ILC applied to a flexible two-link robot model using sensor-fusion-based estimates

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Abstract

Estimates from an extended Kalman filter (EKF) is used in an Iterative Learning Control (ILC) algorithm applied to a realistic two-link robot model with flexible joints. The angles seen from the arm side of the joints (arm angles) are estimated by an EKF in two ways: 1) using measurements of angles seen from the motor side of the joints (motor angles), which normally are the only measurements available in commercial industrial robot systems, 2) using both motor-angle and tool-acceleration measurements. The estimates are then used in an ILC algorithm. The results show that the actual arm angles are clearly improved compared to when only motor angles are used in the ILC update, even though model errors are introduced.

Keywords: Iterative learning control; Robotics; Sensor fusion

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Abstract—Estimates from an extended Kalman filter (EKF) is used in an Iterative Learning Control (ILC) algorithm applied to a realistic two-link robot model with flexible joints. The angles seen from the arm side of the joints (arm angles) are estimated by an EKF in two ways: 1) using measurements of angles seen from the motor side of the joints (motor angles), which normally are the only measurements available in commercial industrial robot systems, 2) using both motor-angle and tool-acceleration measurements. The estimates are then used in an ILC algorithm. The results show that the actual arm angles are clearly improved compared to when only motor angles are used in the ILC update, even though model errors are introduced.

I. INTRODUCTION

The idea in Iterative Learning Control (ILC) is to use the repetitive system dynamics to compensate for errors. Since the first publications [3], [5], [6] in 1984, ILC has become one of the methods to achieve high accuracy in robot control. Normally in commercial industrial robot systems only the angles seen from the motor side of the joints are measured, while the control objective is to follow a desired tool path. ILC experiments performed on a large-size ABB robot indicate that although the performance on the motor side is improved, it is no guarantee that the errors of the tool position decrease [18]. This motivates the need of using additional sensors to improve the estimates of the tool position, velocity and acceleration and hence be able to improve the control performance.

In many publications the subject of estimation and control of flexible robots are discussed, see for example [17]. In [10] the states for a robot with flexible joints are estimated using an observer based on only the motor angles. Examples when the extended Kalman filter (EKF) are used in the estimation are [11] and [12]. In [15] the acceleration of the tool is measured and two solutions for the state estimation problem are discussed, using EKF and particle filters, respectively.

In this paper EKF and ILC are combined for a realistic two-link robot model with mechanical flexibilities. The methods for estimating the robot tool position are further investigated in [9], upon which the work in this paper is based. To the best of the authors knowledge, estimation techniques and ILC have only been combined in a few publications. One example is [8], where the ILC algorithm uses an estimate of

the arm angle, computed using measurements of the motor angle and the arm angular acceleration of a flexible one-link robot arm.

II. TWO-LINK ROBOT MODEL

For the simulation the two-link robot model shown in Fig. 1 is used. The model corresponds to the second and third link of a large industrial serial robot with six motors, and the motion is restricted to the $\hat{x}_0\hat{z}_0$ -plane.

The elasticity in the robot is modelled as flexible joints, while the links are considered as being rigid. In order to describe the deflection in each joint, two angles per joint are needed. The angle seen from the motor side of the joint is referred to as motor angle, while the angle seen from the arm side (link side) of the joint is called arm angle.

Fig. 1: Flexible two-link robot model, which corresponds to the second and third motor on a large industrial serial robot with six motors. The joints (gear transmissions) are described by nonlinear spring torque τ_s , linear damping d and friction torque f .

