Identification of Flexibility Parameters of 6-axis Industrial Manipulator Models

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Abstract
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Keywords: Identification, Robotics, Flexible arms, Manipulators
Identification of Flexibility Parameters of 6-axis Industrial Manipulator Models

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Abstract
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1 Introduction
Dynamics and kinematics models of industrial robots are used for mechanical design, performance simulation, control and supervision, etc., and the needs for detailed flexible models grow as the performance/price ratio increases. The discussion in this report focus on flexible models and identification of flexibility parameters (spring-damper pairs) of industrial robots. The identification is carried out in the frequency domain by comparing experimental Frequency Response Functions (FRF’s) to FRF’s obtained from a nonlinear flexible model. The report is organized in the following way: The analytical model and the modeling procedure are described in Section 2. Estimation of experimental FRF’s is discussed in the Section 3 and a parameter identification method is presented in the Section 4. The discussion is closed by a presentation of some experimental results, conclusions and suggestions for future work. Note that industrial robots are often referred to as manipulators in this text. A common type of industrial robot manufactured by ABB is shown in Figure 6.

2 Manipulator model
The model consists of a kinematic chain of rigid bodies denoted by rbᵢ representing the manipulator arms. Such a rigid body is illustrated in Figure 1 and is described by its mass mi, center of mass location ξᵢ, the position vector li representing the translation from the frame (coordinate system) ai (which is fixed in the rigid body rbᵢ), to the frame ai+¹ (which is fixed in the rigid body rbᵢ+¹), and the inertia tensor (w.r.t. the
The center of mass is given by
\[
J^i = \begin{bmatrix}
J^i_{xx} & J^i_{xy} & J^i_{xz} \\
J^i_{yx} & J^i_{yy} & J^i_{yz} \\
J^i_{zx} & J^i_{zy} & J^i_{zz}
\end{bmatrix}.
\] (1)

The superscript \(i\) indicates that quantities are associated with the rigid body (arm) \(rb^i\), joint \(i\) and motor \(i\).

The arms are linked together by gearboxes and motors and due to flexibilities in the gearboxes, spring-damper pairs are introduced between all the motors and arms in the model. These flexibilities are active in the joint angle direction and the spring is tensed by the difference between arm angle and motor angle, i.e. \(q^i_a - q^i_m\). The gearbox torque, denoted by \(\tau^i_g\), is then a function of \(q^i_a - q^i_m\) and for a real manipulator gearbox this function is nonlinear. However, for the sake of simplicity, it will here be modeled by the linear function
\[
\tau^i_g = k^i_g(q^i_a - q^i_m) + c^i_g(v^i_a - v^i_m)
\]
where the last term represents the damping and where \(v^i_a = \dot{q}^i_a\) and \(v^i_m = \dot{q}^i_m\) are angular speeds of the arm and the motor, respectively. The subscript \(g\) stands for gear, the subscript \(a\) for arm, and the subscript \(m\) for motor.

Although models based on the assumptions above have proved to be adequate for many types of manipulators, experiments and tests have shown that such models are not always able to catch the overall flexibility behavior of some manipulator types and therefore, two additional spring-damper pairs are introduced in some of the links. These two spring-damper pairs mainly represent flexibilities in bearings and they are individually orthogonally oriented and also orthogonally oriented to the spring-damper of the gear. This results in three-dimensional spring-damper pairs in these joints. The Figure 2 illustrates such a manipulator model. The angle displacement caused by the bearing flexibilities, denoted by \(q^i_b\) (\(b\) for bearing), are small compared to the actuated joint rotations \(q^i_a\). By introducing \(q^i_b\) directly after the joint angles \(q^i_a\) in the kinematic chain, a good approximation of the real situation is obtained. The torques causing the displacements \(q^i_b\) are modeled in a similar way as the gear torque, namely by
\[
\tau^i_b = k^i_b q^i_b + c^i_b v^i_b.
\]

The torque/current control of the motor is assumed to be ideal so that the input signals of the model are the motor torque references \(u^i\) which equals the motor torque \(\tau^i_m\).

