0.185, \lambda_2 = 0.704 \) were chosen such that variances \( \sigma_x^2 = \sigma_e^2 = 0.5 \). Then, signal-to-noise ratios are \( SNR = 20 \) and \( SNR = 18 \). The Wiener system’s intermediate and output signals without added noises and in the opposite case are shown in Fig. 9 a, c, respectively. The output \( y(t) \) was calculated according to (57) with such values of constants (see Fig. 2a):

\[
m_1 = 1, m_2 = 1.5, D = 7.5, b = 0.3.
\]

The signal \( \hat{x}_1(t) \) is calculated according to the expression (41) by substituting instead of true values of parameters their LS estimates (22): \( \hat{b}_1 = 0.6761, \hat{a}_1 = \ldots \)
a_2 = 0.5181 in an unnoisy case, and \( b_1 = 0.7695, \; \hat{a}_1 = -1.3315, \; \hat{a}_2 = 0.4410 \) in an opposite one.

Then, the LS problem (27), consisting of the identification of parameters of FIR filter (42) was solved, at first, without rejecting of cut off observations. The whole number of parameters \( \nu = 14 \) was chosen, as usual. Afterwards, the intermediate signal \( x(t) \) was reconstructed according to (43) and the signal \( \hat{x}_2(t) \) was determined. The FIR filter parameters estimation problem (27) was repeated once more. In such a case, the 50 cut off observations from 100 initial ones were rejected beforehand and estimates of FIR filter parameters were calculated, using 50 observations of

\[
\hat{y}(t) = m_1^{-1} y(t) + D \text{sgn}(y(t)) - bm_1^{-1} \text{sgn}(y(t)),
\]

if \( y(t) > 0 \), and

\[
\hat{y}(t) = m_2^{-1} y(t) + D \text{sgn}(y(t)) - bm_2^{-1} \text{sgn}(y(t)),
\]

if \( y(t) < 0 \). They were substituted instead of respective values of \( y(\cdot) \) in (30). Thus, the signal \( \hat{x}_3(t) \) was determined by (43), using above mentioned estimates of FIR parameters.

The intermediate signal \( x(t) \) and three its reconstructed versions \( \hat{x}_1(t), \; \hat{x}_2(t), \; \hat{x}_3(t) \) are shown in Fig. 9 b, d as curves 1, 3, 4, 5, respectively. The values of the signal \( \hat{x}_3(t) \) were substituted into the matrix (46) and vector (47), and estimates of parameters were calculated by (44). For the unnoisy case \( b_1 = 0.9875, \; \hat{a}_1 = -1.3732, \; \hat{a}_2 = 0.4689 \) and for the noisy one \( b_1 = 1.0423, \; \hat{a}_1 = -1.3677, \; \hat{a}_2 = 0.4661 \).

The signal \( \hat{x}_1(t) \) (curve 3), based on the estimates of the ordinary LS (22), as compared with the signals \( \hat{x}_2(t), \; \hat{x}_3(t) \) (curves 4, 5), is the worst estimate of \( x(t) \). Signal \( \hat{x}_2(t) \) is less accurate, comparing with \( \hat{x}_3(t) \) (curves 4 and 5), respectively. The signal \( \hat{x}_3(t) \) (curve 5) deviates from \( x(t) \) (curve 1) slightly, when a transition process of parameters estimation is over.

Afterwards, the values of constants, including the values of linear segment slopes \( m_1, m_2, \) that determine the character of the discontinuous nonlinearity, were changed. For the second set of constants

\[
m_1 = 1, \; m_2 = 0, \; D = 0.5, \; b = 0.5 \tag{65}
\]

and for the third one

\[
m_1 = 0, \; m_2 = 1.5, \; D = 7.5, \; b = 0.3 \tag{66}
\]