A. Original nonlinear robot model

The model used is fully described in [13]. Each link has rigid body characteristics described by mass m , link length l , center of mass ξ and inertia j with respect to center of mass. The links are actuated by electrical motors, connected to the links via elastic joints. Each joint is described by the nonlinear spring torque τ_s , linear damping D , friction torque f and gear ratio n . Due to the deflection mentioned earlier, each joint is described by the motor angle θ_m and the arm angle θ_a . The motor angle is the only measured output, and is subject to measurement noise and time delay. The motor torque u_m is subject to saturation. To summarise, the robot dynamics is given by

$$u = M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) + D(\dot{\theta}) + \tau_s(\theta) + f(\dot{\theta}), \quad (1)$$

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where

$$\theta = (\theta_{a1} \ \theta_{a2} \ \theta_{m1}/n_1 \ \theta_{m2}/n_2)^T, \quad (2)$$

and u is the vector of inputs. Now introduce the vector of states $x = (\theta^T \ \dot{\theta}^T)^T$, and denote $x_1 = (\theta_{a1} \ \theta_{a2})^T$, $x_2 = (\theta_{m1}/n_1 \ \theta_{m2}/n_2)^T$, $x_3 = \dot{x}_1$, $x_4 = \dot{x}_2$. The nonlinear state-space description of (1) is

$$\dot{x} = \begin{pmatrix} x_3 \\ x_4 \\ M_a^{-1}(x_1)(-C(x_1, x_3) - G(x_1) - A(x)) \\ M_m^{-1}(A(x) + f(x_4) + u_m) \end{pmatrix}, \quad (3)$$

where

$$A(x) = D(x_3 - x_4) + \tau_s(x_1, x_2), \quad (4)$$

and M_a and M_m are the inertia matrices for the arm and motor, given from $M = \text{diag}(M_a, M_m)$. The reader is directed to [13] regarding the details of the model.

B. Nonlinear robot system used in the simulation

From the desired tool position the motor angle references r_m are computed using the kinematic and dynamic models. These references are thereafter compared to the actual motor angles in the discrete-time controller, which can be seen in Fig. 2. In the figure it is also illustrated how the ILC update u_k at iteration k is added to r_m , and it can be seen that the ILC algorithm thereby is a complement to the ordinary controller. The robot system is implemented in Simulink.

The input disturbance is set to zero for simplicity. To avoid problems with drift in the estimates as described in [9], the assumption is made that the robot is subject to no gravitation, that is, $G(\theta) = 0$.

The tool motion studied in the simulation is half a circle in the $\hat{x}_0\hat{z}_0$ -plane with radius 5 mm and center coordinates $(x \ z)^T = (1 \ 1.5)^T$ m. The resulting arm angles θ_{ai} and arm angular velocities $\dot{\theta}_{ai}$ can be seen in Fig. 3.

C. Linearised robot system for analysis

The robot system in Sec. II-B need to be linearised both for the EKF in Sec. III and the stability analysis of the ILC algorithm described in Sec. IV.

For the EKF, the nonlinear continuous state-space description (3) is discretised using Euler difference approximation, and thereafter linearised around the previous estimate.

Fig. 2: The robot system used in the simulation. The ILC update u_k at iteration k is added to the motor-angle reference r_m .

Fig. 3: The desired tool motion is half a circle. In (a) the resulting arm angles θ_{a1}, θ_{a2} are shown, and in (b) the arm angular velocities $\dot{\theta}_{a1}, \dot{\theta}_{a2}$.

For the ILC stability analysis, the robot dynamics is linearised around the stationary point $(x \ z)^T = (1 \ 1.5)^T$ m, which means the initial arm angles ($t = 0$) shown in Fig. 3. The feedforward torques u_{ffw} are set to zero, since only the relationship between the references r_{mi} and the outputs θ_{mi}, θ_{ai} are needed as described in Sec. IV.

III. ESTIMATION ALGORITHMS

In this paper the extended Kalman filter (EKF) is used, see for example [2]. The arm-angle estimation is based on the results presented in [9], where different versions of the EKF as well as a deterministic observer described in [7] are investigated and evaluated experimentally. For the details regarding the estimation, tuning process and robustness of the methods, the reader is directed to [9].

