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Energy coupling among the degrees of freedom in an electron–positron plasma

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Abstract. Nonlinear coupling of the motion in the three spatial degrees of freedom of a cold fluid electron–positron plasma is investigated. Exact solutions describing expanding flows with oscillations are obtained. It is found that the energy in the irrotational flow component is in general transferred to the rotational components, but not in the reversed direction. Furthermore, since the density evolution need not be related to all the three flow components, oscillations in one or two of the flow fields can be purely electromagnetic and are not accompanied by density oscillations.

1. Introduction

Nonlinear phenomena in fluids and plasmas are often studied in terms of finite but small perturbations of an equilibrium or steady state [1, 2]. As a result, the nonlinear behavior is in most cases due to wave–particle and/or wave–wave interactions of the participating linear waves and particles. The resulting nonlinear waves or phenomena retain most of the properties of the corresponding linear modes. Such problems are usually investigated using perturbation methods to obtain weakly nonlinear evolution equations. Most linear waves have similar dispersion, dissipation and nonlinear behavior when the latter is weak, so that their evolution can be described by one of the standard nonlinear evolution equations or their modifications, such as the Korteweg–deVries equation, the Burger’s equation, the nonlinear Schrödinger equation, the Hasegawa–Mima equation, the Kadomtsev–Petviashvili equation, etc. [2], whose derivation, properties and solutions are well understood. On the other hand, there also exists a number of nonperturbative plasma wave solutions of relatively simple problems, (see, e.g., [3–11]). Besides useful for understanding quasi-steady dynamic states and fully nonlinear evolution of waves and instabilities, such exact solutions are also important for verifying new analytical and numerical methods for solving nonlinear partial differential equations, as well as in formulating more realistic problems [4, 5]. Recently, an exact

model for cold plasma motion is used to investigate the energy transfer among the different degrees of freedom in a rotating plasma [9–11]. The governing cold plasma equations are solved non-perturbatively by first constructing a basis solution for the inertial, or force free, motion of the electron and ion fluids. It is found that the flow and oscillation energies tend to concentrate into the azimuthal flow component. It is further found that in an asymmetric flow system with two rotational degrees of freedom, the energy can flow into both rotational flow components [9–11]. An electron–positron (EP) plasma differs from an electron–ion plasma in that there does not exist various time scales arising from the large difference in the masses of the electrons and ions. The electrons and positrons react to any field perturbation with comparable strength. As a result, many known results, especially their nonlinear behavior, in plasma physics appear differently in an EP plasma [12, 13]. In this paper we consider exact asymmetric nonlinear wave motion in an EP plasma. It is found that the energy in the various degrees of freedom tend to flow into the rotational degree.

2. Formulation

We consider a cold EP plasma containing only electrons and positrons. The dimensionless cold fluid equations are

$$\partial_t n_j + \nabla \cdot (n_j \mathbf{v}_j) = 0, \quad (1)$$

$$\partial_t \mathbf{v}_j + \mathbf{v}_j \cdot \nabla \mathbf{v}_j = \mu_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}), \quad (2)$$

$$\nabla \cdot \mathbf{E} = n_+ - n_-, \quad (3)$$

$$\partial_t \mathbf{E} = n_- \mathbf{v}_- - n_+ \mathbf{v}_+, \quad (4)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (5)$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad (6)$$

where the time and space have been normalized by the inverse plasma frequency $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ and R_0 , an arbitrary space scale, say the initial size of the plasma, $\mp e$ and m are the charge and mass of the electrons and positrons, $\mu_{\mp} = \mp 1$, n_j and \mathbf{v}_j are the densities and velocities of the particle species $j = -, +$ (for electrons and positrons) normalized by a reference density n_0 and speed $R_0 \omega_p$, respectively. The electric and magnetic fields \mathbf{E} and \mathbf{B} are normalized by $4\pi e n_0 R_0$ and $(4\pi n_0 m)^{1/2} c$, respectively, where c is the light speed.

