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Optimization as a Thermodynamic System

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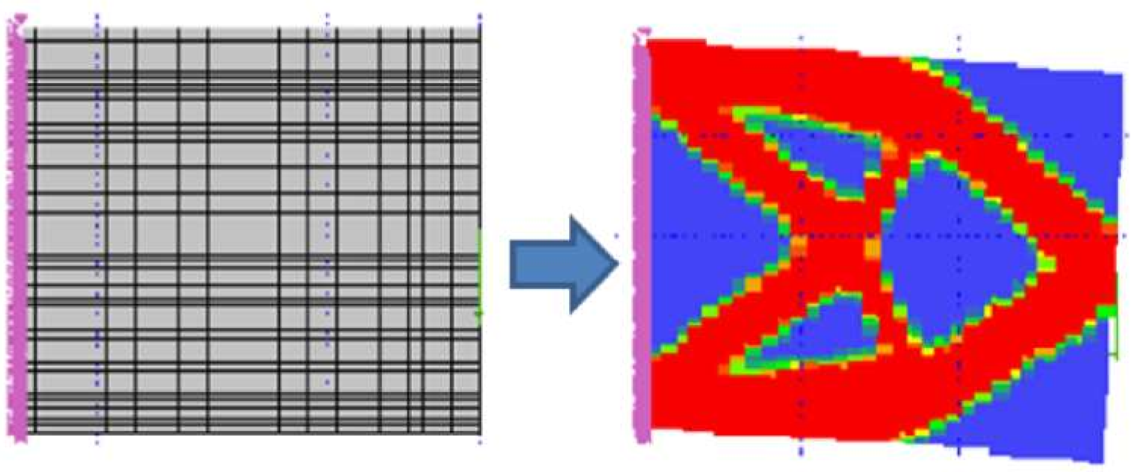

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Abstract

As we know that nature made the things optimized in all point of views, also it is supposed that nature works under some evolutionary process.

Since there was no such Evolutionary Structural Optimization (ESO) method having strong mathematical background, that's why these are not much reliable. The purpose of this thesis work is a little effort to introduce such an ESO method having a strong mathematical background.

In this thesis work *Optimization as a thermodynamic system*, we are introducing a new method for topology optimization by using concept of *Free Energy* and *Dissipation Potential* from non-smooth thermodynamics system. For better understanding we may call it as *Evolutionary Structural Topology Optimization (ESTO)*, and this project work is done in the following steps.

An evolution problem is formulated in terms of free energy and dissipation potential for a non-smooth thermodynamical system. Free energy is taken as an objective function for a general structural optimization problem. Derivation of a well posed evolution problem for which evolution is such that objective function always decreases. An optimality criteria method is derived for given evolution problem and it is implemented in a FEM program *TRINITAS*. And the behaviour of the so called evolutionary parameters such as Forward and Backward plastic constants is analyzed.

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Contents

Chapter 1

Introduction

1.1 Structural optimization

Making an assemblage of materials sustain loads in the best possible way is called *structural optimization* [1]. There are three types of structural optimization problems: size, shape and topology. Topology optimization is a developing technique. The most common algorithms used for topology optimization are Optimality Criteria Method and Methods of moving asymptotes.

In general Evolutionary Structural Optimization(ESO) has no volume constraints. Also ESO can be easily implemented into any general purpose finite element analysis program. In contrast to most other methods, the ESO involves no mathematical programming techniques in the optimization process [2].

1.1.1 Sizing optimization

In size optimization for plates or membrane, thickness is optimized and for beams, height, width and radius of cross section area is optimized.

1.1.2 Shape optimization

In shape optimization inner or outer shape of design domain is optimized.

1.1.3 Topology optimization

In topology optimization, number of holes and their configuration in design domain is discussed. Topology optimization tries to find the best use of material for a body. The objective is to minimize the compliance of structure or maximize stiffness.

In topology optimization, the design variables are the amounts of material in each cell. Material is only added where it is needed to carry loads.

1.2 Why topology optimization?

In general a low weight and high stiffness design requires from structural optimization. But change in shape and size may not lead our design criterion for reduction of structural weight. So one way to achieved this goal is topology optimization. For topology optimization the designer creates only the design space. The efforts for the modelling and preparation are extremely low [3]. Topology optimization is often achieves greater savings and design improvements than shape optimization. The topology optimization problem solves the basic engineering problem of distributing a limited amount of material in a design space, where a certain objective function has to be optimized [4].

Chapter 2

Convex Analysis

2.1 Introduction

In this chapter we are discussing some important topics of convex analysis such as convex set, convex function, optimum solution of convex optimization problem and Lyapunov function for dynamical system which leads us to formulate minimization problems in terms of free energy and dissipation potential in the next chapter.

2.2 Convex set

A set $\mathbb{X} \subset \mathbb{R}^n$ is said to be convex if, for any $x_1, x_2 \in \mathbb{X}$ and for any $\lambda \in [0,1]$, such that

$$\lambda x_1 + (1-\lambda)x_2 \in \mathbb{X}$$

Otherwise $\mathbb{X}$ is non-convex.

2.3 Convex function

Let $\mathbb{X} \subset \mathbb{R}^n$ be a convex set. A function $f : \mathbb{X} \to \mathbb{R}$ is said to be convex if for all $x_1, x_2 \in \mathbb{X}$ and for all $\lambda \in [0,1]$, there exist

$$f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f(x_1) + (1-\lambda)f(x_2)$$

Similarly, f is said to be strictly convex if strict inequality ($<$) holds above instead ($\leq$).

Geometrically, if $x_1, x_2 \in \mathbb{X}$, then the segment in $\mathbb{R}^{n+1}$ joining $(x_1, f(x_1))$ to $(x_2, f(x_2))$ lies above the graph of f.

2.4 Derivative of convex function

Differentiability plays a very important role in optimization for two related reasons

(a) Necessary conditions for optimality involve derivatives.

(b) Optimization algorithms involve derivative.

For a function $f : \mathbb{R}^n \to \mathbb{R}$, first derivative (or gradient) is denoted by $\bigtriangledown f$ and second derivative (or Hessian matrix) of f is denoted as $\bigtriangledown^2 f$.

A convex function needs not be a differentiable. e.g. $f(x) =\mid x \mid$ is not differentiable at $x = 0$.

2.4.1 Non-smooth or non-differentiable function

A function is said to be non-smooth if it is not continuously differentiable in the given interval. The concept of sub-differential is used for such kind of functions.

2.4.2 Sub-differential of a function

For any function f the sub-differential at x is denoted by $\partial f(x)$ and defined as a set of vectors $v \in \mathbb{R}^n$ such that

$$\partial f(x) = \{v : f(y) - f(x) \geq v^T(y - x) \;\; \forall \;\; y \in \mathbb{X}\}$$

For example sub-differential of $f(x) =\mid x \mid$ at $x = 0$ is a closed interval $[-1, 1]$.

2.4.3 Properties of sub-differential

(a) Sub-differential is a non-empty, closed and convex set.

(b) When f is differentiable then $\partial f(x) = \{\bigtriangledown f(x)\}$, a singleton set.

The elements of sub-differential are called sub-gradients.

2.5 Optimization problem

In general a minimization problem under inequality constraints is defined as

$$(\mathbb{P}) \begin{cases} \min\limits_{x} f_0(x) \\ \text{such that} \\ f_i(x) \leq 0 \; ; i = 1, 2, ..., \ell \\ x \in \mathbb{X} \end{cases} \tag{2.5.1}$$

where f_0 is an objective function and $f_i : \mathbb{R}^n \to \mathbb{R}, i = 1, ..., \ell$ are constraint functions. Here $f_i : \mathbb{R}^n \to \mathbb{R}, i = 0, 1, ..., \ell$ are assumed to be continuous differentiable functions, and

$$\mathbb{X} = \{x \in \mathbb{R}^n : x_j^{min} \leq x_j \leq x_j^{max}; j = 1, 2, ..., n\}$$

where $x_j^{min} \leq x_j \leq x_j^{max}; j = 1, 2, ..., n$ are so-called box constraints, if x_j^{min} and x_j^{max} have values $-\infty$ and $+\infty$ respectively, then there will be no box constraints.

$$S = \{\bar{x} \in \mathbb{X} : f_i(\bar{x}) \leq 0; i = 1, 2, ..., \ell\}$$

is called the feasible set for problem $(\mathbb{P})$.

Note:- $\max f_0(x) = -\min(-f_0(x))$

A point x^* is said to be local minimum of f_0 if

$$f_0(x^*) \leq f_0(\bar{x}) \; \forall \; | x^* - \bar{x} | < \varepsilon$$

and it is said to be global minimum of f_0 if

$$f_0(x^*) \leq f_0(\bar{x}) \; \forall \; \bar{x} \in S$$

Note:- The Problem $(\mathbb{P})$ is convex if and only if $f_i; i = 0, 1, ..., \ell$ are convex functions and S is a convex set.

