Frequency-Domain Identification of Continuous-Time ARMA Models from Non-Uniformly Sampled Data

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Abstract
This paper treats direct identification of continuous-time autoregressive moving average (CARMA) time-series models. The main result is a method for estimating the continuous-time power spectral density from non-uniformly sampled data. It is based on the interpolation (smoothing) using the Kalman filter. A deeper analysis is also carried out for the case of uniformly sampled data. This analysis provides a basis for proceeding with the non-uniform case. Numerical examples illustrating the performance of the method are also provided both, for spectral and subsequent parameter estimation.

Keywords: Continuous-time systems; Parameter estimation; Continuous-time ARMA; CARMA; continuous-time noise model; Whittle likelihood estimator; Non-uniform sampling; Irregular sampling; Non-equidistant sampling;
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Abstract

This paper treats direct identification of continuous-time autoregressive moving average (CARMA) time-series models. The main result is a method for estimating the continuous-time power spectral density from non-uniformly sampled data. It is based on the interpolation (smoothing) using the Kalman filter. A deeper analysis is also carried out for the case of uniformly sampled data. This analysis provides a basis for proceeding with the non-uniform case. Numerical examples illustrating the performance of the method are also provided both, for spectral and subsequent parameter estimation.

Recently there has been a renewed interest in continuous-time system identification in general and continuous-time noise models in particular, (Rao and Garnier, 2002), (Ljung, 2003a), (Larsson, 2003). See for instance the articles by Larsson and Söderström on continuous-time AR (Larsson and Söderström, 2002) and ARMA (Larsson and Mossberg, 2003) parameter estimation. The work on hybrid Box-Jenkins and ARMAX modeling by Pintelon et.al (Pintelon and Schoukens, 2000) and Johansson (Johansson, 1994) also concerns this problem. These approaches have in common that they use approximations of the noise or the noise model and consequently suffer from a bias in the model parameters.

The purpose of this paper is to derive a method for the identification of continuous-time autoregressive moving average (CARMA) models from non-uniformly sampled data. The identification is performed in the frequency domain where a continuous-time version of the Whittle likelihood approach is used. This requires an accurate estimate of the continuous-time power spectral density. Such an estimate is, in the case of non-uniformly sampled data, produced by smoothing by Kalman filtering. It is also shown that this method is related to the one derived by the authors for uniform sampling in an earlier paper (Gillberg and Ljung, 2005).

1 Model and Representations

In this paper we shall consider continuous-time ARMA models represented as

\[ y(t) = G_c(p)e(t) \] (1)
where $e(t)$ is continuous time white noise such that

$$E[e(t)] = 0$$
$$E[e(t)e(s)] = \sigma^2 \delta(t - s)$$

The operator $p$ is here the differentiation operator. We assume that $G(p)$ is strictly proper, so $y_t$ itself does not have a white-noise component, but is a well defined second order, stationary process. Its spectrum (spectral density) can be written as

$$\Phi_c(\omega) = \sigma^2 |G_c(i\omega)|^2$$

We shall consider a general model parameterization

$$G_c(p, \theta)$$

where the model parameter vector $\theta$ includes the noise variance $\lambda$ (whose true value is $\sigma^2$). The transfer function $G$ can be parameterized by $\theta$ in an arbitrary way, for example by the conventional numerator and denominator parameters:

$$G(p, \theta) = \frac{B(p)}{A(p)}$$

$$A(p) = p^n + a_1 p^{n-1} + a_2 p^{n-2} + \cdots + a_n$$
$$B(p) = p^m + b_1 p^{m-1} + \cdots + b_m$$
$$\theta = [a_1 \ a_2 \ \cdots \ a_n \ b_1 \ b_2 \ \cdots \ b_m \ \lambda]^T.$$  

We will now show how this model can be represented in state-space form.