the nonlinearities have forms (see Fig. 2c, d), respectively. The experimental investigation, using both sets of constants was provided, and the parameters estimation results were obtained. The signal \( \hat{x}_1(t) \) is calculated according to the expression (41) by substituting instead of true values of parameters their LS estimates (22): \( b_1 = 0.4835, \; \hat{a}_1 = -1.3139, \; \hat{a}_2 = 0.3914 \) in an unnoisy case, and \( b_1 = 0.5554, \; \hat{a}_1 = -1.2051, \; \hat{a}_2 = 0.2907 \) in an opposite one, when the set of constants (65) was applied. On the other hand, for the third set of constants (66) the estimates: \( \hat{b}_1 = 0.4095, \; \hat{a}_1 = -1.5239, \; \hat{a}_2 = 0.6347 \) in an unnoisy case, and \( \hat{b}_1 = 0.4810, \; \hat{a}_1 = -1.4353, \; \hat{a}_2 = 0.5466 \) in an opposite one were determined and were used to calculate the signal \( \hat{x}_1(t) \).
Then for both sets of constants (65), (66) the LS problem (27), in order to calculate parameters of FIR filter (42), was solved, at first, without rejecting of cut off observations. The whole number of parameters $\nu = 14$ was chosen, as usual. Afterwards, the intermediate signal $x(t)$ was reconstructed according to (43) and the signal $\hat{x}_2(t)$ was determined. The FIR filter parameters estimation problem was repeated once more. In such a case the 50 cut off observations from 100 initial ones were rejected beforehand and estimates of FIR filter parameters were calculated, using 50 observations of

$$\hat{y}(t) = m_1^{-1} y(t) + D \operatorname{sgn}(y(t)) - b m_1^{-1} \operatorname{sgn}(y(t)),$$

(67)

if $y(t) > 0$ and (65) is valid, and

$$\hat{y}(t) = m_2^{-1} y(t) + D \operatorname{sgn}(y(t)) - b m_2^{-1} \operatorname{sgn}(y(t)),$$

(68)

if $y(t) < 0$ and (66) is satisfied. Observations $\hat{y}(t)$ were substituted instead of respective values of $y(\cdot)$ in (30). It could be noted, that

$$\hat{y}(t) = b \operatorname{sgn}(y(t)),$$

(69)

when $y(t) < 0$ and (65) or $y(t) > 0$ and (66) are valid, respectively. In both cases observations $\hat{y}(t)$ have to be rejected, too. Thus, the signals $\hat{x}_3(t)$ were determined by (43), using above mentioned estimates of FIR parameters, calculated for both constants sets (65), (66).

The intermediate signal $x(t)$ and three its reconstructed versions $\hat{x}_1(t)$, $\hat{x}_2(t)$, $\hat{x}_3(t)$ are shown in Fig. 10 and 11 b, d as curves 1, 3—5, respectively. The values

![Figure 9: The intermediate signal $x(t)$, output signal $y^{dn}(t)$ and the reconstructed versions of $x(t)$ for the discontinuous nonlinearity with (62). Unnoisy environment (a, b), noisy one (c, d). Curves: $x(t)|1$, $y^{dn}(t)|2$, $\hat{x}_1(t)|3$, $\hat{x}_2(t)|4$, $\hat{x}_3(t)|5$.](image)
Figure 10: The intermediate signal $x(t)$, output signal $y^{in}(t)$ and the reconstructed versions of $x(t)$ for the discontinuous nonlinearity with constants (65). Unnoisy environment (a, b), noisy one (c, d). Curves: $x(t)\rightarrow 1$, $y^{in}(t)\rightarrow 2$, $\hat{x}_1(t)\rightarrow 3$, $\hat{x}_2(t)\rightarrow 4$, $\hat{x}_3(t)\rightarrow 5$.