Given a general nonlinear discrete-time system

$$\begin{aligned} x_{t+1} &= F(x_t, u_t) + v_t, & v_t &\sim \mathcal{N}(0, R_{v,t}), \\ z_t &= h(x_t, u_t) + w_t, & w_t &\sim \mathcal{N}(0, R_{w,t}). \end{aligned} \quad (5)$$

The state estimation is performed in two steps; measurement update

$$\begin{aligned} \hat{x}_{t|t} &= \hat{x}_{t|t-1} + K_t(z_t - h(\hat{x}_{t|t-1}, u_t)), \\ P_{t|t} &= P_{t|t-1} - K_t H_t P_{t|t-1}, \end{aligned} \quad (6)$$

and time update

$$\begin{aligned} \hat{x}_{t+1|t} &= F(\hat{x}_{t|t}, u_t), \\ P_{t+1|t} &= A_t P_{t|t} A_t^T + R_{v,t}, \end{aligned} \quad (7)$$

with

$$\begin{aligned} K_t &= P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + R_{w,t})^{-1}, \\ A_t &= \frac{\partial F(x, u_t)}{\partial x} \Big|_{x=\hat{x}_{t|t}}, \\ H_t &= \frac{\partial h(x, u_t)}{\partial x} \Big|_{x=\hat{x}_{t|t-1}}. \end{aligned} \quad (8)$$

An accelerometer attached to the second link close to the tool moves with the robot and the measurements are given in the coordinate frame of the accelerometer. Two ways of producing the estimate $\hat{\theta}_a$ is used, called:

1) *EKF Motor*: θ_a is estimated from measurements of θ_m , the estimate is denoted $\hat{\theta}_{a,m}$. The measurement equation is

$$h(x_t, u_t) = x_{2,t}. \quad (9)$$

2) *EKF Complete*: θ_a is estimated from measurements of θ_m and acceleration of the tool, resulting in $\hat{\theta}_{a,ma}$. The measurement equation now includes both motor-angle measurements as well as accelerometer measurements, giving

$$h(x_t, u_t) = \begin{pmatrix} x_{2,t} \\ \ddot{\rho}_s(x_t) \end{pmatrix}, \quad (10)$$

with

$$\ddot{\rho}_s(x_t) = R_w^s(x_{1,t})(J(x_{1,t})\dot{x}_{3,t} + \dot{J}(x_{1,t})x_{3,t}), \quad (11)$$

where the rotation from the world coordinate system w to the sensor coordinate system s is R_w^s , and J is the Jacobian of the robot kinematics.

The tuning of the noise covariances in the EKF implementations are performed automatically, as is fully described in [9]. The objective function is to minimise the prediction error on a set of measurement data consisting of motor angles, tool acceleration, and true tool position. Data are from simulations where the robot performs various movements near the working point used in this paper.

In Fig. 4 the measured arm angles θ_a from Fig. 3 are compared to the estimated arm angles $\hat{\theta}_{a,m}$ and $\hat{\theta}_{a,ma}$ when the robot system performs a circular motion as described in Sec. II-B. The case with nominal parameters is compared to when model errors are introduced by modifying the joint stiffness parameters by -30% . It can be seen that the estimate $\hat{\theta}_{a,ma}$ from EKF Complete performs better than the estimate $\hat{\theta}_{a,m}$ from EKF Motor, as is also noted in [9]. For the case with model errors, EKF Motor gives a worse estimate, as could be expected, since it relies on only motor-angle measurements and the dynamic model.

It should be noted that considerable more computational resources are required for EKF Complete than for EKF Motor. Although the EKF in combination with ILC is applied offline, this will become an important issue for the experimental implementation for problems with larger dimensions.

(a) EKF with motor measurements (b) EKF with motor and tool measurements

Fig. 4: EKF based on motor-angle measurements, EKF Motor (a), and EKF based on motor-angle measurements and measurements of tool acceleration, EKF Complete (b). In each subfigure, the estimation error $\theta_a - \hat{\theta}_a$ are shown for the two arm angles for the case with nominal parameters (left) and -30% model error (right).

