Uncertain LTI-models for Linear Control Design of Non-Linear Systems

Torkel Glad, Anders Helmersson, Lennart Ljung

Division of Automatic Control

E-mail: torkel@isy.liu.se, andersh@isy.liu.se,
ljung@isy.liu.se

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Address:

Department of Electrical Engineering
Linköpings universitet
SE-581 83 Linköping, Sweden

WWW: <http://www.control.isy.liu.se>

AUTOMATIC CONTROL
REGLERTEKNIK
LINKÖPINGS UNIVERSITET

Abstract

Much attention in robust identification and control has been focused on linear low order models approximating high order linear systems. We consider the more realistic situation with a linear model approximating a non-linear system. We describe how a linear time invariant (LTI) model with unstructured uncertainty, i.e. a “band” of Nyquist curves can be developed using a non-linear model error model. Applying standard linear robust control design to this uncertain LTI model will lead to a (non-linear) closed loop system with performance robustness guarantees (in terms of gain from disturbances to output) well in line with the objectives of the linear design. Clearly the design can be successful only if the linear model is a reasonable good approximation of the system. A particular aspect of the design process is to define a workable definition of “practical stability” for robust control design, with possibly non-linear model errors. We use affine power norms for that purpose.

Keywords: robustness, nonlinear system, model approximation

Uncertain LTI-models for Linear Control Design of Non-Linear Systems

S. T. Glad, A. Helmersson and L. Ljung
Department of Electrical Engineering
Linköping University
S-581 83 Linköping, Sweden

Abstract. Much attention in robust identification and control has been focused on linear low order models approximating high order linear systems. We consider the more realistic situation with a linear model approximating a non-linear system. We describe how a linear time invariant (LTI) model with unstructured uncertainty, i.e. a “band” of Nyquist curves, can be developed using a non-linear model error model. Applying standard linear robust control design to this uncertain LTI model will lead to a (non-linear) closed loop system with performance robustness guarantees (in terms of gain from disturbance to output) well in line with the objectives of the linear design. Clearly the design can be successful only if the linear model is a reasonably good approximation of the system. A particular aspect of the design process is to define a workable definition of “practical stability” for robust control design, with possibly non-linear model errors. We use affine power norms for that purpose.

1 Introduction

In this paper we consider the estimation of linear models, of the type

$$y(t) = G(q, \theta)u(t) + H(q, \theta)e(t) \quad (1)$$

while keeping in mind that the true system will probably lie outside the model class, for instance because it contains a nonlinearity. The standard LTI identification machinery, e.g., [6], gives a transfer function estimate

$$\hat{G}_N(q) = G(q, \hat{\theta}_N) \quad (2)$$

along with an uncertainty region $\mathcal{G}$ defined by a transfer function $W(q)$:

$$\mathcal{G} = \{G \| G(e^{i\omega}) - \hat{G}_N(e^{i\omega}) \| \leq W(e^{i\omega}), \forall \omega\} \quad (3)$$

This can be used to design a regulator

$$u(t) = F(q)(r(t) - y(t)) \quad (4)$$

so that the closed loop system

$$y(t) = \frac{F(q)\hat{G}_N(q)}{1 + F(q)\hat{G}_N(q)} \quad (5)$$

behaves well. The design has to take the uncertainty region $\mathcal{G}$ into account. The extensive literature on robust control design, e.g., [15], [12] deals with the question to ensure good behavior of the closed loop system, despite inaccuracies in the model $\hat{G}_N$.

An important question is how to find the uncertainty region. The standard LTI-machinery delivers bounds that are based on estimates of the statistical uncertainty, i.e. the variance of the estimated parameters. This assumes that the model class indeed contains a “true” description of the system.

The interplay between the estimation and the design processes has received substantial attention over the past years, e.g., [4], [11], [3], [5], [13], [10].

Part of this problem is to estimate, in reliable way, the model error or model uncertainty, not assuming that the true system can be described within the chosen structure (1). See, e.g., [9], [8]. Then it is mostly assumed that the true system is linear, but of higher order than the model. In this contribution we shall consider the more general situation of possibly non-linear model errors, and discuss the roles that an explicit model error model will play in such a set-up, to define an equivalent uncertain LTI model (3). For simplicity we confine ourselves to single-input-single-output systems. See also [7]

2 Model Error Models and Their Gains

A model error model is a description of how the input u affects the output error of the nominal model ε . We will typically not be interested in a detailed such description – that should rather be done as a more detailed nominal model. Primarily we focus on bounds on the gain of such a model. To go well together with linear control design, we shall work with combinations of frequency weightings and unstructured, unknown models with bounded (estimated) gain. That means that the general structure of the model from u to ε can be depicted as in Figure 1. Formally we have

Figure 1: The model error model

$$\varepsilon_F(t) = W_2^{-1}(q)\varepsilon(t), \quad u_F(t) = W_1(q)u(t) \quad (6)$$

$$\varepsilon_F(t) = \hat{g}_{mem}(u_F^{t-1}) + w(t) \quad (7)$$

Here, W_1 and W_2 are given linear filters, superscript $t-1$ denotes all the signal values up to time $t-1$, and $\hat{g}_{mem}$ is a general dynamic model. The bottom line is that, with some amount of confidence, based on the measured data, and possibly on prior information, we should be able to estimate its gain.