2.5.1 Necessary and sufficient conditions

For convex $(\mathbb{P})$, necessary and sufficient condition for x^* to be the optimal point is

$$\bigtriangledown f_0(x^*)^T (x - x^*) \geq 0 \;\; \forall \;\; x \in \mathbb{X}$$

Also for unconstrained convex optimization problems, local (hence global) optima are located at stationary point x^*. i.e. a points for which the gradient of f is zero [5].

$$\bigtriangledown f_0(x^*) = \left[\frac{\partial f_0(x^*)}{\partial x_1}, ..., \frac{\partial f_0(x^*)}{\partial x_n}\right]^T = 0 \tag{2.5.2}$$

Example 1:- A convex problem needs not have a solution, unless the feasible set $\mathbb{X}$ is compact. i.e. $\mathbb{X}$ is bounded and closed. For example if $f_0 = 1/x$ is minimized subject to the closed, but unbounded set $x \geq 1$, then no solution exist, but if the same function is minimized subject to the compact set $[1, 2]$ then solution will be $x^* = 1/2$.

Example 2:- Convexity of feasible set $\mathbb{X}$ is also very important, because if the strictly convex function $x_1^2 + x_2^2$ is minimized subject to non-convex and compact set $1 \leq x_1^2 + x_2^2 \leq 2$, then there are infinite number of global minima are all points (x_1^*, x_2^*) with $x_1^2 + x_2^2 = 1$ [1].

2.5.2 Karush-Kuhn-Tucker (KKT) conditions

To identify a local (hence global) minimum of a convex problem ($\mathbb{P}$), first we define the **Lagrangian function** $\mathcal{L} : \mathbb{R}^n \times \mathbb{R}^l \to \mathbb{R}$ of problem ($\mathbb{P}$)

$$\mathcal{L}(x, \lambda) = f_0(x) + \sum_{i=1}^{\ell} \lambda_i f_i(x) \tag{2.5.3}$$

where $\lambda_i; i = 1, 2, ..., \ell$ are called Lagrange multipliers. The KKT-conditions of problem ($\mathbb{P}$) are defined as

$$\frac{\partial \mathcal{L}(x, \lambda)}{\partial x_j} \leq 0 \ , \ \ if \ \ x_j = x_j^{max} \tag{2.5.4}$$

$$\frac{\partial \mathcal{L}(x, \lambda)}{\partial x_j} = 0 \ , \ \ if \ \ x_j^{min} \leq x_j \leq x_j^{max} \tag{2.5.5}$$

$$\frac{\partial \mathcal{L}(x, \lambda)}{\partial x_j} \geq 0 \ , \ \ if \ \ x_j = x_j^{min} \tag{2.5.6}$$

$$\lambda_i f_i(x) = 0 \tag{2.5.7}$$

$$f_i(x) \leq 0 \tag{2.5.8}$$

$$\lambda_i \geq 0 \tag{2.5.9}$$

$$x \in \mathbb{X} \tag{2.5.10}$$

Also

$$\frac{\partial \mathcal{L}(x, \lambda)}{\partial x_j} = \frac{\partial f_0(x)}{\partial x_j} + \sum_{i=1}^{\ell} \lambda_i \frac{\partial f_i(x)}{\partial x_j} \ \ \forall \ j = 1, 2, ..., n \tag{2.5.11}$$

or

$$\nabla\mathcal{L}(x,\lambda) = \nabla f_0(x) + \sum_{i=1}^{\ell} \lambda_i \nabla f_i(x) \tag{2.5.12}$$

Each point $(x^*, \lambda^*) \in \mathbb{R}^n \times \mathbb{R}^\ell$ that satisfies all conditions (2.5.4) to (2.5.10) is said to be a *KKT point* . Box constrains can also be included in $f_i(x) \leq 0; i = 1, 2, ..., \ell$ by writing $x_j - x_j^{max} \leq 0$ and $x_j^{min} - x_j \leq 0; j = 1, 2, ..., n$.

2.5.3 KKT conditions and convex programming problems

In general, KKT conditions are not sufficient for local minimality, but for convex programming problem following properties hold.

(a) Local and global minima are same

(b) KKT point is always an optimal point

(c) Karush-Kuhn-Tucker (KKT) Conditions are necessary and sufficient for local (and hence global) minimality, provided that constraints are differentiable, but the differentiability assumption can be dropped, because we can use sub-gradients in place of derivatives [6].

2.6 Dynamical system

A dynamical system consists of a set of all possible states, together with rules that define the present state in term of past states [9].

2.7 Lyapunov function

A Lyapunov function for a dynamical system is a special function having following properties

(a) It maps any state of a particular dynamical system into a real number

(b) Its values, as a dynamical system evolves in time, is non-increasing on the dynamical system trajectories

Lyapunov functions are used for studying the stability properties of dynamical systems and are specially useful for the analysis of high-dimensional non-linear dynamical systems [8].

Chapter 3

Problem Formulation

3.1 Introduction

In this chapter first of all, simultaneous and nested formulation for general structural optimization are presented. Then an evolution problem is formulated in terms of free energy and dissipation potential for non-smooth thermodynamical system by means of dynamical system approach. Optimality criteria method is used to generate a sequence of subproblems for given problem. Since for any structure, plastic evolution of material can be interpolated between solid and void, so Solid Isotropic Material with Penalization (SIMP) approach is used for topology optimization, since this approach has been proven to generalize easily to the alternative applications [7].

3.1.1 Simultaneous formulation

In general, structural optimization problem in simultaneous formulation can be expressed as

$$(\mathbb{SO})_{sf} \begin{cases} \min\limits_{\rho,u} f_0(\rho,u) \\ \text{such that} \\ K(\rho)u = F \\ f_i(\rho,u) \leq 0 \; ; i = 1,2,........,\ell \\ \rho \in \mathbb{X} = \{\rho \in \mathbb{R}^n : \rho_j^{min} \leq \rho_j \leq \rho_j^{max}; j = 1,2,....,n\} \end{cases} \tag{3.1.1}$$

Where $f_0(\rho,u)$ is an objective function, $f_i(\rho,u) \leq 0 \; ; i = 1,2,........,\ell$ are constraints functions, $F = K(\rho)u$ is quasi-static equilibrium equation, $\rho = (\rho_1,\rho_2,......\rho_n)^T$ is a vector of design variable, u is vector of nodal displacement, F is corresponding force vector and $K(\rho)$ is symmetric positive-semidefinite stiffness matrix that depends on ρ.

$$C = \{\rho^* \in \mathbb{X} : f_i(\rho^*) \leq 0 \,; i = 1, 2, ..., \ell\}$$

is called feasible set for problem $(\mathbb{SO})_{sf}$.

3.1.2 Nested formulation

By using $u = u(\rho) = K(\rho)^{-1}F$ in $(\mathbb{SO})_{sf}$ we get nested formulation of structural optimization as

$$(\mathbb{SO})_{nf} \begin{cases} \min\limits_{\rho} \hat{f}_0(\rho, u(\rho)) \\ \text{such that} \\ \hat{f}_i(\rho) \leq 0 \,; i = 1, ..., \ell \\ \rho \in \mathbb{X} \end{cases} \tag{3.1.2}$$

where $\hat{f}_i(\rho) = \hat{f}_i(\rho, u(\rho)), i = 0, ..., \ell$.

Nested formulation for variable thickness sheet problem can be written as

$$(\mathbb{P}_s^{sheet})_{nf} \begin{cases} \min\limits_{\rho} f_0(\rho, u(\rho)) \\ \text{such that} \\ \displaystyle\int_{\Omega} \rho \, d\Omega = \sum_{e=1}^{N} \int_{\Omega_e} \rho \, dA \\ \displaystyle\approx \sum_{e=1}^{N} \rho_e a_e = \rho^T a = V \\ \underline{\rho} \leq \rho_e \leq \overline{\rho}, \quad e = 1, ..., N \end{cases} \tag{3.1.3}$$

Where $a = [a_1, ..., a_N]^T$ is an area vector for discrete structural domain Ω_e and V be the total volume.

3.1.3 SIMP method

The global stiffness matrix $K(\rho)$ for this method can be expressed as

$$K(\rho) = \sum_{e=1}^{N} \rho_e^q K_e$$

where N is the number of elements in the discretized structure and K_e is element stiffness matrix for single value of design variable ρ_e. In this method we have values $\rho_e = \epsilon \approx 0$ or $\rho_e = 1$, 0 means hole and 1 means material in the structure and q is SIMP exponent or relative volume exponent. To make the global stiffness matrix $K(\rho)$ non-singular, the set of design variable C can be defined as

$$C = \{\rho : \epsilon \leq \rho_e \leq 1, e = 1, ..., N\}$$

where ϵ is smallest positive real number used to make the global stiffness matrix $K(\rho)$ non-singular.

For structural optimization problems, it requires to introduce a volume constraint, which can be defined in the following set as

$$C_V = \{\rho \in C : \sum_{e=1}^{N} a_e \rho_e \leq V\}$$

Note: Volume constraint does not require for evolutionary structural optimization.