IV. ILC ALGORITHMS

It is assumed that a general description of a linear discrete-time system can be formulated as

$$y_k = T_r r + T_u u_k, \quad (12)$$

where u_k is the ILC input signal, y_k is the system output, k denotes the iteration number and r is the reference. The signals are defined on a finite time interval $t = 0, \dots, N-1$. Finally, T_r and T_u are stable, causal discrete-time filters. System and measurement disturbances are not included here, but can easily be treated in the framework. The update equation for a general ILC algorithm is

$$u_{k+1} = Q(u_k + L e_k), \quad (13)$$

where the linear filters Q and L are possibly non-causal and

$$e_k = r - y_k, \quad (14)$$

is the error at iteration k .

The system (12) can be described in matrix form, [16] was among the first users of the description in the ILC community. Let

$$\mathbf{y}_k = (y_k(0) \dots y_k(N-1))^T, \quad (15)$$

and define $\mathbf{r}$ and $\mathbf{u}_k$ similarly, which gives

$$\mathbf{y}_k = \mathbf{T}_r \mathbf{r} + \mathbf{T}_u \mathbf{u}_k. \quad (16)$$

This system description is more general than the LTI-system representation presented above, because $\mathbf{T}_r$ and $\mathbf{T}_u$ can be linear time-variant. The matrix $\mathbf{T}_u$ is formed by the impulse response coefficients of the transfer function T_u and is described by the Toeplitz matrix

$$\mathbf{T}_u = \begin{pmatrix} g_{T_u}(0) & 0 & \dots & 0 \\ g_{T_u}(1) & g_{T_u}(0) & & 0 \\ \vdots & & \ddots & \vdots \\ g_{T_u}(N-1) & g_{T_u}(N-2) & \dots & g_{T_u}(0) \end{pmatrix}, \quad (17)$$

and $\mathbf{T}_r$ is defined analogously. Using this form, the ILC update (13) can be written

$$\begin{aligned} \mathbf{u}_{k+1} &= \mathbf{Q}(\mathbf{u}_k + \mathbf{L} \mathbf{e}_k), \\ \mathbf{e}_k &= \mathbf{r} - \mathbf{y}_k. \end{aligned} \quad (18)$$

Combining (16) and (18) then gives

$$\mathbf{u}_{k+1} = \mathbf{Q}(\mathbf{I} - \mathbf{L} \mathbf{T}_u) \mathbf{u}_k + \mathbf{Q} \mathbf{L}(\mathbf{I} - \mathbf{T}_r) \mathbf{r} = \mathbf{F} \mathbf{u}_k + \mathbf{F}_r \mathbf{r}, \quad (19)$$

using the notation in [14]. Now, if

$$\bar{\sigma}(\mathbf{F}) = \bar{\sigma}(\mathbf{Q}(\mathbf{I} - \mathbf{L} \mathbf{T}_u)) < 1, \quad (20)$$

where $\bar{\sigma}(\cdot)$ denotes the largest singular value, the system is stable and has monotone convergence, see [14].

A. Previous works on ILC algorithms for MIMO systems

In this section the choice of ILC algorithm for the two-input-two-output system is related to previous works regarding ILC for MIMO systems.

As pointed out in [4], extensions of the matrix form to square MIMO systems are generally straightforward, which will be illustrated in the example in next section. The work [19] discusses various aspects regarding MIMO ILC algorithms applied to nonlinear non-affine-in-input systems. Although the system in this paper is affine with respect to the input, the result in [19] is worth noting. They investigate the performance of the linear ILC algorithm $u_{k+1} = u_k + \gamma e_k$ as a comparison to a more complicated ILC algorithm (called Newton-type in [19]), which requires more system knowledge. One result, that motivates the choice of ILC algorithm in this paper seen in Sec. V, is that the simple linear algorithm can achieve asymptotic convergence also for quite complicated nonlinear non-affine MIMO systems.