3. Basis flow structure

For our basis flow field we introduce the reduced velocity field [11] $\mathbf{V} = V_r(t, r, z) \mathbf{e}_r + V_\phi(t, r, z) \mathbf{e}_\phi + V_z(t, r, z) \mathbf{e}_z$, where $V_r = A(t)r$, $V_z = B(t)r$, and $V_\phi = C(t)r$, and the flow variables $A(t)$, $B(t)$ and $C(t)$ are associated with the temporal evolution of the radial, axial and azimuthal velocity components, respectively. The corresponding flow vorticity is then $\boldsymbol{\Omega} = -B(t) \mathbf{e}_\phi + 2C(t) \mathbf{e}_z$. We note that the azimuthal component of the vorticity is non-zero, so that the flow is not axisymmetric, although axisymmetric basis flows can also exist. A simple non-trivial case is when A , B and C are homogeneous in space. The force-free momentum equation

$\partial_t \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V} = 0$, then leads to

$$d_t A + A^2 - C^2 = 0, \tag{7}$$

$$d_t B + AB = 0, \tag{8}$$

$$d_t C + 2AC = 0, \tag{9}$$

which yield the basis solutions that can be used for constructing exact solutions of the cold EP plasma equations [12, 13]. The density does not directly affect the flow since the plasma is cold. We also note that the purely inertial basis flow is independent of the mass and charge of the particle species.

4. Dynamics of the EP plasma

Using the basis flow field obtained in the last section, one can investigate (1)–(6) for a cold EP plasma. For example, the velocity fields of the electrons and positrons can be represented by [11] $v_{jr} = rA_j(t)$, $v_{jz} = rB_j(t)$ and $v_{j\phi} = rC_j(t)$. For a spatially uniform EP plasma the electric field can be written as

$$\mathbf{E} = r\varepsilon_r(t)\mathbf{e}_r + r\varepsilon_\phi(t)\mathbf{e}_\phi + r\varepsilon_z(t)\mathbf{e}_z. \tag{10}$$

From the conservation equations we obtain after equating terms with the same dependence the ordinary differential equations

$$d_t n_\mp + 2A_\mp n_\mp = 0, \tag{11}$$

$$d_t A_\mp + A_\mp^2 - C_\mp^2 = \mu_\mp(\varepsilon_r + C_\mp B_z - B_\mp B_\phi), \tag{12}$$

$$d_t B_\mp + A_\mp B_\mp = \mu_\mp(\varepsilon_z + A_\mp B_\phi), \tag{13}$$

$$d_t C_\mp + 2A_\mp C_\mp = \mu_\mp(\varepsilon_\phi - A_\mp B_z), \tag{14}$$

where \mathbf{B} is the magnetic field, which should not be confused with the velocity variable B , (they can easily be distinguished from their subscripts). Recall that we have assumed that the plasma is homogeneous, or $n_\mp = n_\mp(t)$. From the Maxwell's equations we get

$$n_+ = n_-(t) + 2\varepsilon_r(t), \tag{15}$$

$$d_t \varepsilon_r = n_-(A_- - A_+) - 2\varepsilon_r A_+, \tag{16}$$

$$d_t \varepsilon_\phi = n_-(C_- - C_+) - 2\varepsilon_r C_+, \tag{17}$$

$$d_t \varepsilon_z = n_-(B_- - B_+) - 2\varepsilon_r B_+, \tag{18}$$

$$d_t B_z = -2\varepsilon_\phi, \tag{19}$$

$$d_t B_\phi = \varepsilon_z, \tag{20}$$

where (15) is from the Poisson's equation. The ion and electron continuity equations (11) are thus satisfied automatically. It follows that the temporal and spatial variations of all the flow and field parameters are separated, but they are nonlinearly coupled through the above equations. The ordinary differential equations (12)–(20) determine in a mathematically exact manner the evolution of the flow field. Since the electron and positron vorticities are given by $\mathbf{\Omega}_\mp = -B_\mp(t)\mathbf{e}_\phi + 2C_\mp(t)\mathbf{e}_z$, we see that the flows of the electrons and positrons are rotational in both the axial and azimuthal directions.