3.2 Objective function, free energy and dissipation potential

It is clear from $(\mathbb{SO})_{sf}$ that, in general the objective function depends on both design variable ρ and displacement u, that is

$$f = f(\rho, u)$$

where

$$u = u(\rho) = K(\rho)^{-1} F$$

Objective and constraints function can also be written as

$$f_e = f_e(u, \rho) = f(u, \rho) + I_C(\rho)$$

$$C = \{\rho \mid \underline{\rho} \leq \rho \leq \overline{\rho}\}$$

where I_C is an indicator function over convex set C such that

$$I_C(\rho) = \begin{cases} 0 & \text{if} \quad \rho \in C \\ \infty & \text{if} \quad \rho \notin C \end{cases}$$

To define a Lyapunov function for dynamical system, let us construct an equation defining an evolution $\rho = \rho(t)$ such that

$$\frac{d}{dt}\widetilde{f}(\rho(t)) \leq 0 \qquad (3.2.1)$$

where

$$\widetilde{f}(\rho(t)) = f(u(\rho), \rho)$$

3.2.1 Free energy

Consider a closed thermodynamic system, *free energy* ψ for the system is defined as the difference between internal energy (U) and product of entropy (η) and temperature (θ).

$$\psi = U - \eta\theta \tag{3.2.2}$$

That is,

Free Energy (Useful Energy) = Total Energy - Unusable Energy

Differentiating (3.2.2) with respect to t (time) we get

$$\dot{\psi} = \dot{U} - \dot{\eta}\theta - \eta\dot{\theta} \tag{3.2.3}$$

By the first law of thermodynamics

$$\dot{U} = X \cdot \dot{x} + Q \tag{3.2.4}$$

The dot product $X \cdot \dot{x}$ represents the rate of work done on the system, where X is a total thermodynamical force applied to the system, conjugate with the kinematical vector x and Q is heat supply per unit time to the system.

By 2nd law of thermodynamics

$$\dot{\eta} \geq \frac{Q}{\theta} \tag{3.2.5}$$

where η is *entropy* of the system

By using (3.2.4) and (3.2.5) in (3.2.3) we have

$$\dot{\psi} \leq X \cdot \dot{x} - \eta\dot{\theta}$$

For $X = 0$ and $\dot{\theta} = 0$

$$\Rightarrow \dot{\psi} \leq 0 \tag{3.2.6}$$

From (3.2.1) and (3.2.6) we can suppose that $\widetilde{f}$ is a free energy of a thermodynamical system.

By the chain rule

$$\frac{d}{dt}\widetilde{f} = \sum_i \frac{\partial \widetilde{f}}{\partial \rho_i}\dot{\rho}_i \tag{3.2.7}$$

Since

$$-r_i \in \frac{\partial \widetilde{f}}{\partial \rho_i} + \partial I_{C_i}(\rho_i) \tag{3.2.8}$$

if and only if

$$\begin{cases} -r_i = \frac{\partial \widetilde{f}}{\partial \rho_i} + \underline{\lambda}_i + \overline{\lambda}_i \\ \underline{\lambda}_i \geq 0, \ \ \underline{\rho} \leq \rho_i, \ \ \underline{\lambda}_i(\underline{\rho} - \rho_i) = 0 \\ \overline{\lambda}_i \geq 0, \ \ \overline{\rho} \geq \rho_i, \ \ \overline{\lambda}_i(\rho_i - \overline{\rho}) = 0 \end{cases} \tag{3.2.9}$$

Where r_i is thermodynamic force.

(3.2.8) and (3.2.9) resembles KKT conditions in section 2.5.3.

Since

$$\underline{\lambda}_i \dot{\rho}_i = 0 \tag{3.2.10}$$

and

$$\overline{\lambda}_i \dot{\rho}_i = 0 \tag{3.2.11}$$

when (3.2.10) satisfied by the following

$$\underline{\lambda}_i(\underline{\rho} - \rho_i) = 0 \Rightarrow \dot{\underline{\lambda}}_i(\underline{\rho} - \rho_i) - \underline{\lambda}_i \dot{\rho}_i = 0$$

when $\underline{\rho} = \rho_i$ the first term is zero and we are done. when $\underline{\rho} < \rho_i$ this inequality holds in a neighborhood of "t" and therefore $\dot{\underline{\lambda}}_i = 0$ and again we are done. Similarly (3.2.11) can also be proved.

3.2.2 Dissipation inequality

By using (3.2.7) and (3.2.9) we get

$$-\frac{d}{dt}\widetilde{f} = \sum_i (\underline{\lambda}_i + \overline{\lambda}_i + r_i)\dot{\rho}_i \tag{3.2.12}$$

By using (3.2.10) and (3.2.11) in (3.2.12) we get

$$-\frac{d}{dt}\widetilde{f} = \sum_i r_i \dot{\rho}_i \tag{3.2.13}$$

from (3.2.6) and (3.2.13) we got

$$\sum_i r_i \dot{\rho}_i \geq 0$$

that is

$$r^T \dot{\rho} \geq 0 \tag{3.2.14}$$

which is dissipation inequality and we construct the system so that it holds for all thermodynamics process at any time t.

Moreover (3.2.14) holds if

$$r_i \in \partial D_i(\dot{\rho}_i) \tag{3.2.15}$$

where D is dissipative potential with $0 = D_i(0), 0 \in \partial D_i(0)$ and it is convex function.

From (3.2.8) and (3.2.15) we got the dynamical system

$$0 \in \frac{\partial \widetilde{f}}{\partial \rho_i} + \partial I_{C_i}(\rho_i) + \partial D_i(\dot{\rho}_i) \tag{3.2.16}$$

For numerical integration of time dependent equation (3.2.16), we need to discretized it with respect to time t into n steps of length Δt

$$0 \in T(\rho_i) = \frac{\partial \widetilde{f}}{\partial \rho_i} + \partial I_{C_i}(\rho_i) + \partial D_i(\frac{\rho_i - \rho_i^n}{\Delta t}) \tag{3.2.17}$$

Solution $\rho^{n+1}(t_{n+1})$ is obtained by inserting $\rho^n(t_n)$ in (3.2.17).

Since $T(\rho_i)$ resembles to the unconstrained convex optimization problem and $0 \in T(\rho_i)$, so by definition of section 2.5.1, we can say (3.2.17) defines the unilateral stationary point for the following minimization problem

$$\min_{\rho \in C} G(\rho) \tag{3.2.18}$$

where

$$G(\rho) = \widetilde{f}(\rho) + \Delta t \sum_i D_i(\frac{\rho_i - \rho_i^n}{\Delta t}) \tag{3.2.19}$$

So we can say that a general objective function can be written in terms of free energy and dissipation potential.

3.3 Particular example

To solve (3.2.18) with SIMP topology optimization, we consider a classical example of topology optimization. that is,

$$\widetilde{f}(\rho) = \frac{1}{2}F^T u(\rho) + \mu \sum_i a_i \rho_i; \text{ where } F = Ku \text{ and } K = \sum_i \rho_i^q K_i \tag{3.3.1}$$

Here F is a constant force, $\frac{1}{2}F^T u(\rho)$ is a strain energy similar to thermodynamic free energy of the system, the positive constant μ controls the relative importance of two terms in (3.3.1)[10]. Since it is a simple form of the compliance minimization problem, so this problem was used as fundamental test case in the initial development of the topology optimization method [7].

$$D_i(\dot{\rho}) = \frac{1}{2}c_i\dot{\rho}_i{}^2 + \begin{cases} d_+\dot{\rho}_i \text{ if } \dot{\rho} \geq 0 \\ -d_-\dot{\rho}_i \text{ if } \dot{\rho} < 0 \end{cases} \tag{3.3.2}$$

where d_+, d_- and c_i are so called *Forward Plastic Constant*, *Backward Plastic Constant* and *Viscosity Constant* respectively. The first term of D_i represents viscous behaviour while the second term shows plastic behaviour.

3.4 Optimality criteria (OC) method

In general, structural optimization problems are non-convex, also for larger problems it is impossible to write objective and constraints functions explicitly as a function of design variable, so it requires to generate sequence of convex explicit subproblems that are approximations of original problem and solve these subproblem instead [1].

There are a number of sequential convex approximation methods, but we are using here *Classical Optimality Criteria Method*, which is one of particular case, the OC method is most suitable for the given problem.