Figure 1: A general description of a rigid body.

The model equations are derived by computing the linear and angular momentum. By using Kane’s method the projected equations of motion are derived to yield a system of ordinary differential equations (ODE) with minimum number of DOF’s, see [1] and [2]. It should be pointed out that this procedure is not trivial, and limited computer capacity may cause trouble for long kinematical chains. These potential problems will,

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1These two spring-damper pairs can also catch flexibilities of the arms which are in fact not ideal rigid bodies.
2Since the purpose of the model used in the experiments is to study and control effects caused by elasticity, the friction is omitted.
however, not be addressed in this report. The final model can be described by a system of first order ODE’s

\[ M_m(q_m, q_a) \begin{pmatrix} \dot{v}_m \\ \dot{v}_a \end{pmatrix} = c(q_m, q_a, v_m, v_a) + g(q_m, q_a) - \begin{pmatrix} \tau_m \\ \tau_g \end{pmatrix} \]  

(2a)

\[ M_m \dot{v}_m = \tau_g + u \]  

(2b)

\[ \tau_g = K_g(q_a - q_m) + C_g(v_a - v_m) \]  

(2c)

\[ \tau_b = K_b q_b + C_b v_b \]  

(2d)

\[ v_a = \dot{q}_a \]  

(2e)

\[ v_m = \dot{q}_m. \]  

(2f)

If the number of arms, motors, and joints are \( N \), and if two orthogonally oriented bearing spring-damper pairs have been added to \( M \) of the \( N \) joints, then \( q_a \in \mathbb{R}^{N \times 1} \) is a vector of the arm angle positions \( q_a^i \), \( q_m \in \mathbb{R}^{N \times 1} \) is a vector of motor angle positions \( q_m^i \), and \( q_b \in \mathbb{R}^{2M \times 1} \) is a vector of angle displacements \( q_b^i \) caused by the flexible bearings. The corresponding vectors of speeds are \( v_a, v_m, v_b \). The input vector \( u \) contains the applied motor torques \( \tau_m \). The matrix \( M_m(q_b, q_a) \in \mathbb{R}^{(N+2M) \times (N+2M)} \) is the inertia matrix for the arms and \( M_m \in \mathbb{R}^{N \times N} \) is the diagonal inertia matrix for the motors. The Coriolis and centrifugal torques are described by the function \( c(q_b, q_a, v_a, v_m) \in \mathbb{R}^{(N+2M) \times 1} \), and \( g(q_b, q_a) \in \mathbb{R}^{(N+2M) \times 1} \) represents gravity torque.\(^3\) The time \( t \) is omitted in the expressions. The matrices \( K_g, C_g \) and \( K_b, C_b \) are here diagonal and given by

\[
K_g = \begin{pmatrix} k_{g1} & 0 & \ldots & 0 \\ 0 & k_{g2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & k_{gN} \end{pmatrix}, \quad C_g = \begin{pmatrix} c_{g1} & 0 & \ldots & 0 \\ 0 & c_{g2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & c_{gN} \end{pmatrix}, \quad K_b = \begin{pmatrix} k_{b1} & 0 & \ldots & 0 \\ 0 & k_{b2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & k_{b2M} \end{pmatrix}, \quad C_b = \begin{pmatrix} c_{b1} & 0 & \ldots & 0 \\ 0 & c_{b2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & c_{b2M} \end{pmatrix}.
\]

(3)

For a model including the cartesian position and orientation of the tool, \( Z \), the forward kinematic model of the robot must be added. This model is a mapping

\[ Z = \Gamma(q_b, q_a). \]  

(4)

The cartesian position of the tool is often referred to as the tool center point (TCP), see Figure 2. Kinematic models for the most common types of manipulators are discussed in standard literature on robotics, see e.g. [4] and [5]. Methods for modeling of robot dynamics in general are discussed in e.g. [3].