Figure 11: The intermediate signal $x(t)$, output signal $y^{in}(t)$ and the reconstructed versions of $x(t)$ for the discontinuous nonlinearity with constants (66). Unnoisy environment (a, b), noisy one (c, d). Curves: $x(t)\rightarrow 1$, $y^{in}(t)\rightarrow 2$, $\hat{x}_1(t)\rightarrow 3$, $\hat{x}_2(t)\rightarrow 4$, $\hat{x}_3(t)\rightarrow 5$. 
of the signal \(\hat{x}_3(t)\) were substituted into the matrix (46) and vector (47), and estimates of parameters were calculated by (44). For the unnoisy case \(b_1 = 0.8834, \hat{a}_1 = -1.4404, \hat{a}_2 = 0.5353\) and for the noisy one \(b_1 = 0.9537, \hat{a}_1 = -1.3935, \hat{a}_2 = 0.4981\), when the set of constants (65) were applied. For the set of constants (66): \(\hat{b}_1 = 0.9537, \hat{a}_1 = 1.3935, \hat{a}_2 = 0.5353\) and \(\hat{b}_1 = 1.2772, \hat{a}_1 = -1.1115, \hat{a}_2 = 0.2386\) (noisy one).

For both cases of constants the signal \(\hat{x}_3(t)\) (curve 5), as compared with the signals \(\hat{x}_1(t), \hat{x}_2(t)\) (curve 3, 4), deviates from \(x(t)\) (curve 1) less than previous two signals. However it is less accurate than \(\hat{x}_3(t)\), calculated for the set of constants (62).

### 6 A Monte-Carlo simulation

In order to establish how different measurement noise realizations affect the intermediate signal reconstruction and true linear part parameters estimation we have used a Monte Carlo simulation, with 10 data sets, each containing 100 input-output observations pairs. 10 experiments with different realizations of additive measurements noise \(e(t)\) and different levels of its intensity were carried out in order to investigate more precisely the accuracy of estimates of parameters of (36). As inputs for all given nonlinearities the periodical signal (35) and white Gaussian noise with variance 1 were chosen. For the periodical signal the threshold of the saturation, dead-zone and preload was \(a = 7.5\), while for the hysteresis — \(a = 3.5\). In a case of the Gaussian noise the threshold \(a\) was chosen 4.5 for the saturation, 3.5 for the dead-zone, 3.5 for the preload, 0.5 and 3.5 for the hysteresis, respectively. For the discontinuous nonlinearity—

\[
m_1 = 1, m_2 = 1.5, D = 7.5, b = 0.3,\text{ when input is the periodical signal (35)}
\]

and

\[
m_1 = 1, m_2 = 1.5, D = 1.25, b = 0.3,\text{ when input is Gaussian noise with variance 1.}
\]

In each ith experiment the estimates of parameters \(b_1 = 1, a_1 = -1.4, a_2 = 0.49\) were calculated, using approach with rejection of incorrect observations, which is presented in the Section 4. Figures (12)–(32) present input-output signals and \(\hat{x}_1(t)\), that is calculated, substituting in (41) instead of true values of parameters their LS estimates (22), determined by processing one realization of \(u(t), y(t)\). In Figures (18)–(32) the signal \(\hat{x}_1(t)\), that is calculated substituting in (41) instead of true parameter values their estimates, averaged by 10 experiments, is shown, too. Here dashed lines correspond to the confidence intervals

\[
\Delta = \pm t_{\alpha} \frac{\hat{\sigma}_{x(k)}}{\sqrt{M}} \quad \forall \ k = \frac{1}{100},
\]

which for the respective values of averaged \(\hat{x}_1(k) \forall k = \frac{1}{100}\) are shown; \(\hat{\sigma}_{x(k)}\) are estimate of the standard deviation \(\sigma_{x(k)}\); \(\alpha = 0.05\) is the significance level; \(t_{\alpha} = 2.26\) is the 100(1 − \(\alpha\))% point of Student’s distribution with \(M − 1\) degrees of freedom; \(M = 10\) is the number of experiments.

During a Monte Carlo simulation \(\sigma_v^2 = \sigma_e^2 = 0.5\) and \(\sigma_v^2 = \sigma_e^2 = 1\) were chosen and averaged values of estimates of parameters and their confidence
Figure 12: The signals of the simulated Wiener system for the saturation, when $v(t) \equiv e(t) \equiv 0$. Input (white Gaussian noise with variance 1) (a), true intermediate signal $x(t)$ and output $y(t)$ (b), true intermediate signal $x(t)$ and $\hat{x}_1(t)$, determined by (41), substituting instead of true values of parameters their estimates $\hat{b}_1 = 0.9735$, $\hat{a}_1 = -1.4128$, $\hat{a}_2 = 0.5026$ (c). Curves: 1—$x(t)$, 2—$y(t)$, 3—$\hat{x}_1(t)$.