### 1.1 Continuous-Time State Space Form

The model in (1) can be formally represented in a continuous-time controller canonical form

$$\begin{cases}
\dot{x}(t) = Ax(t)dt + Be(t) \\
y(t) = Cx(t)
\end{cases}$$

where

$$A = \begin{bmatrix}
-a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0
\end{bmatrix}$$
$$B = \begin{bmatrix}
1 & 0 & \cdots & 0
\end{bmatrix}^T$$
$$C = [0 \ \cdots \ 0 \ 1 \ b_1 \ b_2 \ \cdots \ b_m]$$

if $m < n$. The exact solution to the state-space representation in (10) can be written as

$$y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-s)}Be(t)dt.$$  

2
Since the systems in this paper are all linear, the treatment of \( e(t) \) in (6) will serve our purpose. The state covariance matrix \( \Pi(t) \) will obey the differential equation

\[
\dot{\Pi}(t) = A\Pi(t) + \Pi(t)A^T + \sigma^2 BB^T
\]  

(7)

If the process is to be stationary, the initial states must have a Gaussian distribution with zero mean and covariance \( \Pi \) which satisfies the Lyapunov equation

\[
A\Pi + \Pi A^T + \sigma^2 BB^T = 0.
\]  

(8)

The autocorrelation function for the process \( y(t) \) will then be

\[
r(\tau) = E[y(t)y(t+\tau)] = Ce^{A\tau}\Pi C^T.
\]  

(9)

We will now give a discrete-time representation of this process.

### 1.2 Discrete-Time State Space Form

In this paper the discrete-time version

\[
\begin{align*}
\begin{cases}
x(t_{k+1}) & = F(h_k)x(t_k) + u(t_k) \\
y(t_k) & = Cx(t_k) \\
Eu(t_k)u^T(t_k) & = Q(h_k)
\end{cases}
\end{align*}
\]  

(10)

at the time instances \( \{t_1, t_2, \ldots, t_N\} \) of the continuous-time model will also be of interest. Here the inter-sample distances are denoted \( h_k \triangleq t_{k+1} - t_k, k = 1 \ldots N - 1 \). The familiar state transition matrix will be

\[
F(h_k) = e^{Ah_k}.
\]  

(11)

The sequence \( u(t_k), k = 1 \ldots N \) is as set of independent vector-valued Gaussian variables, each with the covariance matrix

\[
Q(h_k) = \sigma^2 \int_0^{h_k} e^{At}BB^Te^{AT}dt
\]  

(12)

In this case the discrete-time state covariance matrix will be governed by the relationship

\[
\Pi(t_{k+1}) = F(h_k)\Pi(t_{k+1})F^T(h_k) + Q(h_k)
\]  

(13)

where, for equidistant sampling, stationarity is guaranteed if the covariance matrix satisfies the discrete-time Lyapunov equation

\[
F\Pi F^T - \Pi + Q = 0
\]  

(14)

### 2 The estimation problem

The common way of modeling a general time series of the unstructured form (4) is to estimate a discrete-time model in the time domain and then transform
it to continuous time. If the parameterization (3) is tailor-made it would have to be transformed to discrete time

\[ G_d(q, \theta) \]  

by the well known sampling formulas, (Söderström, 1991), retaining the original parameters. Then the discrete time grey box model \( y_t = G_d(q, \theta) e_d^t \) can be estimated by a straightforward prediction error method. See, e.g (Ljung, 1999), and implementations of this approach in (Ljung, 2003b). These methods work well and for Gaussian distributed signals they constitute the Maximum likelihood approach. Thus, in theory and asymptotically as the number of data tends to infinity, no method can perform better. However, for fast sampled data, they could be subject to numerically ill-conditioned calculations. Also, for irregularly sampled data, the computational burden could be substantial.

2.1 Continuous time signals

Let us for a moment assume that we have available the whole, continuous time signal \( y \) over the time interval \([0, T]\) \( (T = NT_s) \). Then the periodogram estimate of the continuous time spectrum could be computed as

\[ \hat{\Phi}_T^c(i\omega) = \left| Y_T^c(i\omega) \right|^2 \]  

where the truncated continuous-time Fourier transform is

\[ Y_T^c(i\omega) = \frac{1}{\sqrt{T}} \int_0^T y(t) e^{-i\omega t} dt. \]  

Just as in the sampled case \( Y_T^c \) will have a Gaussian distribution with zero mean and variance equal to the continuous time spectrum (once \( T \) is large enough so that transients and non-periodic effects can be neglected). Moreover the Fourier transforms will be asymptotically independent for frequencies that are further apart than the frequency resolution \( \frac{2\pi}{T} \). See e.g. (Brillinger, 1981) and (Gillberg, 2004), Section 3. Just as for the discrete-time case we thus have the continuous-time Whittle-type estimator

\[ \hat{\theta} \triangleq \arg \min_{\theta} V_T^c(\theta, \hat{\Phi}_T^c) \]  

where

\[ V_T^c(\theta, \hat{\Phi}_T^c) \triangleq \sum_{k=1}^{N_c} \frac{\hat{\Phi}_T^c(i\omega_k)}{\Phi_c(i\omega_k, \theta)} + \log \Phi_c(i\omega_k, \theta). \]  

See also (Gillberg, 2004), Chapter 3 for a more detailed description.