To solve (3.2.18) we linearize the first term of (3.3.1), in the intervening variable $\rho_i^{-\alpha}, \alpha > 0$. This gives

$$\widetilde{f}(\rho) \approx \text{constant} + \sum_i (b_i^k \rho_i^{-\alpha} + \mu a_i \rho_i) \tag{3.4.1}$$

Where

$$b_i^k = \frac{1}{\alpha}(q(\rho_i^k)^{q-1}\frac{1}{2}u^{k^T}K_i u^k)(\rho_i^k)^{1+\alpha} \geq 0 \tag{3.4.2}$$

The subproblem becomes

$$\min_{\underline{\rho} \leq \rho_i \leq \overline{\rho}} \varphi_i(\rho_i)$$

where

$$\varphi_i(\rho_i) = b_i^k \rho_i^{-\alpha} + \mu a_i \rho_i + \frac{1}{2}c_i\frac{(\rho_i - \rho_i^n)^2}{\Delta t} + \begin{cases} d_+(\rho_i - \rho_i^n) \text{ if } \rho_i \geq \rho_i^n \\ -d_-(\rho_i - \rho_i^n) \text{ if } \rho_i \leq \rho_i^n \end{cases} \tag{3.4.3}$$

To find stationary point of (3.4.3), differentiating it with respect to ρ_i and equating it to zero. There are five cases as follow,

3.4.1 Case 1

For $\rho_i \geq \rho_i^n$

$$\frac{\partial\varphi(\rho_i)}{\partial\rho_i} = \mu a_i + c_i(\frac{\rho_i - \rho_i^n}{\Delta t}) + d_+ - \alpha b_i^k \rho_i^{-\alpha-1} = 0 \tag{3.4.4}$$

3.4.2 Case 2

When $\rho_i \leq \rho_i^n$

$$\frac{\partial\varphi(\rho_i)}{\partial\rho_i} = \mu a_i + c_i(\frac{\rho_i - \rho_i^n}{\Delta t}) - d_- - \alpha b_i^k \rho_i^{-\alpha-1} = 0 \tag{3.4.5}$$

For special case $c_i = 0$ we get two explicit solutions for two cases

$$\rho_i = \big(\frac{\alpha b_i^k}{\mu a_i + d_+}\big)^{\frac{1}{1+\alpha}} \text{ when } \quad \rho_i \geq \rho_i^n \tag{3.4.6}$$

and

$$\rho_i = \big(\frac{\alpha b_i^k}{\mu a_i - d_-}\big)^{\frac{1}{1+\alpha}} \text{ when } \quad \rho_i \leq \rho_i^n \tag{3.4.7}$$

by using (3.4.2) in (3.4.6) and (3.4.7) we get

$$\rho_i^{k+1} = \big[\frac{1}{\mu a_i + d_+}\big\{q(\rho_i^k)^{q-1}\frac{1}{2}(u^k)^T K_i u^k\big\}\big]^{\frac{1}{1+\alpha}} \rho_i^k \;; \quad \rho_i \geq \rho_i^n$$

and

$$\rho_i^{k+1} = \big[\frac{1}{\mu a_i - d_-}\big\{q(\rho_i^k)^{q-1}\frac{1}{2}(u^k)^T K_i u^k\big\}\big]^{\frac{1}{1+\alpha}} \rho_i^k \;; \quad \rho_i \leq \rho_i^n$$

For $\rho_i^n \leq \rho_i \leq \overline{\rho}$ we call the solution of (3.4.1) as ${}^1\hat{\rho}_i{}^{k+1}$
i.e.

$${}^1\hat{\rho}_i{}^{k+1} = \big[\frac{1}{\mu a_i + d_+}\big\{q(\rho_i^k)^{q-1}\frac{1}{2}(u^k)^T K_i u^k\big\}\big]^{\frac{1}{1+\alpha}} \rho_i^k$$

And for $\underline{\rho} \leq \rho_i \leq \rho_i^n$, solution of (3.4.1) is $\hat{\rho_i^2}^{k+1}$
i.e.

$${}^2\hat{\rho}_i{}^{k+1} = \big[\frac{1}{\mu a_i - d_-}\big\{q(\rho_i^k)^{q-1}\frac{1}{2}(u^k)^T K_i u^k\big\}\big]^{\frac{1}{1+\alpha}} \rho_i^k$$

3.4.3 Case 3

When ${}^{1}\hat{\rho}_i^{k+1} \leq \rho_i^n \leq {}^{2}\hat{\rho}_i^{k+1}$ then

$$\rho_i^{k+1} = \rho_i^n$$

3.4.4 Case 4

When ${}^{1}\hat{\rho}_i^{k+1} \geq \overline{\rho}$ then

$$\rho_i^{k+1} = \overline{\rho}$$

3.4.5 Case 5

When ${}^{2}\hat{\rho}_i^{k+1} \leq \underline{\rho}$ then

$$\rho_i^{k+1} = \underline{\rho}$$

For $c_i \neq 0$ (3.4.1) can not be solved explicitly, so it is solved by using some numerical approach such as Bisection method. The updating formula for given problem now becomes

$$\rho_i^{k+1} = \begin{cases} \overline{\rho} & \text{if} \quad {}^{1}\hat{\rho}_i^{k+1} \geq \overline{\rho} \\ \underline{\rho} & \text{if} \quad {}^{2}\hat{\rho}_i^{k+1} \leq \underline{\rho} \\ {}^{1}\hat{\rho}_i^{k+1} & \text{if} \quad \rho_i^n < {}^{1}\hat{\rho}_i^{k+1} < \overline{\rho} \\ {}^{2}\hat{\rho}_i^{k+1} & \text{if} \quad \underline{\rho} < {}^{2}\hat{\rho}_i^{k+1} < \rho_i^n \\ \rho_i^n & \text{if} \quad {}^{1}\hat{\rho}_i^{k+1} \leq \rho_i^n \leq {}^{2}\hat{\rho}_i^{k+1} \end{cases} \tag{3.4.8}$$

Also we can observe that at least for $c_i = 0$ it holds that ${}^{1}\hat{\rho}_i^{k+1} < {}^{2}\hat{\rho}_i^{k+1}$, so the situations above are disjoint.

3.5 Standard optimality criteria method

Since

$$D_i \equiv 0 \iff c_i = d_\pm = 0$$

So for $D_i \equiv 0$ we come to the solution of standard OC method as given below

$$\rho_i^{k+1} = \begin{cases} \underline{\rho} & \text{if} \quad \hat{\rho}_i < \underline{\rho} \\ \hat{\rho}_i & \text{if} \quad \underline{\rho} \leq \hat{\rho}_i \leq \overline{\rho} \\ \overline{\rho} & \text{if} \quad \hat{\rho}_i > \overline{\rho} \end{cases}$$

where

$$\hat{\rho}_i = \left(\frac{\alpha b_i^k}{\mu a_i}\right)^{\frac{1}{1+\alpha}}$$

SIMP method gives the values $\overline{\rho} = 1$ (material) and $\underline{\rho} = 0$ (hole) for each cell in the discretized structure.

For standard OC method $\frac{1}{2}(u^k)^T K_i u^k$ (i.e. Specific strain energy) is constant for every finite element at convergence, so to get such a state iterative method tries to modify thickness, less stiff element expected to have high strain energy, so by making these elements thicker we get required stiffness [1].

3.6 Flow chart

Since n is number of time step but k is used for iteration number, ρ^n value of design variable at nth time step, ρ_i^k shows value of ith component of density vector $\rho^k = [\rho_1^k, ...\rho_i^k, ..., \rho_N^k]$ at kth iteration.

Initial guess for design variable $\rho^n = \rho^*$; $\epsilon \leq \rho^* \leq 1$, plastic evolution of design variable can be utilized in the way that it can be started from filled design domain and takes away parts that are not needed or conversely it can be started from an empty design domain (i.e. $\rho^n = \epsilon$) and adds or removes material under evolutionary constants (d_+ and d_-).

When dissipation potential is zero (i.e. $D \equiv 0$) given problem is solved by using standard OC Method.

The algorithm for given problem is implemented in TRINITAS by using FORTRAN programming language in Microsoft visual studio environment and it works according to the flow chart as shown in the figure 3.6.1.

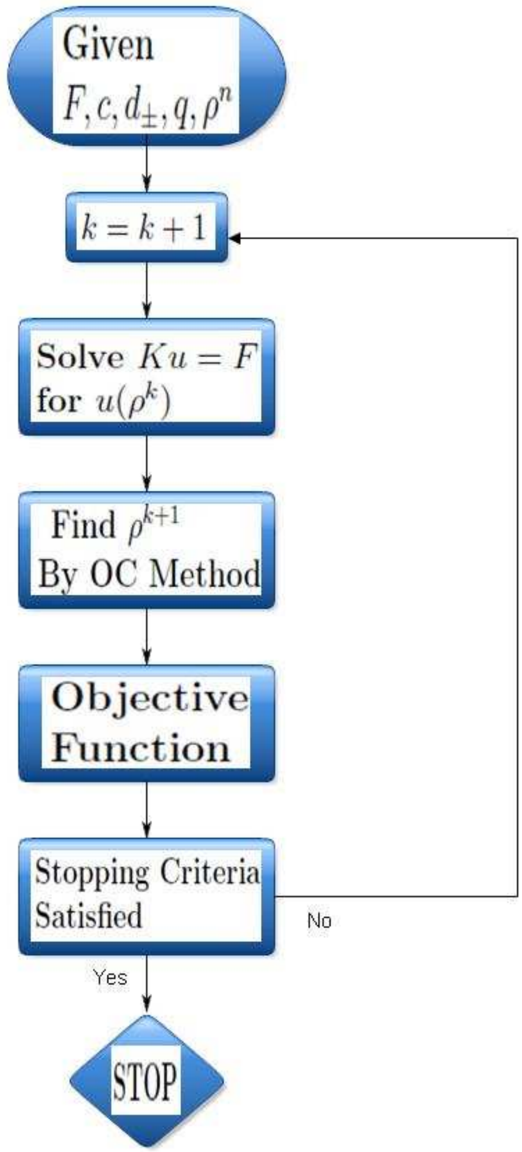


Figure 3.6.1: Flow Chart

Chapter 4

Discussion and conclusion

4.1 Introduction

In this chapter we are discussing and concluding about the results obtained by using different values of underlying parametric values and their relation with others parameters. The constraints on these parameters and conclusions about the affected results for specific parametric values are also part of this chapter. The FEM program TRINITAS is used for parametric study. TRINITAS developed by Bo Torstenfelt.