Figure 13: The signals of the simulated Wiener system for the dead-zone, when $v(t) \equiv e(t) \equiv 0$. The signal $\hat{x}_1(t)$ is calculated by (41), substituting instead of true values of parameters their estimates $\hat{b}_1 = 1.3111$, $\hat{a}_1 = -1.2790$, $\hat{a}_2 = 0.4115$. Other values and markings are the same as in Fig. 12.
Figure 14: The signals of the simulated Wiener system for the preload, when $v(t) \equiv e(t) \equiv 0$. The signal $\hat{x}_1(t)$ is calculated by (41), substituting instead of true values of parameters their estimates $\hat{b}_1 = 1.0643, \hat{a}_1 = -1.1216, \hat{a}_2 = 0.2635$. Other values and markings are the same as in Fig. 12.

Figure 15: The signals of the simulated Wiener system for the hysteresis ($a=0.5$), when $v(t) \equiv e(t) \equiv 0$. The signal $\hat{x}_1(t)$ is calculated by (41), substituting instead of true values of parameters their estimates $\hat{b}_1 = 0.8920, \hat{a}_1 = -1.2263, \hat{a}_2 = 0.3308$. Other values and markings are the same as in Fig. 12.
Figure 16: The signals of the simulated Wiener system for the hysteresis \( a=3.5 \), when \( v(t) \equiv c(t) \equiv 0 \). The signal \( \hat{x}_1(t) \) is calculated by (41), substituting instead of true values of parameters their estimate \( \hat{b}_1 = -2.7502, \hat{a}_1 = -0.3954, \hat{a}_2 = -0.1097 \). Other values and markings are the same as in Fig. 12.

Figure 17: The signals of the simulated Wiener system for the discontinuous nonlinearity \( (m_1 = 1, m_2 = 1.5, D = 1.25, b = 0.3) \), when \( v(t) \equiv c(t) \equiv 0 \). The signal \( \hat{x}_1(t) \) is calculated by (41), substituting instead of true values of parameters their estimates \( \hat{b}_1 = 1.0266, \hat{a}_1 = -1.3768, \hat{a}_2 = 0.4652 \). Other values and markings are the same as in Fig. 12.
intervals were obtained by the formulas
\[ \Delta_1 = \pm t_{M} \frac{\hat{\sigma}_b}{\sqrt{M}}, \Delta_2 = \pm t_{M} \frac{\hat{\sigma}_a}{\sqrt{M}}, \Delta_3 = \pm t_{M} \frac{\hat{\sigma}_b}{\sqrt{M}} \quad \forall \ k = \frac{1}{100}. \] (71)

In Tables for each input the first line corresponds to \( \sigma_v^2 = \sigma_e^2 = 0.5 \), while the second line — to \( \sigma_v^2 = \sigma_e^2 = 1 \).

Figure 18: The signals of the Wiener system for the saturation with \((\sigma_v^2 = \sigma_e^2 = 1)\), when input is periodical signal (35). Input \( u(t) \) (a), noisy \( x(t) \) and noisy \( y(t) \) (b), true \( x(t) \) and \( \hat{x}_1(t) \), determined using (41) and estimates \( \hat{b}_1 = 1.0766, \hat{a}_1 = -1.3931, \hat{a}_2 = 0.4805 \), which are calculated by processing one realization of \( u(t), y(t) \) (c), true intermediate signal \( x(t) \) and \( \hat{x}_1(t) \), determined using (41) and estimates \( \hat{b}_1 = 1.0682, \hat{a}_1 = -1.3784, \hat{a}_2 = 0.4673 \), which are calculated by processing 10 realizations of \( u(t), y(t) \) (d). Curves: 1—noisy \( x(t) \), 2—noisy \( y(t) \), 3—true \( x(t) \), 4, 5—\( \hat{x}_1(t) \), dashed—confidence intervals of \( \hat{x}_1(t) \).