In [1] three different approaches; heuristic, model-based, and frequency-response based method are discussed when deriving the linear ILC algorithm with the structure $u_{k+1} = u_k + QLe_k$. The heuristic ILC design is very similar to the one used in this paper. The methods are compared in experiments with a parallel kinematic manipulator (PKM), which is a highly nonlinear and coupled MIMO system, and all approaches yield an important improvement of accuracy.

B. Example

In this paper an ILC algorithm with a diagonal structure is assumed. For the two-input-two-output system in Sec. II-B, the ILC update

$$u_{k+1,i} = Q_i(u_{k,i} + L_i e_{k,i}), \quad i = 1, 2 \quad (21)$$

is added to the motor angle reference r_{mi} for each motor, as could be seen in Fig. 2. The error (14) refers to either motor angles or arm angles. In this example the system description (12) is

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} T_{u,11} & T_{u,12} \\ T_{u,21} & T_{u,22} \end{pmatrix} \begin{pmatrix} u_{k,1} \\ u_{k,2} \end{pmatrix} + \begin{pmatrix} T_{r,11} & T_{r,12} \\ T_{r,21} & T_{r,22} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}. \quad (22)$$

The relation T_u between ILC update u_k and y and T_r from reference r to y , are in this example both equal to the closed-loop system, since r_m and u_k affect the system from the same point, see Fig. 2. It then gives

$$\begin{aligned} \begin{pmatrix} u_{k+1,1} \\ u_{k+1,2} \end{pmatrix} &= \begin{pmatrix} Q_1(1 - L_1 T_{u,11}) & -Q_1 L_1 T_{u,12} \\ -Q_2 L_2 T_{u,21} & Q_2(1 - L_2 T_{u,22}) \end{pmatrix} \begin{pmatrix} u_{k,1} \\ u_{k,2} \end{pmatrix} \\ &\quad + \begin{pmatrix} Q_1 L_1(1 - T_{r,11}) & -Q_1 L_1 T_{r,12} \\ -Q_2 L_2 T_{r,21} & Q_2 L_2(1 - T_{r,22}) \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \\ &= \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} \begin{pmatrix} u_{k,1} \\ u_{k,2} \end{pmatrix} + \begin{pmatrix} F_{r,11} & F_{r,12} \\ F_{r,21} & F_{r,22} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}, \end{aligned} \quad (23)$$

which also can be written

$$u_{k+1} = Q(I - LT_u)u_k + QL(I - T_r)r, \quad (24)$$

$$Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}, \quad L = \begin{pmatrix} L_1 & 0 \\ 0 & L_2 \end{pmatrix}.$$

Using the matrix description for the reference r gives

$$r = \begin{pmatrix} r_1(0) & \dots & r_1(N-1) & r_2(0) & \dots & r_2(N-1) \end{pmatrix}^T, \quad (25)$$

the signals u_k, u_{k+1} are defined analogously. The Toeplitz matrices $F_{ij}, F_{r,ij}$ are described similarly to (17), noting that the systems $F_{ij}, F_{r,ij}$ are non-causal. This results in the ILC update (23) on matrix form,

$$u_{k+1} = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} u_k + \begin{pmatrix} F_{r,11} & F_{r,12} \\ F_{r,21} & F_{r,22} \end{pmatrix} r = F u_k + F_r r. \quad (26)$$

As mentioned before, the ILC system has monotone exponential convergence if $\bar{\sigma}(F) < 1$.

V. SIMULATION RESULTS

The overall idea in this paper is to investigate whether it is possible to improve the robot tool performance using estimates of the arm angles in the ILC update, instead of measured motor angles that normally are the only measurements available in commercial industrial robot systems. The flexible nonlinear two-input-two-output robot model described in Sec. II-B is used in the simulation.

In the simulation the ILC algorithm (21) is used, where

- Q is a low-pass second-order Butterworth filter with cutoff frequency ω_n above the resonance frequency of the controlled system. The filter is applied by filtering the signal forwards and backwards to give zero-phase characteristics.
- L is given by $L = \gamma q^\delta$, where γ is the learning gain, q is the discrete time-shift operator and δ is the time shift.