3 Estimating the Continuous-Time Spectrum

The natural way to form an estimate of the continuous-time spectrum would be to use the continuous-time periodogram as illustrated in (16). Unfortunately we are unable to compute the continuous-time Fourier transform in (17) since we do not know the appropriate intersample behavior of the process \( y(t) \). In the sections below we will show how this can be accomplished for both uniformly and non-uniformly sampled data.
3.1 Revisiting the Uniformly Sampled case

In the paper (Gillberg and Ljung, 2005) a way to estimate the continuous-time spectrum from the discrete-time one

\[
\hat{\Phi}_c(i\omega) = \Phi_{f}(e^{i\omega T_s}) \hat{\Phi}_d(e^{i\omega T_s})
\]  

(19)

was presented for the case of uniformly sampled data \( t_k = kT_s, \ k = 1 \ldots N \).

Here

\[
\Phi_{f}(e^{i\omega T_s}) \triangleq \frac{\epsilon^{i\omega T_s - 1}}{B_{2\ell-1}(e^{i\omega T_s})} \left( 2\ell - 1 \right)^{2\ell}
\]  

(20)

where \( B_{2\ell-1}(z) \) were the so called Euler-Frobenius polynomials (Weller et al., 2001). This relationship was found by approximating the system by a set of integrators

\[
C_{c}(p) = \frac{1}{p^\ell}
\]  

(21)

where \( \ell \) is the same as the relative degree of the original system. In that case we will have

\[
\Phi_{f}(e^{i\omega T_s}) = \frac{\Phi_{c}^\ell(i\omega)}{\Phi_{d}^\ell(e^{i\omega T_s})}.
\]  

(22)

This expression can be interpreted another way which opens for estimators for the case of non-uniformly sampled data.

Assume that we want to create a linear estimator of the continuous-time process \( y(t) \) when we only know equidistantly distributed samples. Such an estimator would then be expressed as

\[
\hat{y}(t) = \sum_{k=-\infty}^{\infty} f(t, t_k)y(t_k).
\]

The best such estimator, in a mean square sense, would be characterized by the coefficients \( f(t, t_k) \): \( k = -\infty \ldots \infty \) minimizing

\[
E \left[ y(t) - \sum_{k=-\infty}^{\infty} f(t, t_k)y(t_k) \right]^2.
\]

The solution to this problem are the classical Yule-Walker equations

\[
E \left[ \left( y(t) - \sum_{k=-\infty}^{\infty} f(t, t_k)y(t_k) \right) y(t_l) \right] \quad \forall l \in \mathbb{Z}
\]

which are equivalent to

\[
\rho(t - t_l) = \sum_{k=-\infty}^{\infty} f(t, t_k)\rho(t_k - t_l) \quad \forall l \in \mathbb{Z}
\]  