It should be noted, that from the simulation results (see Figures (12)—(32), Tables (1—5)) imply that the accuracy of the parametric estimation depends on many factors, such as input signal, intensity of process and measurement noises, the type of nonlinearity, the threshold of the nonlinearity, etc. The additive noise in observations to be processed influences the quality of estimates of parameters even for the same nonlinearity quite different. It depends on the input signal to be excited the Wiener system. The value of a threshold strongly affects the accuracy of estimates, sometimes destroying the calculation process (see Fig. 16) completely. In spite of this, in general, a Monte Carlo simulation results maintains the efficiency of the proposed approach to the identification of parameters of the Wiener system. On the other hand, there arise here some problems, such as, the identification of the Wiener system in a case of unknown thresholds of hard nonlinearities, requiring further investigations.
Figure 19: The signals of the simulated Wiener system for the dead-zone with $(\sigma^2_v = \sigma^2_e = 1)$, when input is periodical signal (35). For the calculation of the signal $\hat{x}_1(t)$, which is shown in (c, d), estimates $\hat{b}_1 = 1.1033$, $\hat{a}_1 = -1.3411$, $\hat{a}_2 = 0.4329$ and $\hat{b}_1 = 1.0589$, $\hat{a}_1 = -1.3811$, $\hat{a}_2 = 0.4765$ are used, respectively. Other values and markings the same as in Fig. 18.

Figure 20: The signals of the simulated Wiener system for the preload with $(\sigma^2_v = \sigma^2_e = 1)$, when input is periodical signal (35). For the calculation of the signal $\hat{x}_1(t)$, which is shown in (c, d) estimates $\hat{b}_1 = 1.1081$, $\hat{a}_1 = -1.2725$, $\hat{a}_2 = 0.3695$ and $\hat{b}_1 = 1.0681$, $\hat{a}_1 = -1.2883$, $\hat{a}_2 = 0.3842$ are used, respectively. Other values and markings the same as in Fig. 18.
Figure 21: The signals of the simulated Wiener system for the hysteresis with \( (\sigma_1^2 = \sigma_2^2 = 1) \), when input is periodical signal (35). For the calculation of the signal \( \hat{x}_1(t) \), which is shown in (c, d), estimates \( \hat{b}_1 = 0.9369, \hat{a}_1 = -1.2132, \hat{a}_2 = 0.2973 \) and \( \hat{b}_1 = 0.9275, \hat{a}_1 = -1.3840, \hat{a}_2 = 0.4754 \) are used, respectively. Other values and markings the same as in Fig. 18.

Figure 22: The signals of the simulated Wiener system for the discontinuous nonlinearity \( n_1 = 1, n_2 = 1.5, D = 7.5, b = 0.3 \), with \( (\sigma_1^2 = \sigma_2^2 = 1) \), when input is periodical signal (35). For the calculation of the signal \( \hat{x}_1(t) \), which is shown in (c, d), estimates \( \hat{b}_1 = 1.0351, \hat{a}_1 = -1.3993, \hat{a}_2 = -0.4937 \) and \( \hat{b}_1 = 1.0082, \hat{a}_1 = -1.4193, \hat{a}_2 = 0.5165 \) are used, respectively. Other values and markings the same as in Fig. 18.
Figure 23: The signals of the simulated Wiener system for the saturation with $(\sigma_v^2 = \sigma_e^2 = 0.5)$, when input is white Gaussian noise with variance 1. For the calculation of the signal $\hat{x}_1(t)$, which is shown in (c, d), estimates $\hat{b}_1 = 0.7870$, $\hat{a}_1 = -1.2467$, $\hat{a}_2 = 0.3374$ and $\hat{b}_1 = 0.9087$, $\hat{a}_1 = -1.2378$, $\hat{a}_1 = 0.3285$ are used, respectively. Other values and markings the same as in Fig. 18.