In the simulation the same ILC filter is used for both motors for simplicity, that is, $Q_1 = Q_2$ and $L_1 = L_2$.

To evaluate performance and robustness when using estimated arm angles in the ILC update (21), the stiffness parameters of the joints are changed by -30% in the robot model.

A. Categorisation of the cases

The conditions can be divided into:

- 1) θ_m is the measured variable. The ILC update uses $e_k = r_m - \theta_m$.
- 2) θ_a is estimated based on measurements of θ_m (EKF Motor), resulting in $\hat{\theta}_{a,m}$. The ILC update uses $e_k = N(r_a - \hat{\theta}_{a,m})$, where N is a matrix of gear ratios.
- 3) θ_a is estimated based on measurements of θ_m and tool acceleration (EKF Complete), resulting in $\hat{\theta}_{a,ma}$. The ILC update uses $e_k = N(r_a - \hat{\theta}_{a,ma})$.

The ILC update is added to the motor reference r_m as in Fig. 2. The reference signals r_m are computed from the nominal system, and can be seen as filtered versions of the arm-angle references r_a and given beforehand.

In the simulation the actual arm angles θ_a are used as evaluation variables that should follow the references r_a .

B. ILC using motor-angle error (case 1)

The ILC algorithm uses the motor-angle error $e_m = r_m - \theta_m$. The ILC parameters are gain $\gamma = 0.5$, time shift $\delta = 70$, and $\omega_n = 8$ Hz for the cutoff frequency of the Q filter, which well fulfil the criterion (20) since $\bar{\sigma} \approx 0.9 < 1$ for both the nominal and model-error case.

In Fig. 5 the motor-angle errors and arm-angle errors are shown for ILC iteration $k = 0$, together with the errors after iteration $k = 50$. The case with nominal model can be compared to the case when a model error of -30% for the stiffness parameters are introduced. In Fig. 6 the 2-norm of the motor-angle errors and arm-angle errors are shown, when expressed in percent of the error for iteration $k = 0$.

Figs. 5 – 6 illustrate what happens when model errors are introduced. As is not surprising, the error reduction on the arm side for $k = 50$ is smaller compared to the nominal case. It can be explained by the fact that the motor-angle reference r_m is a filtered version of the arm-angle reference r_a . This works as long as the model from actual motor angle θ_m to actual arm angle θ_a is correct, so that controlling θ_m towards r_m will give θ_a close to r_a . In the case of model errors, θ_m is still controlled towards r_m , but θ_m now does not result in correct θ_a .

Fig. 5: Case 1: Resulting motor and arm-angle errors for ILC iteration $k = 50$ compared to the errors for ILC iteration $k = 0$. Nominal system (a) and system with -30% model errors (b).

Fig. 6: Case 1: Error norm expressed in percent of the error norm for ILC iteration $k = 0$, for the nominal system (black) and the system with -30% model errors (grey).

C. ILC using estimated arm-angle error (case 2 – 3)

Case 2: The ILC update is now made in another way; namely by using the estimate $\hat{\theta}_{a,m}$ from EKF Motor in Sec. III. The ILC parameters used here are $\omega_n = 8$ Hz for the cutoff frequency of the Q filter, gain $\gamma = 0.5$ and time shift $\delta = 220$. The parameters are tuned by simulation so that they give a stable behaviour for case 2 and case 3, for both the nominal and model-error case.

The simulation result can be seen in Figs. 7 – 8. It is not obvious how to compare the result to case 1 shown in Figs. 5 – 6. However, one can see that the gap between the error reduction for the nominal case and the model-error case is reduced. This is also seen in Table I, where the 2-norm and ∞ -norm of the arm-angle errors for iteration $k = 50$ are given in percent of the error for iteration $k = 0$.