(23)
If we define \( t' = t - t_l \) and \( t'_\tau = t_k - t_l \) the expression will become
\[
r(t') = \sum_{\tau=-\infty}^{\infty} f(t' + t_l, t'_\tau + t_l) r(t'_\tau) \quad \forall l \in \mathbb{Z}
\]
Since the left side is independent of \( t_l \) the same must be true for the right. Hence
\[
f(t' + t_l, t_l + t_l) = f(t', t'_\tau) \quad \forall l \in \mathbb{Z}
\]
which in turn means that
\[
f(t', t_l) = f(t' - t_l).
\]
The expression in (23) will therefore become
\[
r(t') = \sum_{\tau=-\infty}^{\infty} f(t' - t'_\tau) r(t'_\tau).
\]
Taking the continuous-time Fourier transform of this expression will lead to
\[
\Phi_c(i\omega) = \int_{-\infty}^{\infty} r(t) e^{-i\omega t'} dt'
\]
\[
= \int_{-\infty}^{\infty} \sum_{\tau=-\infty}^{\infty} f(t' - t'_\tau) r(t'_\tau) e^{-i\omega t'} dt'
\]
\[
= \sum_{\tau=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t' - t'_\tau) e^{-i\omega t'} dt' r(t'_\tau)
\]
\[
= \sum_{\tau=-\infty}^{\infty} \Phi_f(e^{i\omega T_s}) e^{-i\omega t'_\tau} r(t'_\tau)
\]
\[
= \Phi_f(e^{i\omega T_s}) \sum_{\tau=-\infty}^{\infty} r(t'_\tau) e^{-i\omega t'_\tau}
\]
\[
= \Phi_f(e^{i\omega T_s}) \Phi_d(e^{i\omega T_s}).
\]
Therefore we will have the expression
\[
\Phi_f(e^{i\omega T_s}) = \frac{\Phi_c(i\omega)}{\Phi_d(e^{i\omega T_s})}
\]
Using the approximations in (21) the above expression will immediately yield (22) and (20).

4 Spectral Estimation for Non-Uniform Sampling

The method described above assumes an infinite amount of uniformly sampled data, and yields a frequency domain expression for a smoother. The generalization to the case of non-uniformly sampled data over a limited time interval is of course to use smoothing based on the Kalman filter. In the sections below we will explain forwards and backwards Markovian models; the Kalman filter and how these objects can be used to estimate the intersample behavior of the process \( y(t) \).
4.1 Backwards Markovian Models

The state space representation of the process (1) provided by the state-space representation in (10) is a so called forwards Markovian model. Given such a model, the so called backwards Markovian representation e.g.

\[
\begin{aligned}
    x(t_k) &= F_b(h_k)x(t_{k+1}) + u_b(t_{k+1}) \\
    y(t_k) &= Cx(t_k) \\
    Eu_b(t_k)u_b^T(t_k) &= Q_b(h_k)
\end{aligned}
\]

with reversed time-direction can be found. Here the new state transition matrix is

\[
F_b(h_k) = F^{-1}(h_k) - F^{-1}(h_k)Q(h_k)\Pi^{-1}(t_{k+1})
\]

and the new noise covariance is

\[
Q^b(h_k) = F^{-1}(h_k)Q(h_k) + Q(h_k)\Pi^{-1}(t_{k+1})Q(h_k)F^{-T}(h_k)
\]

See e.g. the book (Kailath et al., 2000) or the paper by Verghese and Kilath (Verghese and Kailath, 1979).

4.2 Intersample Smoothing by Forward and Backward Kalman Filters

Assuming that the parameters of the CARMA models are known, the discrete-time forward Kalman filter will be

\[
\begin{aligned}
    e(t_{k+1}) &= y(t_{k+1}) - CF(h_k)\hat{x}(t_k|t_k) \\
    \hat{x}_f(t_{k+1}|t_{k+1}) &= F(h_k)\hat{x}(t_k|t_k) + K(t_{k+1})e(t_{k+1}) \\
    \hat{x}_f(t_0|t_0) &= \Pi(t_0)C^TR_e^{-1}(t_0)y(t_0)
\end{aligned}
\]

where

\[
\begin{aligned}
    K(t_{k+1}) &= (F(h_k)P(t_k|t_k)F^T(h_k) + Q(h_k))CR_e^{-1}(t_{k+1}) \\
    R_e(t_{k+1}) &= R(t_{k+1}) + C(F(h_k)P(t_k|t_k)F^T(h_k) + Q(h_k))C^T \\
    P_f(t_{k+1}|t_{k+1}) &= F(h_k)P(t_k|t_k)F^T(h_k) + Q(h_k) \\
    & - K(t_{k+1})R_e(t_{k+1})K^T(t_{k+1}) \\
    P_f(t_0|t_0) &= \Pi_0 - \Pi_0C^TR_e^{-1}(t_0)C\Pi_0
\end{aligned}
\]