Figure 24: The signals of the simulated Wiener system for the saturation with $(\sigma_v^2 = \sigma_e^2 = 1)$, when input is white Gaussian noise with variance 1. For the calculation of the signal $\hat{x}_1(t)$, which is shown in (c, d), estimates $\hat{b}_1 = 0.7485$, $\hat{a}_1 = -1.1237$, $\hat{a}_2 = 0.2173$ and $\hat{b}_1 = 0.8988$, $\hat{a}_1 = -1.1299$, $\hat{a}_2 = 0.2237$ are used, respectively.
Figure 25: The signals of the simulated Wiener system for the dead-zone with $(\sigma_v^2 = \sigma_e^2 = 0.5)$, when input is white Gaussian noise with variance 1. For the calculation of the signal $\hat{x}_1(t)$, which is shown in (c, d), estimates $\hat{b}_1 = 1.3390$, $\hat{a}_1 = -0.4922$, $\hat{a}_2 = -0.1151$ and $b_1 = 1.4648$, $\hat{a}_1 = -0.5079$, $\hat{a}_2 = -0.1304$ are used, respectively. Other values and markings the same as in Fig. 18.

Figure 26: The signals of the simulated Wiener system for the dead-zone with $(\sigma_v^2 = \sigma_e^2 = 1)$, when input is white Gaussian noise with variance 1. For the calculation of the signal $\hat{x}_1(t)$, which is shown in (c, d), estimates $\hat{b}_1 = 1.2183$, $\hat{a}_1 = -0.4339$, $\hat{a}_2 = -0.1572$ and $b_1 = 1.4899$, $\hat{a}_1 = -0.5032$, $\hat{a}_2 = -0.0958$ are used, respectively.
Figure 27: The signals of the simulated Wiener system for the preload with \((\sigma_v^2 = \sigma_e^2 = 0.5)\), when input is white Gaussian noise with variance 1. For the calculation of the signal \(\hat{x}_1(t)\), which is shown in (c, d), estimates \(\hat{b}_1 = 1.1365, \hat{a}_1 = -1.0520, \hat{a}_2 = 0.1881\) and \(\hat{b}_1 = 1.1271, \hat{a}_1 = -1.0798, \hat{a}_2 = 0.2178\) are used, respectively. Other values and markings the same as in Fig. 18.

Figure 28: The signals of the simulated Wiener system for the preload with \((\sigma_v^2 = \sigma_e^2 = 1)\), when input is white Gaussian noise with variance 1. For the calculation of the signal \(\hat{x}_1(t)\), which is shown in (c, d), estimates \(\hat{b}_1 = 1.1738, \hat{a}_1 = -1.0107, \hat{a}_2 = 0.1459\) and \(\hat{b}_1 = 1.1616, \hat{a}_1 = -1.0428, \hat{a}_2 = 0.1808\) are used, respectively.
Figure 29: The signals of the simulated Wiener system for the hysteresis with \((\sigma_v^2 = \sigma_e^2 = 0.5)\), when input is white Gaussian noise with variance 1. The threshold is 0.5. For the calculation of the signal \(\hat{x}_1(t)\), which is shown in (c, d), estimates \(\hat{b}_1 = 0.8239, \hat{a}_1 = -1.0218, \hat{a}_2 = 0.1155\) and \(\hat{b}_1 = 1.0324, \hat{a}_1 = -1.0941, \hat{a}_2 = 0.1879\) are used, respectively. Other values and markings the same as in Fig. 18.