4.2 Parametric study

There are four parameters taken under discussion and these are

- Lagrange Multiplier (μ)
- Forward Plastic Constant (d_+)
- Backward Plastic Constant (d_-)
- Initial Guess (ρ^n)

These are non-negative constants. We are using just two values for relative volume exponent (i.e. $q = 1$ and $q = 3$) for our analysis. But we need to recommend the optimal values for given parameters. Also there are some constraints on these parameters, such as for any value of Lagrange Multiplier and Backward Plastic Constant, following inequality should be confirmed.

$$\mu a_i = \frac{\mu L W}{n} > d_- \tag{4.2.1}$$

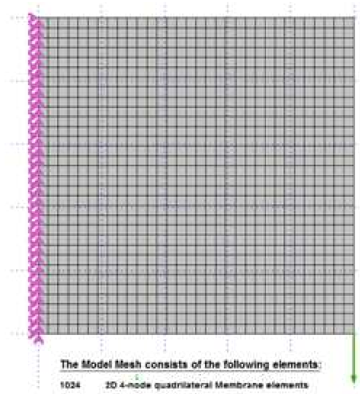


Figure 4.3.1: Design Domain

Where L, W, n and a_i are length, width, number of elements and element volume respectively for a design domain. There is no such constraint on Forward Plastic Constant. We are considering ρ^n in following way

$$\underline{\rho} \leq \rho^n \leq \overline{\rho}$$

where $\underline{\rho}$, $\overline{\rho}$ are lower and upper bounds for design variable.

4.3 Example 1

4.3.1 Geometry

Consider a square as design domain ($L = 0.5$ and $W = 0.5$) to analyze these parameters.

4.3.2 Material properties

Material symmetry: Isotropic, Youngs Modulus = $0.20E + 12$, Poisson Ratio = 0.3.

4.3.3 Boundary conditions

Fixed left end line in both x and y direction, Point Load =-10 unit applied on right end corner.

4.3.4 Mesh properties

We are using here 1024 2D 4-node quadrilateral membrane elements as shown in the figure 4.3.1.

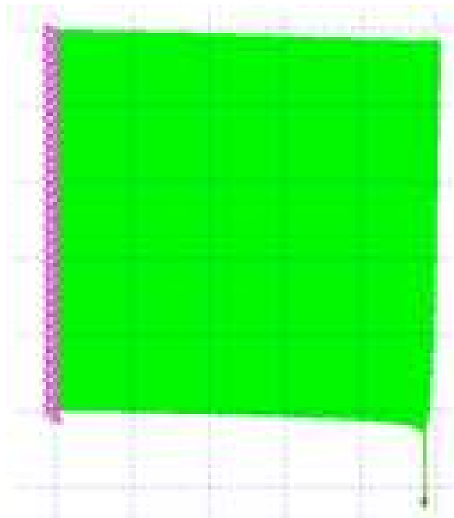


Figure 4.5.1: Special case

4.4 Behavioral study of parameters

We are analyzing the behaviour of these parameters one by one in such a way that the values of one parameter is changed but fixed the others parametric values.

4.5 Behaviour of μ

First of all we are discussing the affect of μ on design domain for both case that is when $q = 1$ and $q = 3$.

Let the following parametric values

$\underline{\rho} = \epsilon = 0.001$

$\overline{\rho} = 1$

$\rho^n = 0.333$

$d_+ = d_- = 0$

are fixed under the variation of μ. But $\underline{\rho} = 0.001$ is supposed to be fixed through out this parametric study.

4.5.1 Case 1

When $\mu < 0.1E - 11$ or $\mu = 0$ and for any value of q, d_+ and ρ^n we observe that all elements have same values as shown in the figure 4.5.1.

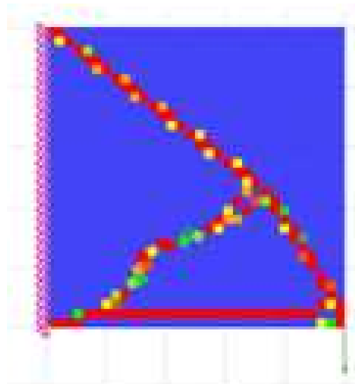


Figure 4.5.2: $q = 3$, $FR = 0$

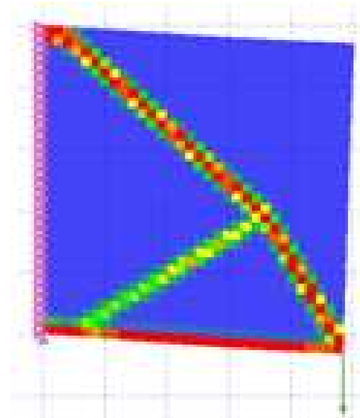


Figure 4.5.3: $q = 3$, $FR = 0.02$

4.5.2 Case study for filter radius (FR)

When $q = 3$ with Filter Radius $(FR) = 0$, the results obtained are not so good because there exist two numerical problems, one is oscillation of elements as shown in the figure 4.5.2 and other problem called as checkerboard as shown in the figures 4.5.5, so to overcome these problems at the same time, we need to choose the suitable value of FR. Thus we got oscillating and checkerboard free design by using $FR = 0.02$ as shown in the figures 4.5.2 and 4.5.7 respectively. The concept of filtering was taken from image processing techniques. It is a most efficient technique to remove checkerboards [1].

It is observed that there is no such difference in the results when $FR = 0$ or $FR = 0.01$ as shown in the figures 4.5.5 and 4.5.6. For $FR = 0.02$ the result is presented in figure 4.5.7. We have to choose a suitable value for FR so that we can control the loss of useful information. We can observe from figure 4.5.7 that by using $FR = 0.02$ we are losing less amount of information and can overcome checkerboard problem. So we are choosing $FR = 0.02$ for further analysis.

Note:- When $q = 1$, then there is no need to choose $FR > 0$, because there is no such checkerboard problem as shown in the figure 4.5.4.

4.5.3 Case 2

For $q = 1$ and $\mu = 0.1E - 07$, result is shown in figure 4.5.4, when $q = 3$, results are shown in the figures 4.5.5 to 4.5.7.