### 2.1 A flexible robot model with 18 DOF’s

The model used in the experiments presented below have three-dimensional spring-damper pairs in the three main axis joints (1-3) and one-dimensional spring-damper pairs (gearbox) in the joints (4-6), see Figure 2. Then, \( N = 6, M = 3 \) and the total number of DOF’s is \( 6 + 6 + 2 \cdot 3 = 18 \) (6 arms + 6 motors + 2 · 3 joints having bearing flexibilities). Minimal state-space realizations have 36 states. As the discussion now turns from mechanical modeling over to identification, it is appropriate to stack the spring-damper parameters in a vector

\[
\theta = \begin{bmatrix}
    k_{bx1}^1, k_{by1}^1, k_{bz1}^1, k_{bx1}^2, k_{by1}^2, k_{bz1}^2, k_{bx1}^3, k_{by1}^3, k_{bz1}^3, k_{gx1}^1, k_{gy1}^1, k_{gz1}^1, k_{gx1}^2, k_{gy1}^2, k_{gz1}^2, k_{gx1}^3, k_{gy1}^3, k_{gz1}^3, k_{bx2}^1, k_{by2}^1, k_{bz2}^1, k_{bx2}^2, k_{by2}^2, k_{bz2}^2, k_{bx2}^3, k_{by2}^3, k_{bz2}^3, k_{gx2}^1, k_{gy2}^1, k_{gz2}^1, k_{gx2}^2, k_{gy2}^2, k_{gz2}^2, k_{gx2}^3, k_{gy2}^3, k_{gz2}^3, \\
    c_{bx1}^1, c_{by1}^1, c_{bz1}^1, c_{bx1}^2, c_{by1}^2, c_{bz1}^2, c_{bx1}^3, c_{by1}^3, c_{bz1}^3, c_{bx2}^1, c_{by2}^1, c_{bz2}^1, c_{bx2}^2, c_{by2}^2, c_{bz2}^2, c_{bx2}^3, c_{by2}^3, c_{bz2}^3, c_{gx1}^1, c_{gy1}^1, c_{gz1}^1, c_{gx1}^2, c_{gy1}^2, c_{gz1}^2, c_{gx1}^3, c_{gy1}^3, c_{gz1}^3, c_{gx2}^1, c_{gy2}^1, c_{gz2}^1, c_{gx2}^2, c_{gy2}^2, c_{gz2}^2, c_{gx2}^3, c_{gy2}^3, c_{gz2}^3
\end{bmatrix}.
\]

(5)

Note that the extra index \( x, y \) or \( z \) indicates the orientation of each spring-damper pair. The rigid body parameters are known from a CAD (Computer Aided Design) model.

\(^3\)All torques, positions and inertias are described either on the motor side or on the arm side of the gear box (scaling by the gear ratio).
Figure 2: A 6-axis model consisting of six rigid bodies connected in a serial chain by three-dimensional spring-damper pairs in the joints (1-3) and by one-dimensional spring-damper pairs in the joints (4-6). For the sake of keeping the picture clean all the illustrations of the spring-damper pairs are, however, one-dimensional and lacks the damper. The dot on axis six marks the tool center point (TCP).

3 FRF estimation

As an intermediate step in the parameter identification, estimates of the frequency response function (FRF) in a number of positions are needed. These can be obtained from experiments and calculations carried out in the following way:

1. Generate appropriate speed reference signals of multisine type from Matlab™, and load them into the robot controller.
2. Take the robot into position \( q_r \) and run it in closed loop using these signals as speed references. After a few seconds, this will result in motor torques and speeds of multisine type which are sampled and stored.
3. Import the measurements into Matlab™ and calculate the FRF.