Figure 30: The signals of the simulated Wiener system for the hysteresis with \((\sigma_v^2 = \sigma_e^2 = 1)\), when input is white Gaussian noise with variance 1. For the calculation of the signal \(\hat{x}_1(t)\), which is shown in (c, d), estimates \(\hat{b}_1 = 0.8810, \hat{a}_1 = -0.8426, \hat{a}_2 = -0.0533\) and \(\hat{b}_1 = 1.0520, \hat{a}_1 = -1.0454, \hat{a}_2 = 0.1387\) are used, respectively.
Figure 31: The signals of the simulated Wiener system for the discontinuous nonlinearity with \((m_1 = 1, m_2 = 1.5, D = 1.25, b = 0.3)\), when input is white Gaussian noise with variance 1. Variances of noises \((\sigma_v^2 = \sigma_e^2 = 0.5)\). For the calculation of the signal \(\hat{x}_1(t)\), which is shown in (c, d), estimates \(\hat{b}_1 = 1.1103, \hat{a}_1 = -1.2676, \hat{a}_2 = 0.3612\) and \(b_1 = 1.1125, a_1 = -1.2476, a_2 = 0.3375\) are used, respectively. Other values and markings the same as in Fig. 18.

Figure 32: The signals of the simulated Wiener system for the discontinuous nonlinearity with \((m_1 = 1, m_2 = 1.5, D = 1.25, b = 0.3)\), when input is white Gaussian noise with variance 1. Variances of noises \((\sigma_v^2 = \sigma_e^2 = 1.0)\). For the calculation of the signal \(\hat{x}_1(t)\), which is shown in (c, d), estimates \(\hat{b}_1 = 1.1378, \hat{a}_1 = -1.2337, \hat{a}_2 = 0.3279\) and \(b_1 = 1.1355, a_1 = -1.1844, a_2 = 0.2740\) are used, respectively.
Table 1: The averaged estimates of parameters and their confidence intervals for different inputs in a case of the saturation

<table>
<thead>
<tr>
<th>Input signal with SNRs</th>
<th>Estimates of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(35) with SNR(^y) = 20, SNR(^e) = 9</td>
<td>1.05 ± 0.03, -1.38 ± 0.02, 0.46 ± 0.02</td>
</tr>
<tr>
<td>(35) with SNR(^y) = 14, SNR(^e) = 6</td>
<td>1.07 ± 0.05, -1.37 ± 0.03, 0.46 ± 0.03</td>
</tr>
<tr>
<td>noise with SNR(^y) = 5.5, SNR(^e) = 4.5</td>
<td>0.90 ± 0.10, -1.23 ± 0.05, 0.32 ± 0.05</td>
</tr>
<tr>
<td>noise with SNR(^y) = 4, SNR(^e) = 3</td>
<td>0.89 ± 0.13, -1.12 ± 0.07, 0.22 ± 0.08</td>
</tr>
</tbody>
</table>

Table 2: A case of the dead-zone

<table>
<thead>
<tr>
<th>Input signal with SNRs</th>
<th>Estimates of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(35) with SNR(^y) = 20, SNR(^e) = 12</td>
<td>1.03 ± 0.06, -1.38 ± 0.03, 0.47 ± 0.03</td>
</tr>
<tr>
<td>(35) with SNR(^y) = 14, SNR(^e) = 9</td>
<td>1.05 ± 0.06, -1.38 ± 0.03, 0.47 ± 0.03</td>
</tr>
<tr>
<td>noise with SNR(^y) = 5.5, SNR(^e) = 1.6</td>
<td>1.46 ± 0.30, -0.50 ± 0.13, -0.13 ± 0.17</td>
</tr>
<tr>
<td>noise with SNR(^y) = 4, SNR(^e) = 1.2</td>
<td>1.48 ± 0.29, -0.50 ± 0.16, -0.09 ± 0.19</td>
</tr>
</tbody>
</table>

Table 3: A case of the preload

<table>
<thead>
<tr>
<th>Input signal with SNRs</th>
<th>Estimates of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(35) with SNR(^y) = 20, SNR(^e) = 27.5</td>
<td>1.06 ± 0.01, -1.29 ± 0.01, 0.38 ± 0.01</td>
</tr>
<tr>
<td>(35) with SNR(^y) = 14, SNR(^e) = 21</td>
<td>1.06 ± 0.01, -1.28 ± 0.01, 0.38 ± 0.01</td>
</tr>
<tr>
<td>noise with SNR(^y) = 5.5, SNR(^e) = 14.5</td>
<td>1.12 ± 0.05, -1.07 ± 0.02, 0.21 ± 0.02</td>
</tr>
<tr>
<td>noise with SNR(^y) = 4, SNR(^e) = 11</td>
<td>1.16 ± 0.07, -1.04 ± 0.03, 0.18 ± 0.03</td>
</tr>
</tbody>
</table>