Case 3: Now the ILC update uses the estimated tool angle $\hat{\theta}_{a,ma}$ from EKF Complete, described in Sec. III. Since the result only show a minor difference compared to the corresponding result for case 2 shown in Fig. 7, this figure is omitted here. As is shown in Fig. 9 and Table. I, the error reduction is slightly better for EKF Complete than for EKF Motor in Fig. 6, originating from the behaviour shown in Fig. 4. This fits the result shown in [9], where EKF Motor and EKF Complete have similar performance in some of the evaluation measures.

To summarise, the overall conclusion is that using an estimate of the arm angle in the ILC update it is possible to achieve better performance of the tool.

Fig. 7: Case 2: Resulting motor and arm-angle errors for ILC iteration $k = 50$ compared to the errors for ILC iteration $k = 0$. Nominal system (a) and system with -30% model errors (b).

VI. CONCLUSIONS AND FUTURE WORK

In this paper the extended Kalman filter (EKF) and the Iterative Learning Control (ILC) algorithm are combined for a realistic flexible two-link robot model. Estimates of the arm angles are derived using the EKF in two ways; 1) using only motor-angle measurements, and 2) using both motor-angle measurements and tool-acceleration measurements. The case where the ILC update is based on only motor angles is compared to the case where estimated arm angles are used in the ILC update. Robustness issues and performance are discussed for the cases. The results show that it is possible

Fig. 8: Case 2: Error norm expressed in percent of the error norm for ILC iteration $k = 0$, for the nominal system (black) and the system with -30% model errors (grey).

Fig. 9: Case 3: Error norm expressed in percent of the error norm for ILC iteration $k = 0$, for the nominal system (black) and the system with -30% model errors (grey).

TABLE I: Arm-angle errors for iteration $k = 50$, given in percent of the error for iteration $k = 0$ in different norms, for the cases with nominal parameters and modified joint stiffness parameters by -30% .

Conditions	Nom.		-30%	
	$\ e_{a1}\ $	$\ e_{a2}\ $	$\ e_{a1}\ $	$\ e_{a2}\ $
Case 1, $\ \cdot\ _2$	14.55 %	16.27 %	34.69 %	38.19 %
	$\ \cdot\ _\infty$	11.74 %	15.60 %	21.12 %
Case 2, $\ \cdot\ _2$	11.66 %	5.80 %	21.78 %	18.84 %
	$\ \cdot\ _\infty$	9.55 %	6.53 %	14.48 %
Case 3, $\ \cdot\ _2$	7.55 %	5.55 %	17.45 %	14.55 %
	$\ \cdot\ _\infty$	6.19 %	6.03 %	11.86 %

to improve the performance of the tool when the estimate is used in the ILC algorithm, compared to when only available measurements of motor angles are used.

Next step is to do an experimental comparison of ILC using estimates, using the results presented in [9]. It is then important to achieve good estimates without drift. Methods for tuning the ILC algorithm that works in practice will also be an important part of the work, as well as the computational issues. The stability of the ILC system also needs to be analysed in more detail when the (nonlinear) estimate is used in the ILC update.

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Titel ILC applied to a flexible two-link robot model using sensor-fusion-based estimates Title					
Författare Johanna Wallén, Svante Gunnarsson, Robert Henriksson, Stig Moberg, Mikael Norrlöf Author					
Sammanfattning Abstract Estimates from an extended Kalman filter (EKF) is used in an Iterative Learning Control (ILC) algorithm applied to a realistic two-link robot model with flexible joints. The angles seen from the arm side of the joints (arm angles) are estimated by an EKF in two ways: 1) using measurements of angles seen from the motor side of the joints (motor angles), which normally are the only measurements available in commercial industrial robot systems, 2) using both motor-angle and tool-acceleration measurements. The estimates are then used in an ILC algorithm. The results show that the actual arm angles are clearly improved compared to when only motor angles are used in the ILC update, even though model errors are introduced.					
Nyckelord Keywords Iterative learning control; Robotics; Sensor fusion					