A analogous backwards Kalman filter can of course also be derived

\[
\begin{aligned}
    e(t_k) &= y(t_k) - CF_b(h_k)\hat{x}(t_{k+1}|t_{k+1}) \\
    \hat{x}_b(t_k|t_k) &= F_b(h_k)\hat{x}(t_{k+1}|t_{k+1}) + K_b(t_k)e(t_k) \\
    \hat{x}_b(t_{N+1}|t_{N+1}) = 0
\end{aligned}
\]
where
\[ K(t_k) = (F_b(h_k)P_b(t_{k+1}|t_{k+1})F_b^T(h_k) + Q_b(h_k))CR_b^{-1}c(t_k) \]
\[ R_b(h_k) = R(t_k) + C(F_b(h_k)P(t_{k+1}|t_{k+1})F_b^T(h_k) + Q_b(h_k))C^T \]
\[ P_b(t_k|t_k) = F_b(h_k)P(t_{k+1}|t_{k+1})F_b^T(h_k) + Q_b(h_k) \]
\[ - K(t_{k+1})R_b(t_{k+1})K^T(t_{k+1}) \]
\[ P_b(t_N|t_N) = \Pi(t_{N+1}) \]

These filters provide forward and backwards state estimates, \( \hat{x}_b(t_k|t_k) \) and \( \hat{x}_b(t_{k+1}|t_{k+1}) \), together with their error covariance matrices \( P_f(t_k|t_k) \) and \( P_b(t_{k+1}|t_{k+1}) \). In order to estimate the process in between samples at time \( t_k \) and \( t_{k+1} \), dead reckoning (time-updating) of the estimates and covariance matrices
\[
\hat{x}_f(t|t_k) = F(t - t_k)\hat{x}(t_k|t_k) \\
\hat{x}_b(t|t_{k+1}) = F_b(t_{k+1} - t)\hat{x}(t_{k+1}|t_{k+1}) \\
P_f(t|t_k) = F(t - t_k)P(t_k|t_k)F^T(t - t_k) + Q(t - t_k) \\
P_b(t|t_{k+1}) = F_b(t_{k+1} - t)P(t_{k+1}|t_{k+1})F_b^T(t_{k+1} - t) + Q_b(t_{k+1} - t)
\]
are be used. These estimates can then be fused together using the classical formulas
\[
P^{-1} = P_f^{-1}(t|t_k) + P_b^{-1}(t|t_{k+1}) - \Pi^{-1}(t) \\
P^{-1}\hat{x}_x(t) = P_f^{-1}(t|t_k)\hat{x}_f(t|t_k) + P_b^{-1}(t|t_{k+1})\hat{x}_b(t|t_{k+1}) \\
\hat{y}_m(t) = C\hat{x}_x(t)
\]
in order to provide a smoothed estimate \( \hat{y}_m(t) \) of the process at time \( t \).

### 4.3 Approximations

The purpose of estimating the continuous-time spectrum in this paper is to use it for frequency-domain system identification. Therefore, it is unrealistic to assume too much knowledge about the system in beforehand. We are therefore content with knowing the relative degree \( \ell \) of (11) and to approximate the system with integrations as in (21). This means the continuous- and discrete-time state transition matrices will be
\[
A = \begin{bmatrix}
0 & 0 & \cdots & 0 & 0 \\
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0
\end{bmatrix}
\]
Since \( A^T = 0 \) we get
\[
F(h_k) = e^{Ah_k} = \sum_{m=0}^{l-1} A^m \frac{h_k^m}{m!} = \begin{bmatrix}
1 & 0 & \cdots & 0 & 0 \\
\frac{h_k}{1!} & 1 & \cdots & 0 & 0 \\
\frac{h_k^2}{2!} & t & \cdots & 1 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\frac{h_k^l}{l!} & \frac{h_k^{l-1}}{(l-1)!} & \cdots & h_k & 1
\end{bmatrix}
\]
and the discrete time noise covariance matrix will be

\[ Q(h_k) = \sigma^2 \int_0^h e^{At} BB^T e^{A^T t} dt \]

\[ = \sum_{m=0}^{l-1} \sum_{n=0}^{l-1} \frac{A^m BB^T A^T n}{m! n!} \int_0^h t^{m+n} dt \]

\[ = \sum_{m=0}^{l-1} \sum_{n=0}^{l-1} \frac{A^m BB^T A^T n}{m! n!} \frac{h^{m+n+1}}{m + n + 1} \]

\[ = \begin{bmatrix} h_k^2 & h_k^3 & \cdots \\ h_k^3 / 2 & h_k^4 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \]