Table 4: A case of the hysteresis

<table>
<thead>
<tr>
<th>Input with SNRs</th>
<th>Estimates of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(35) with SNR(^y) = 20, SNR(^e) = 21.5</td>
<td>0.92 ± 0.01, -1.38 ± 0.01, 0.48 ± 0.01</td>
</tr>
<tr>
<td>(35) with SNR(^y) = 14, SNR(^e) = 15</td>
<td>0.92 ± 0.01, -1.38 ± 0.01, 0.47 ± 0.01</td>
</tr>
<tr>
<td>noise with SNR(^y) = 5.5, SNR(^e) = 7</td>
<td>1.03 ± 0.05, -1.09 ± 0.03, 0.18 ± 0.03</td>
</tr>
<tr>
<td>noise with SNR(^y) = 4, SNR(^e) = 5</td>
<td>1.05 ± 0.07, -1.04 ± 0.05, 0.13 ± 0.06</td>
</tr>
</tbody>
</table>

Table 5: A case of the discontinuous nonlinearity

<table>
<thead>
<tr>
<th>Input with SNRs</th>
<th>Estimates of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(35) with SNR(^y) = 20, SNR(^e) = 17</td>
<td>0.99 ± 0.04, -1.42 ± 0.04, 0.51 ± 0.04</td>
</tr>
<tr>
<td>(35) with SNR(^y) = 14, SNR(^e) = 3</td>
<td>1.01 ± 0.04, -1.41 ± 0.04, 0.51 ± 0.04</td>
</tr>
<tr>
<td>noise with SNR(^y) = 5.5, SNR(^e) = 5</td>
<td>1.11 ± 0.04, -1.24 ± 0.03, 0.33 ± 0.03</td>
</tr>
<tr>
<td>noise with SNR(^y) = 4, SNR(^e) = 4</td>
<td>1.13 ± 0.05, -1.18 ± 0.05, 0.27 ± 0.05</td>
</tr>
</tbody>
</table>
7 Conclusions

Troubles arising by the identification of the Wiener systems with hard and discontinuous nonlinearities, using input-output observations, are because of that such types of nonlinearities can not be described by polynomials and are noninvertible in general. That is why there are only several reports, which are devoted to investigations, concerning such a problem. On the other hand, abovementioned nonlinearities are common in computer controlled systems and have to be identify in order to determine their describing functions, needed for nonlinear systems control. However, one have to deal with an identification problem, consisting of inherent features, as well as of external ones, such as process and measurements noises and missing observations, that are cut off by some of hard nonlinearities, i.e., the saturation or dead-zone. Therefore the well-known identification techniques, based on the ordinary LS appeared to be inefficient. In such a case, it is important to solve the generalized problem of the identification of Wiener systems in an absence of some sets of observations to be processed. This work extends and measurably develops Ljung’s and Hagenblad Wiener models identification ideas and investigations, presented in [1],[5],[6],[7].

It is obvious, that the complicated model of observations requires a new and an efficient approach, that could be used in processing observations with missing data in order to obtain the estimates of unknown parameters. In our work such an approach has been worked out. Theoretically it is based on an important relation between the true linear system (linear part of the Wiener system) and the subsystem from the auxiliary network of subsystems, acting in parallel to the Wiener system. In practice, our approach is realized by means of EM algorithm, consisting of the expectation and maximization steps. During successive steps missing values of observations are replaced by their estimates, determined at E-step of EM, using ordinary LS, but tuned for the missing data. Various results of numerical simulation, including a Monte Carlo one (see Figures (12)−(32) and Tables 1−5), prove the efficiency of the proposed approach for the identification of Wiener systems.
References