If the sampled signals are periodic, the following linear mapping will hold exactly

\[
Y(\omega_k) = \tilde{G}^r(\omega_k)U(\omega_k)
\]

where \( \tilde{G}^r(\omega_k) \in \mathbb{C}^{N \times N} \) is the FRF and \( U(\omega_k) \) and \( Y(\omega_k) \) are DFT:s of the input and output, respectively. To be able to extract \( \tilde{G}(\omega_k) \) from data, at least \( N \) different experiments are needed. The data vectors from the different experiments can then be collected into matrices (bold faced in the sequel) where each column corresponds to one experiment. The relation between the input and output can then be written as

\[
Y(\omega_k) = \tilde{G}(\omega_k)U(\omega_k)
\]

If \( U(\omega_k) \) has full rank, \( \tilde{G}(\omega_k) \) can be calculated as

\[
\tilde{G}(\omega_k) = Y(\omega_k)U^{-1}(\omega_k)
\]
Identification of the (globally valid) flexibility parameters stacked in

\[ \theta \]

Identification of spring-damper parameters

As excitation signals, orthogonal random phase multisines [7] will be used. For each block of \( N \) experiments, the DFT of the reference signal is chosen as

\[
R(\omega_k) = \begin{pmatrix}
    w_{11} R_1(\omega_k) & w_{12} R_1(\omega_k) & \cdots & w_{1N} R_1(\omega_k) \\
    w_{21} R_2(\omega_k) & w_{22} R_2(\omega_k) & \cdots & w_{2N} R_2(\omega_k) \\
    \vdots & \vdots & \ddots & \vdots \\
    w_{N1} R_N(\omega_k) & w_{N2} R_N(\omega_k) & \cdots & w_{NN} R_N(\omega_k)
\end{pmatrix}
\]

where \( w_{kp} \) are elements of an arbitrary, deterministic, orthogonal matrix \( W \), here chosen as \( w_{kp} = e^{\frac{i2\pi k(1-p)}{N}} \). Each \( R_j(\omega_k) \) is the DFT of a random phase multisine. In time domain, random phase multisines are defined as

\[
r(t) = \sum_{k=1}^{N_s/2} A_k \sin(\omega_k t + \phi_k)
\]

with amplitudes \( A_k \), phases \( \phi_k \), and frequencies \( \omega_k = \frac{2\pi k}{N_s} f_s \), where \( f_s \) is the sample frequency. The phases \( \phi_k \) are randomly chosen such that \( E\{e^{i\phi_k}\} = 0 \), for example uniformly distributed in \([0, 2\pi)\). Each period of data has \( N_s \) samples and a period time \( T_0 = \frac{N_s}{f_s} \).4

4 The number of samples \( N_s \) should be distinguished from \( N \), the number of joints, motors and arms, above.

4 Identification of spring-damper parameters

Identification of the (globally valid) flexibility parameters stacked in \( \theta \) (5) is considered next. For our purpose the model given by the equations in (2) and the description in Section 2.1, is linearized at \( Q \) different positions (configurations), \( q_r \), according to the following:

\[
q_m = q_r, \quad r = 1..Q \\
q_a = q_m \\
q_b = 0 \\
v_m = v_u = v_b = 0.
\]

Taking the Fourier transform of the linear model gives the model FRF:s

\[
G^r(\omega_k, \theta), \quad r = 1..Q.
\]

Here, \( G^r(\omega_k, \theta) \in \mathbb{C}^{(N \times N)} \) is a matrix which elements will be denoted by \( G_{nm}^r(\omega_k, \theta) \) in the sequel.

The identification is carried out by searching for the minimal sum of weighted differences between the experimental FRF:s in (6) and the model FRF:s (11), while varying the numerical values of the model parameters in \( \theta \). The following set of expressions define the problem:

\[
\min_{\theta} V(\theta)
\]

\[
V(\theta) = \sum_{r=1}^{Q} \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{\sum_{k=L_0(n,m)}^{U_k(n,m)} \alpha(n, m, k) |E_{nm}(\omega_k, \theta)|^p}{\sum_{k=L_0(n,m)}^{U_k(n,m)} \alpha(n, m, k)} \right)^{\frac{1}{p}}
\]