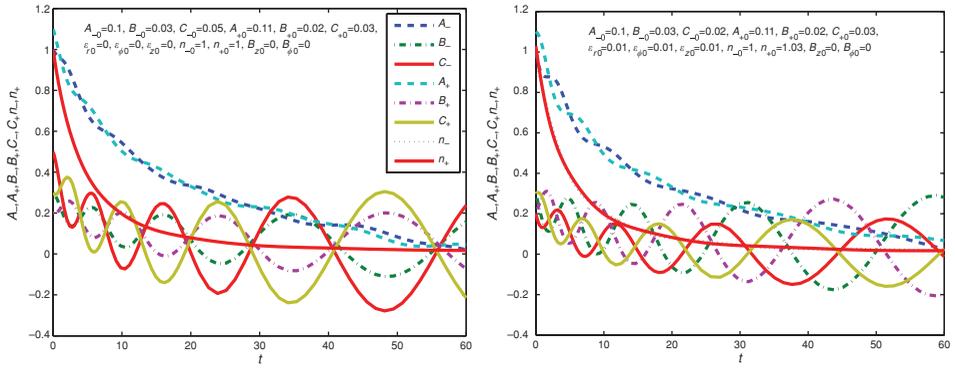


Figure 1. (Color online) Evolution of the flow quantities in an expanding EP plasma. For visual clarity, the curves for A_j , B_j and C_j have been normalized by A_{-0} . (a) The flow oscillation is electromagnetic since there is no density oscillation. (b) A weak density oscillation is started by a small difference in the initial electron and positron densities. The two solid red curves (n_{+} and n_{-}) can be distinguished by the initial conditions.

5. Numerical solutions

Depending on the initial conditions, (12)–(20) admit a rich variety of solutions. For simplicity we shall show six typical results by considering initial states that are near the inertially expanding basis flows.

Figure 1 shows a solution where oscillations occur in all velocity components. The initial conditions, which determine the solutions, are given in the figures. As the plasma expands, the amplitude and period of the oscillations increase with time. We can see that the oscillations in the axial (B_j) and azimuthal (C_j) flow components gain energy from the latter, but the magnitudes of the flow components decrease as the plasma expands. The electron and positron densities overlap, indicating no charge separation, and they also decrease with time as expected. There is almost no oscillation in the densities, so that the oscillations in the flow velocities are mainly electromagnetic. Figure 1(b) is similar to Fig. 1(a), except that initially the electron and positron densities are slightly different. This small difference induces a small charge separation, leading to a small out-of-phase oscillation in the electron and positron densities. The oscillation frequency is time dependent and is larger for larger-amplitude oscillations.

Figure 2(a) shows a case in which there are no oscillations. This case corresponds to the basis flow, which describes a freely expanding EP plasma. Here, as expected, since the basic flow is charge and mass independent, the densities and all the flow components of the electrons and positrons overlap. Figure 2(b) shows a case where the density and radial-flow oscillations decrease, but the axial- and azimuthal-flow oscillations increase with time. The oscillations exhibit both electrostatic and electromagnetic properties, and there is evidence that the oscillation energy in the radial flow is being transferred to the axial and azimuthal components as the plasma expands. Within the time of interest, the flow eventually becomes purely electromagnetic and the density oscillations vanish.

Figures 3(a) and (b) show the effects of strong initial axial and azimuthal magnetic field, respectively. We see that in both cases the oscillations in the azimuthal and radial flows are of large amplitude and that in the axial flow remain relatively

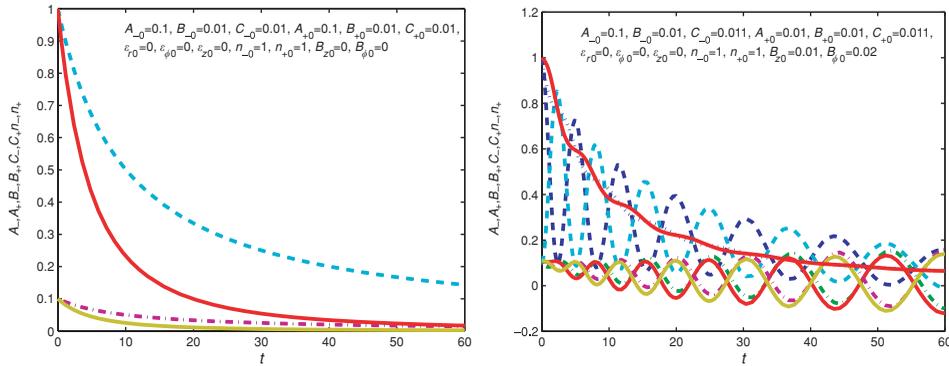


Figure 2. (Color online) Same (including the color code) as in Fig. 1. (a) A purely expanding flow with no oscillations. (b) An expanding flow with both electrostatic and electromagnetic oscillations. As t increases, the axial- and azimuthal-flow oscillations (B_j and C_j , respectively) increase, but the radial oscillations (A_j) decreases. Within the time of interest the flow field becomes electromagnetic and the density oscillation vanishes.