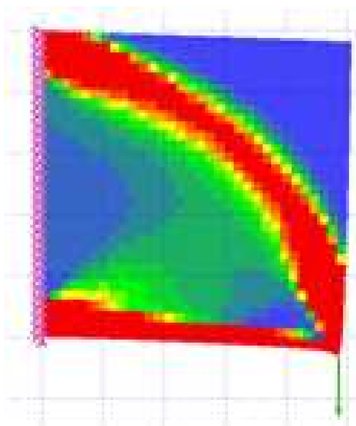


Figure 4.5.4: $q = 1$, $FR = 0$, $\mu = 0.1E - 07$

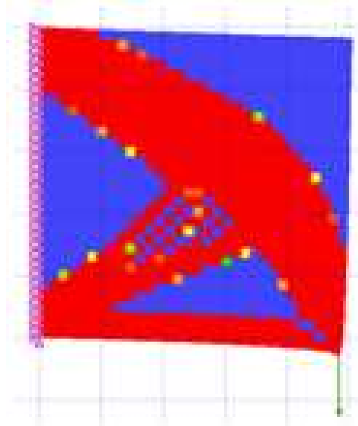


Figure 4.5.5: $q = 3$, $FR = 0$, $\mu = 0.1E - 07$

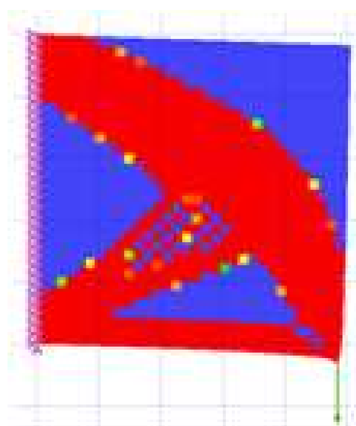


Figure 4.5.6: $q = 3$, $FR = 0.01$, $\mu = 0.1E - 07$

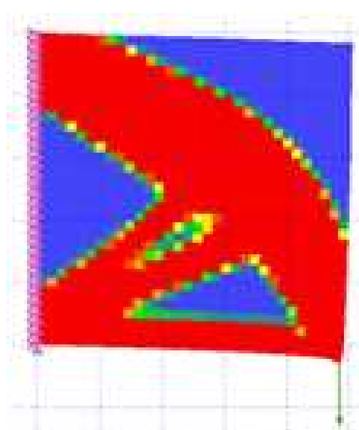


Figure 4.5.7: $q = 3$, $FR = 0.02$, $\mu = 0.1E - 07$

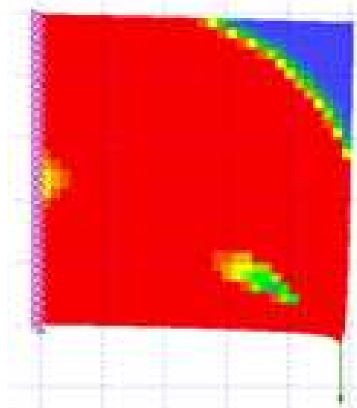


Figure 4.5.8: $q = 1$, $\mu = 0.1E - 08$

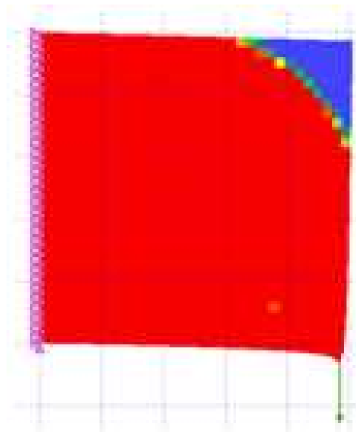


Figure 4.5.9: $q = 3$, $\mu = 0.1E - 08$

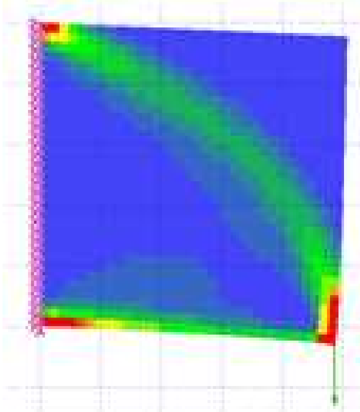


Figure 4.5.10: $q = 1$, $\mu = 0.1E - 06$

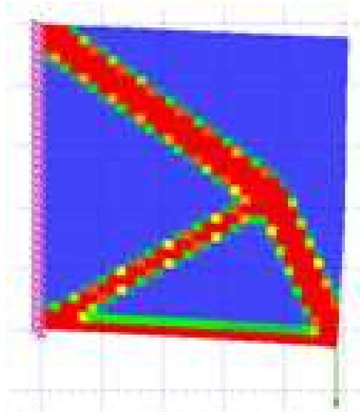


Figure 4.5.11: $q = 3$, $\mu = 0.1E - 06$

4.5.4 Case 3

When $\mu = 0.1E - 08$, the results are given in the figures 4.5.8 and 4.5.9.

4.5.5 Case 4

When $\mu = 0.1E - 06$, the results are given in the figures 4.5.10 and 4.5.11.

Remark:- It is clear from figures 4.5.9, 4.5.7 and 4.5.11, that by increasing value of μ, the thickness of structural topology decreasing.

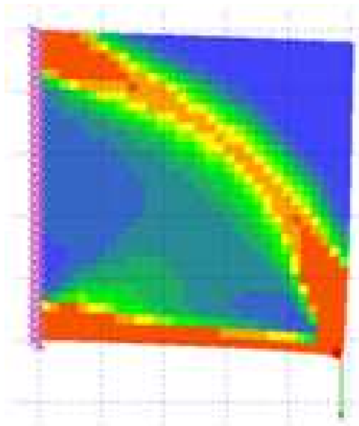


Figure 4.6.1: $q = 1$, $d_+ = 0.1E - 08$

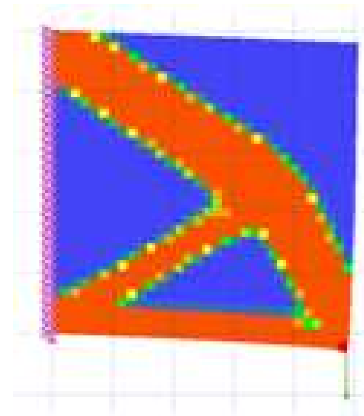


Figure 4.6.2: $q = 3$, $d_+ = 0.1E - 08$

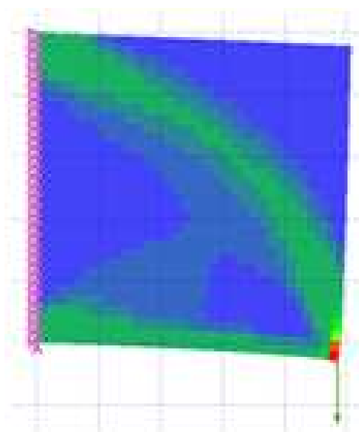


Figure 4.6.3: $q = 1$, $d_+ = 0.1E - 09$

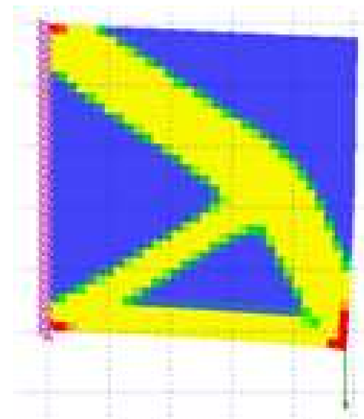


Figure 4.6.4: $q = 3$, $d_+ = 0.1E - 09$

4.6 Behaviour of d_+

Let $\mu = 0.1E-06$ and $d_- = 0$ are fixed, but d_+ varies as shown in the figures 4.6.1 to 4.6.10

Remark:- The results obtained by choosing $d_+ \geq 0.1E-09$ having same structural topology and thickness, but having different color as shown in the 4.6.1 to 4.6.6. Also the results obtained by choosing $0 \leq d_+ \leq 0.1E - 10$ topology have thinner element (as shown in the figures 4.6.7 to 4.6.10) as compared to the results obtained by choosing $d_+ \geq 0.1E - 09$.