3 The nonlinear gain definition

An important aspect of nonlinear model error models is that the standard definition of gain for a nonlinear system with input u and output y :

$$\sup_{u \neq 0} \frac{\|y\|}{\|u\|} \quad (8)$$

is sometimes misleading. Consider for instance a static nonlinearity with a small relay-type nonlinearity at the origin. The gain becomes $+\infty$ no matter how small ϵ is. This type of nonlinearity is quite common in actuators and power amplifiers where positive and negative signals are handled by different physical components, that are not perfectly matched.

We will instead use an affine version of the gain definition. This has been used in the nonlinear control literature, see e.g. Vidyasagar [14]. We use the power gain definition of [2, 1] and say that the nonlinear system has gain β with offset α if there are positive constants α and β such that

$$\int_0^\tau y^2(t) dt \leq \alpha\tau + \beta^2 \int_0^\tau u^2(t) dt \quad (9)$$

for all input-output pairs u and y and all positive τ .

A discussion of the feasibility to estimate α and β in (9) is given in [7].

4 Control design

Consider the system of figure 2. It consists of a nominal linear model G_{nom} together with the nonlinear system

$\hat{g}_{mem}$ and the linear weights (transfer functions) W_1 and W_2 . It is controlled using linear feedback via the transfer function F . We assume that there is a known

Figure 2: Block diagram of feedback loop with model error

affine gain limit on $\hat{g}_{mem}$ so that

$$\int_0^\tau z_2^2(t) dt \leq \alpha\tau + \beta^2 \int_0^\tau z_1^2(t) dt \quad (10)$$

For this linear nominal system with its (possibly) nonlinear model error model we assume that the uncertain LTI model (3) is delivered with $\hat{G}_N = G_{nom}$ and $W = \beta W_1 W_2$. We should also think of W_2 as a shaping filter for the external disturbances at the output.

Applying standard robust linear control design will lead to a regulator F with sensitivity (S) and complementary sensitivity (T) functions of the closed loop system (without the error model):

$$S = \frac{1}{1 + G_{nom}F}, \quad T = \frac{G_{nom}F}{1 + G_{nom}F}$$

such that $\|SW_2\|_\infty$ is small as such that the ratio between the T and the “normalized relative model error”

$$G_w = T \frac{W_1 W_2}{G_{nom}}$$

has a gain less than $1/\beta$. We now have the following property of the closed loop system.

Proposition 1 *Let the system be described by Figure 2. Assume that $\beta\|G_w\| < 1$. Then the output y is bounded by*

$$\int_0^\tau y^2(t) dt \leq \|SW_2\|^2 \frac{\alpha\tau}{1 - \beta^2\|G_w\|^2}$$

Proof: Following the signals around the block diagram of Figure 2 gives the relations

$$z_1 = -G_w z_2 \quad (11)$$

$$y = SW_2 z_2 \quad (12)$$

The relation (10) together with (11) then gives

$$\int_0^\tau z_2^2 dt \leq \frac{\alpha\tau}{1 - \beta^2\|G_w\|^2}$$

from (12) we get

$$\int_0^\tau y^2 dt \leq \|SW_2\|^2 \frac{\alpha\tau}{1 - \beta^2\|G_w\|^2}$$

■

Consequently, the robust linear control objectives to shape S for noise reduction at the same time as T is shaped for robustness will also guarantee a good disturbance rejection in the true, nonlinear closed loop system.

5 Discussion

Of course it is impossible to give absolute robustness guarantees for a design based only on identification data (and hence on a finite number of limited precision measurements) and with no a priori knowledge of the physical system. Proposition 1 shows that a precise robustness result *can* be proven if the parameters α and β of the affine gain of the model error model are known. This shows that one aim of the identification procedure could be to give as reliable estimates as possible for those parameters. Note that the result of the proposition does not give asymptotic stability in the case of $\alpha > 0$. Instead an upper bound on $\|y\|$ is given.

The standard LTI paradigm of estimating linear models always gives an uncertainty set around a linear system (the second order equivalent). The size of this set tends to zero as more and more measurements become available, [7]. This is an unrealistic aspect of LTI-modeling: the system will not be *exactly* known even with very many observations. The way to define an LTI uncertainty set $\mathcal{G}$ as in (3) with $W = \beta W_1 W_2$ from Figure 2 and (10) will allow some *remaining uncertainty* even when the number of data tends to infinity. If this remaining uncertainty is too large to handle with linear robust control theory, we have an indication that a non-linear regulator is required.

On the other hand, if the uncertainty region is small enough to allow for robust linear control, we have seen that applying standard, linear loop shaping of S and T from $\hat{G}_N$, βW and W_2 will lead to a design that also handles the nonlinearities in a nice fashion.

An uncertain LTI model, constructed as described here, can thus very well be used for linear design, even if the true system is nonlinear. This is of course also evident from all real applications done with robust control theory, since no real life system is exactly LTI.

A key problem is to estimate the gains in (10). Some preliminary tests were made in [7], but this is a topic that needs further research.

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