\[
E_{nm}(\omega_k, \theta) = 20 \log_{10} |G_{nm}^r(\omega_k, \theta)| - 20 \log_{10} \left| \hat{G}_{nm}(\omega_k) \right|.
\]
The FRF’s used here for identification have motor torques, \( u \), as inputs and motor speeds, \( v_m \), as outputs. The minimization problem (15) was solved in Matlab\textsuperscript{TM} using the function \textit{fminunc}. The choice of cost function \( V(\theta) \) has shown to be of great importance for the quality of the result and one can expect that local minima and identifiability properties of the different linear models (and combinations of them) are important factors. Good compliance between experimental FRF’s and model FRF’s was obtained by solving (15) for a number of different initial values (grid points) \( \theta_0 \) at properly chosen positions \( q^r \) and for appropriate frequency bounds and weights. The 2-norm were used most of the times, i.e. \( p = 2 \) in (15). The Figure 3 illustrates the model FRF (blue) and the experimental FRF (red) for a successful experiment. Note that the FRF plots in the Figure 3 show motor torque-to-motor acceleration spectra. In this experiment the minimizations started at 60 different grid points \( \theta_0 \) and the final values of \( V(\theta) \) for the 60 results are shown in Figure 5. Note that the 17th run gave the optimal result. The values of the estimated parameters indicate that bearing and arm flexibilities, taken together, are of the same order as the flexibility in the gear, for the same joint.
Figure 3: Frequency response functions of a 6-axis, 18 DOF flexible manipulator model (blue) and a real 6-axis manipulator prototype (red). The FRF’s represent motor torque-to-motor acceleration in dB. The frequency scale is in Hz.

Note also the discrepancies between the model FRF (blue) and the experimental FRF (red) for low frequencies in some of the off-diagonal elements, e.g. the elements \((n, m) = (2, 6), (3, 4), (3, 6)\) and \((4, 3)\). It is, in fact, the model FRF which is most correct! Therefore, the lower frequency bound, \(L_b(n, m)\), should be chosen high, but not so high that the resonance frequencies ends up outside the interval. The FRF’s dependency on the flexibility parameters dominate close to the resonance frequencies. The FRF’s become noisy for high frequencies and the upper frequency bound \(U_b(n, m)\) can be used to cut-off some of the noise. But again, the resonance peaks contains a lot of information about the flexibility parameters. The choice of bounds and weights is a trial-and-error procedure.

The fact that the identification manages to find values of the parameters such that the discrepancies between the model FRF and the experimental FRF are small close to the resonance modes prove that the result shown in Figure 3 is, indeed, satisfactory. It should be pointed out that the identification was carried out by using FRF’s from two different configurations (robot positions \((q_r)\)) and that the result was satisfactory for all twelve different configurations considered. In other words, the model and the estimated parameters are valid globally! In order to evaluate the result further, an accelerometer were placed at the tool center point (TCP) of the robot during the experiment. The FRF’s obtained from these data, and the corresponding model FRF’s (using a kinematic model) are shown in Figure 4. Although the discrepancies are somewhat larger in this case compare to the motor acceleration FRF’s, we think this result also prove the ability of the methods proposed.
Figure 4: Frequency response functions of a 6-axis, 18 DOF flexible manipulator model (blue) and a real 6-axis manipulator prototype (red). The FRF's represent motor torque-to-TCP acceleration in dB. The frequency scale is in Hz.

Figure 5: Final values of $V(\theta)$ for sixty identification runs, started with different initial parameter values $\theta_0$. The 17th run gave the optimal result here.
5 Suggestions for future work

The model could be improved by introducing nonlinear gearbox flexibilities $\tau_g$, and improvements of the identification would probably concern investigations of identifiability properties, which hopefully will determine what robot configurations (positions) give the best results. An investigation of the low frequency bias of the FRF’s estimates could also be of interest. By improving the estimates of the TCP accelerometer FRF’s, these could not only be used for verification but also for identification.

Figure 6: A 6-axis elbow type industrial robot from ABB.

References


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