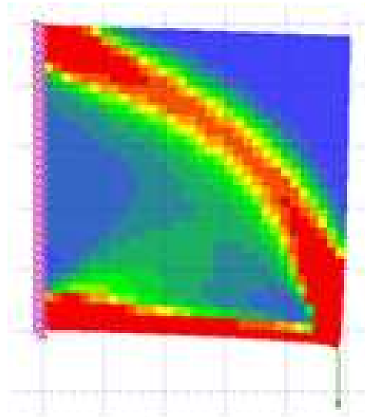


Figure 4.6.5: $q = 1$, $d_+ \geq 0.1E-07$

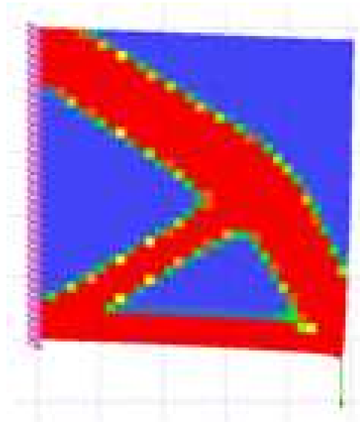


Figure 4.6.6: $q = 3$, $d_+ \geq 0.1E-07$

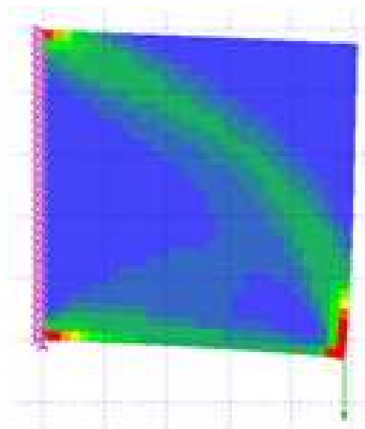


Figure 4.6.7: $q = 1$, $d_+ = 0.1E-10$

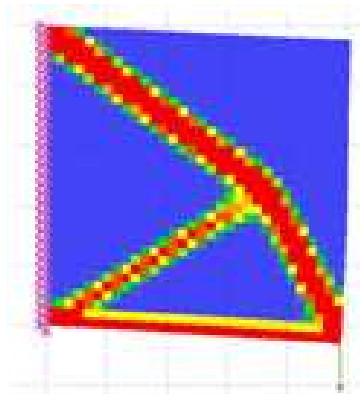


Figure 4.6.8: $q = 3$, $d_+ = 0.1E-10$

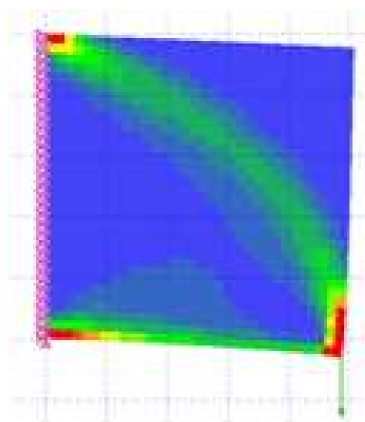


Figure 4.6.9: $q = 1$, $0 \leq d_+ \leq 0.1E-11$

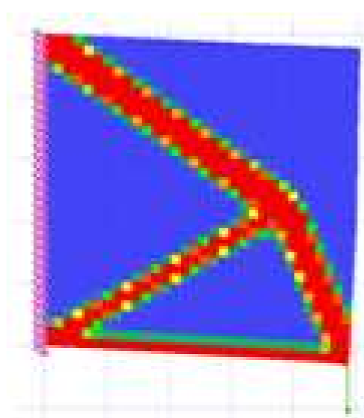


Figure 4.6.10: $q = 3$, $0 \leq d_+ \leq 0.1E-11$

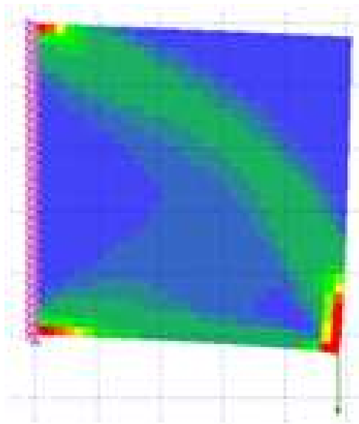


Figure 4.7.1: $q = 1$, $d_- = 0.1E - 10$

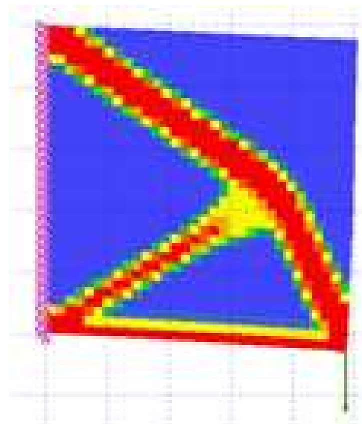


Figure 4.7.2: $q = 3$, $d_- = 0.1E - 10$

4.7 Behaviour of d_-

Its very sensitive to choose the value of d_-, because here it is required to satisfy the inequality (4.2.1), the inequality not only depends on the dimension of design domain, but also depends on the number of elements that divides the design domain. So it is required to follow the inequality during selection of any value of d_-. To present some results the required data is taken from section 4.3.4 and used in inequality (4.2.1).

4.7.1 Case 1

First of all $\mu = 0.1E - 06$ and $d_+ = 0$ are taken fixed but d_- varies as follow from the figures 4.7.1 to 4.7.4.

4.7.2 Case 2

Since it requires to satisfy the inequality 4.2.1, also we supposed a_i and n are constant for this problem, so to use other values for d_-, we need to control value of μ in such that the inequality 4.2.1 satisfied. The results for this case are given in figures 4.7.5 to 4.7.18.

4.7.3 Case 3

When $\mu = 0.1E - 06$, $d_- = 0.1E - 10$ and for any value of $d_+ \geq 0.1E - 08$, the results obtained are shown in figures 4.7.19 and 4.7.20.