The processes defined by \( A, F(h_k) \) and \( B \) are not stationary and the covariance matrix \( \Pi(t) \) will grow as \( t \to \infty \). Therefore we assume that \( \Pi(t) \) is so large that terms involving \( \Pi^{-1}(t) \) can be omitted. This yields simpler expressions for the backwards state transition and process noise matrices \( F_b(h_k) \) and \( Q_b(h_k) \) such that

\[ F_b(h_k) = F^{-1}(h_k) = e^{-Ah_k} \]

\[ = \sum_{m=0}^{l-1} A^m (-1)^m \frac{h_k^m}{m!} \]

\[ = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ -h_k & 1 & \cdots & 0 & 0 \\ \frac{h_k^2}{2} & -h_k & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ (-1)^{l-1} \frac{h_k^l}{l!} & (-1)^{l-1} \frac{h_k^l}{l!} & \cdots & \vdots & \vdots \end{bmatrix} \]

and

\[ Q_b(h_k) = F^{-1}(h_k)Q(h_k)F^{-T}(h_k) \]

\[ = \sigma^2 \int_{-h}^0 e^{A^T t} BB^T e^{At} dt \]

\[ = \sum_{m=0}^{l-1} \sum_{n=0}^{l-1} \frac{A^m BB^T A^T n}{m! n!} (-1)^{m+n} \frac{h^{m+n+1}}{m + n + 1} \]

\[ = \begin{bmatrix} h_k^2 & h_k^3 & \cdots \\ h_k^3 / 2 & h_k^4 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \]
Figure 1: Comparison of the estimated and true spectrum of the system in (28). Estimation has been performed on $N = 400$ samples at nominal sampling time $T_s = 0.5$ with jitter factor $\delta_0 = T_s/3$. The mean value of the estimated spectrum have been generated by $N_{MC} = 250$ Monte-Carlo runs.

5 Numerical Illustration

In the following examples we have used the continuous-time model

$$y(t) = \frac{1}{\rho^2 + 3\rho + 2} e(t).$$  \hspace{1cm} (28)

The output $y$ has been sampled at time instances subject to jitter such that

$$t_k = kT_s + \delta_k, \hspace{0.5cm} k = 1 \ldots N - 1$$

where

$$\delta_k \in U[-\delta_0, \delta_0] \hspace{0.5cm} k = 1 \ldots N - 1.$$  

For the sake of simplicity, the initial and endpoint time instances have been chosen such that $t_0 = 0$ and $t_N = NT_s$.

In Figure 1 we estimated the continuous-time spectrum of the process in (28). The nominal sampling time is $T_s = 0.5$ while the jitter factor is $\delta_0 = T_s/3$. The number of samples that have been used are $N = 400$ which yields a total simulation time of $T = 200$. The spectrum presented in the figure is the average of $N_{MC} = 250$ Monte-Carlo runs. In Table 1 we estimated the parameters of the process in (28). Here, the nominal sampling time is varied between $T_s = 0.5$ and $T_s = 0.25$ while the jitter factor is $\delta_0 = T_s/3$. The number of samples that have been used are $N = T/T_s$ with a total simulation time of $T = 200$. The parameters presented in the figure are the average of $N_{MC} = 250$ Monte-Carlo runs.

6 Conclusions

This paper treats direct identification of continuous-time autoregressive moving average (CARMA) time-series models. The main result is a method for esti-
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Table 1: Results for the system \( \frac{1}{p^3+3p+2} \). This system has a pole excess of 2 and a bandwidth of 0.8358 rad/s.

mating the continuous-time power spectral density from non-uniformly sampled data. It is based on the interpolation (smoothing) using the Kalman filter. A deeper analysis is also carried out for the case of uniformly sampled data. This analysis provides a basis for proceeding with the non-uniform case. Numerical examples illustrating the performance of the method are also provided both, for spectral and subsequent parameter estimation.

References


This paper treats direct identification of continuous-time autoregressive moving average (CARMA) time-series models. The main result is a method for estimating the continuous-time power spectral density from non-uniformly sampled data. It is based on the interpolation (smoothing) using the Kalman filter. A deeper analysis is also carried out for the case of uniformly sampled data. This analysis provides a basis for proceeding with the non-uniform case. Numerical examples illustrating the performance of the method are also provided both, for spectral and subsequent parameter estimation.