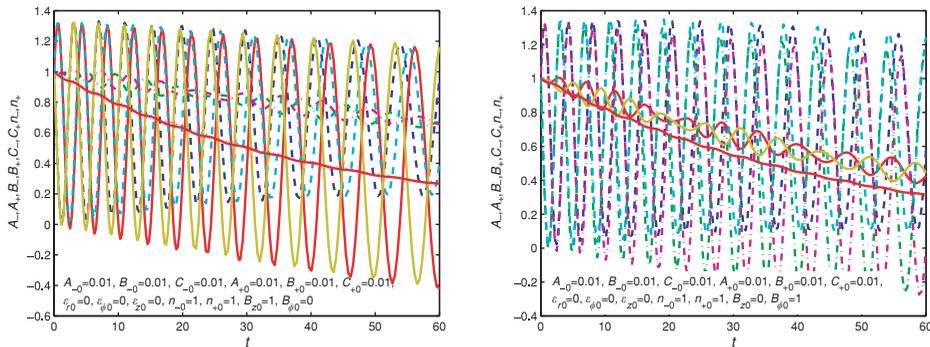


Figure 3. (Color online) Same (including the color code) as in Fig. 1, for cases with (a) strong initial axial magnetic field, and (b) strong initial azimuthal magnetic field. The resulting cyclotron and upper-hybrid frequencies can be inferred from the presence of higher oscillation frequencies

small. Furthermore, in the presence of a strong magnetic field the frequency spectrum is broadened, which can be attributed to the existence of new characteristic frequencies, namely the cyclotron frequency $\Omega = e|\mathbf{B}|/m$ and the upper-hybrid frequency $(\Omega^2 + 2\omega_p^2)^{1/2}$, together with their harmonics and difference frequencies that exist because the oscillations are highly nonlinear.

6. Discussion and conclusion

We have considered three-dimensional cylindrically symmetric nonlinear flows and oscillations in a rotating and expanding EP plasma by first introducing a basis force-free solution for the flow field. The time and the three spatial degrees of freedom of the flow fields are separated and the resulting set of 11 ordinary differential equations are solved numerically. The solutions satisfy in a mathematically exact sense the continuity and momentum equations of cold electrons and positrons, as

well as the Maxwell's equations. Depending on the initial state, even with the simple global flow structure considered here, there can appear a large number of solutions that are of physical interest. Four solutions typical for flows in the freely expanding plasma are presented. These include the expanding field-free basic flow, as well as flows with electrostatic and electromagnetic oscillations.

Clearly, the basis flow and the oscillating solutions discussed here are special cases. More general basis flow structures, such as that for a contracting plasma or other non-symmetric flows, can also be constructed by choosing appropriate spatial variations of the velocity and electric/magnetic fields by trial and error. All such solutions are expected to be mathematically exact and thus inherently stable [7]. Despite their simplicity, they can be of physical interest, and important conclusions can be drawn from the results. In particular, for the cases calculated, we found that energy in the rotational flow components cannot be converted to the irrotational degrees of freedom, but the reverse is possible. Depending on the initial conditions, oscillations can also appear only in certain degrees of freedom. The electrostatic oscillations can be attributed to nonlinear plasma and/or upper-hybrid waves in the EP plasma. There can also exist purely electromagnetic or hybrid oscillations that can be associated with the other normal modes in EP plasmas. However, since the oscillations are highly nonlinear and the oscillation frequencies are amplitude dependent, it is difficult to identify the modes in terms of the well-known linear modes. As expected because of their equal masses and equal but opposite charges, the electrons and positrons always oscillate with the same magnitude and out of phase with each other.

Our results may be useful in the study of large-scale expanding and rotating EP plasmas, such as that might be found in the early universe [12, 13]. The exact solutions can also be useful as bench tests for novel analytical and numerical methods for solving nonlinear partial differential equations [1, 2] and as a guide for numerical study of more complex and realistic problems [5, 8].

Acknowledgements

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