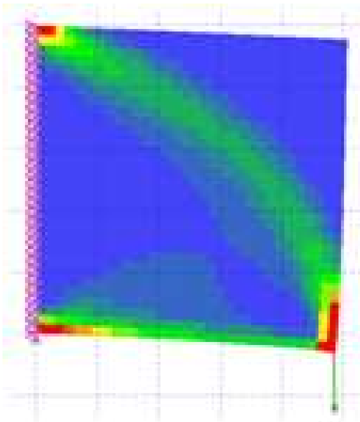


Figure 4.7.3: $q = 1$, $d_- = 0.1E - 11$

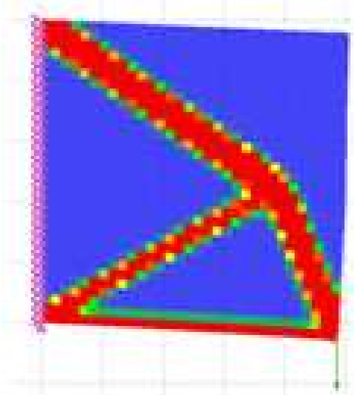


Figure 4.7.4: $q = 3$, $d_- = 0.1E - 11$

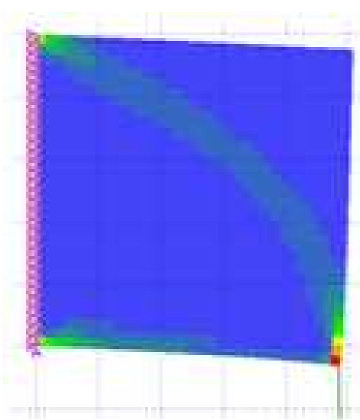


Figure 4.7.5: $q = 1$, $d_- = 0.1E - 06$, $\mu = 0.01$

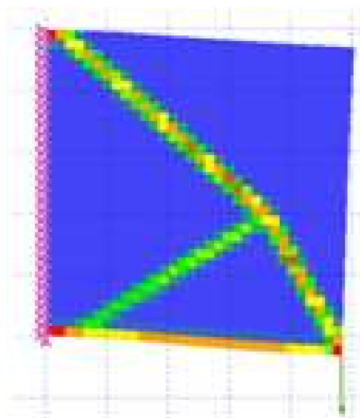


Figure 4.7.6: $q = 3$, $d_- = 0.1E - 06$, $\mu = 0.01$

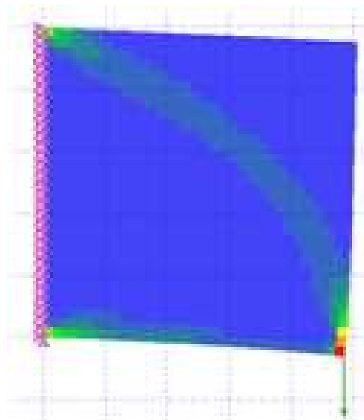


Figure 4.7.7: $q = 1$, $d_- = 0.1E-05$, $\mu = 0.01$

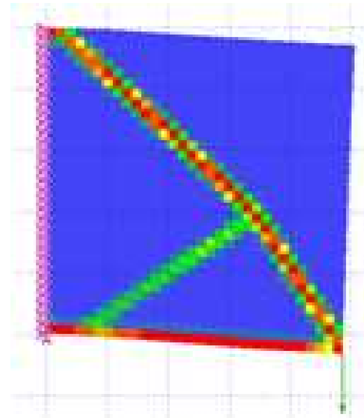


Figure 4.7.8: $q = 3$, $d_- = 0.1E-05$, $\mu = 0.01$

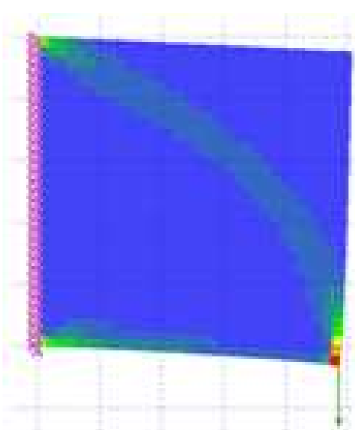


Figure 4.7.9: $q = 1$, $d_- = 0.1E-06$, $\mu = 0.1$

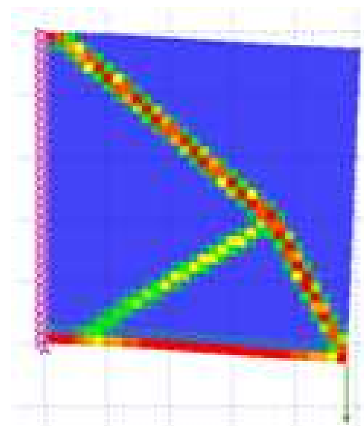


Figure 4.7.10: $q = 3$, $d_- = 0.1E-06$, $\mu = 0.1$

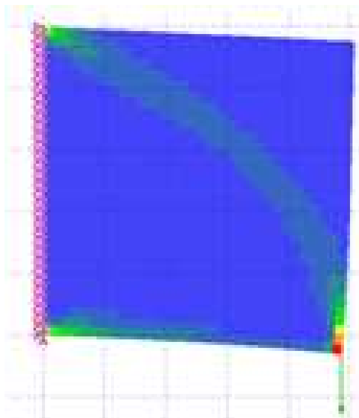


Figure 4.7.11: $q = 1$, $d_{-} = 0.1E - 05$, $\mu = 0.1$

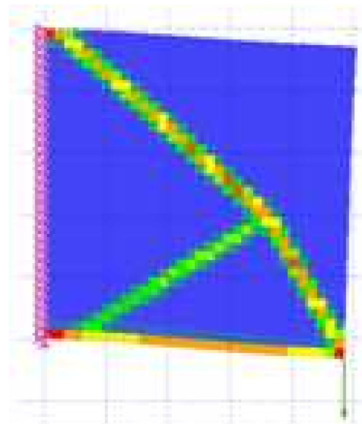


Figure 4.7.12: $q = 3$, $d_{-} = 0.1E - 05$, $\mu = 0.1$

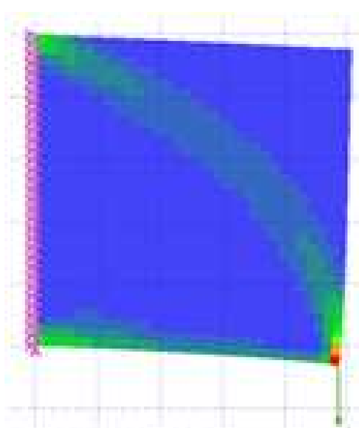


Figure 4.7.13: $q = 1$, $d_{-} = 0.1E - 09$, $\mu = 0.1E - 05$

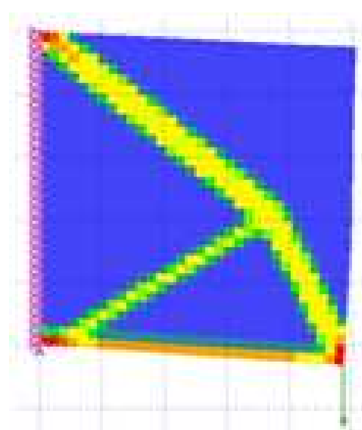


Figure 4.7.14: $q = 3$, $d_{-} = 0.1E - 09$, $\mu = 0.1E - 05$

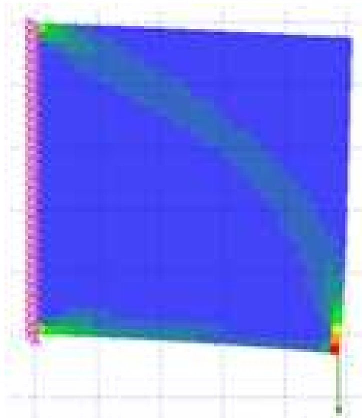


Figure 4.7.15: $q = 1$, $d_- = 0.1E - 08$, $\mu = 0.1E - 04$

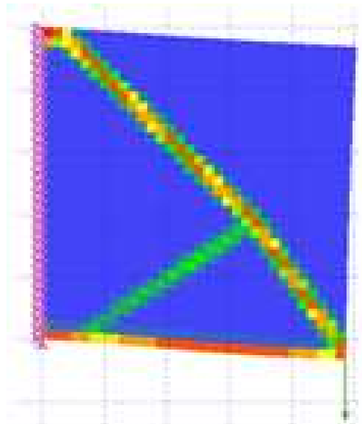


Figure 4.7.16: $q = 3$, $d_- = 0.1E - 08$, $\mu = 0.1E - 04$

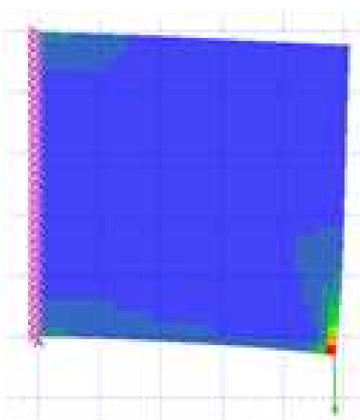


Figure 4.7.17: $q = 1$, $d_- = 0.1E + 04$, $\mu = 0.1E + 08$

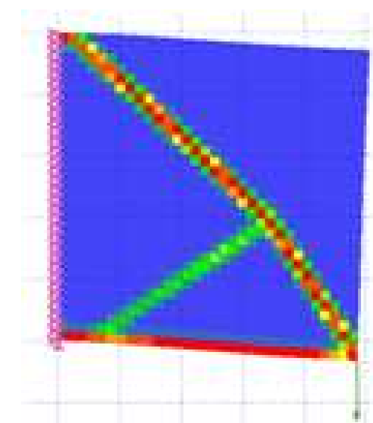


Figure 4.7.18: $q = 3$, $d_- = 0.1E + 04$, $\mu = 0.1E + 08$

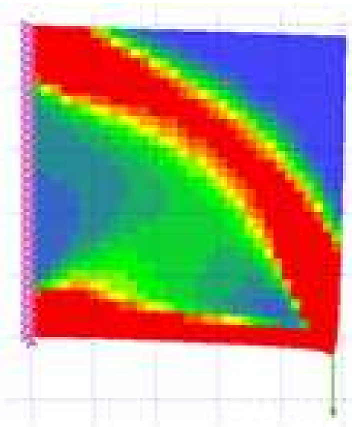


Figure 4.7.19: $q = 1$, $d_- = 0.1E - 10$, $d_+ \geq 0.1E - 08$

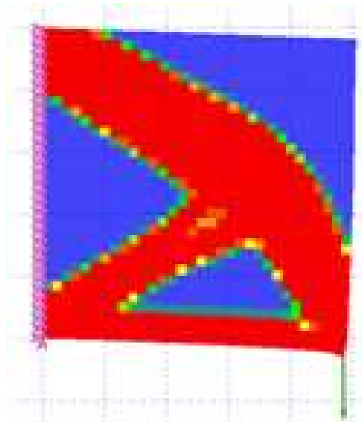


Figure 4.7.20: $q = 3$, $d_- = 0.1E - 10$, $d_+ \geq 0.1E - 08$

4.7.4 Case 4

When $\mu = 0.1E-06$, $d_- = 0.1E-10$ and for any value of $0 < d_+ \leq 0.1E-09$, the results obtained are shown in figures 4.7.21 to 4.7.26.

Remark:- It is observed from case study of d_- that μ value is always greater than d_- value and the exponential difference between d_- and μ values is at least 4 to satisfy the inequality 4.2.1. For example from figure 4.7.18, $d_- = 0.1E + 04$ and $\mu = 0.1E + 08$.

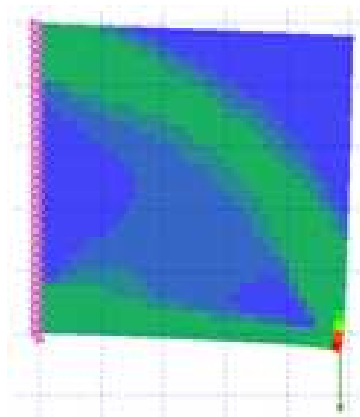


Figure 4.7.21: $q = 1$, $d_- = 0.1E - 10$, $d_+ = 0.1E - 09$

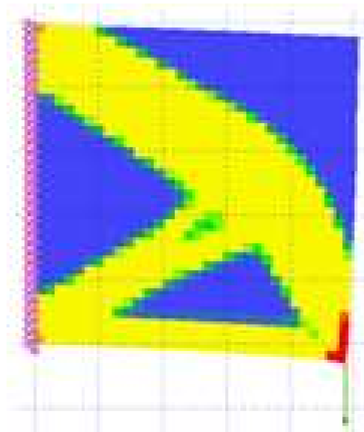


Figure 4.7.22: $q = 3$, $d_- = 0.1E - 10$, $d_+ = 0.1E - 09$

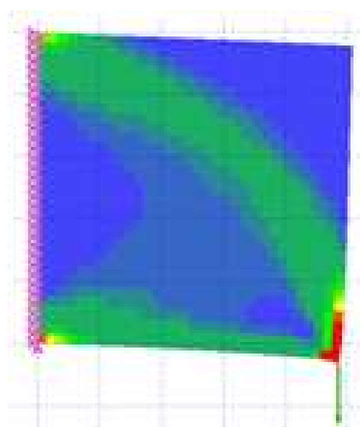


Figure 4.7.23: $q = 1$, $d_- = 0.1E - 10$, $d_+ = 0.1E - 10$

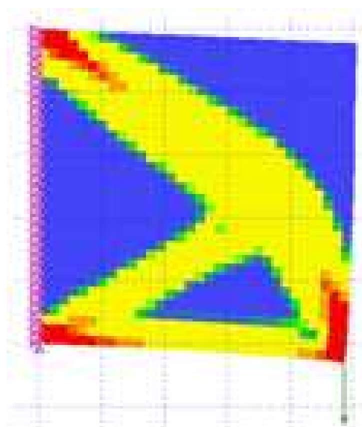


Figure 4.7.24: $q = 3$, $d_- = 0.1E - 10$, $d_+ = 0.1E - 10$

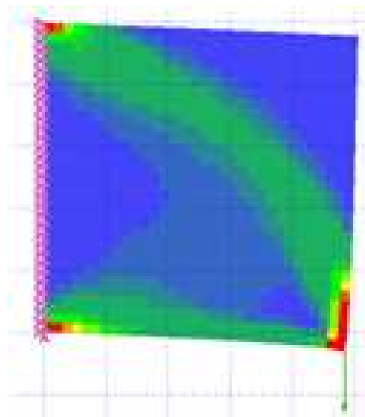


Figure 4.7.25: $q = 1$, $d_{-} = 0.1E-10$, $d_{+} \leq 0.1E-11$

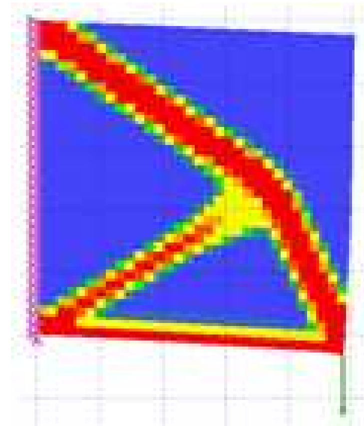


Figure 4.7.26: $q = 3$, $d_{-} = 0.1E-10$, $d_{+} \leq 0.1E-11$

4.8 Behaviour of ρ^n

Let $\mu = 0.1E-06$, $FR = 0.02$ and $\underline{\rho} = 0.001$ are taken to be fixed under the study of ρ^n.

4.8.1 Case 1

When $\overline{\rho} = \rho^n = 1$, the results are shown in the figures 4.8.1 to 4.8.4

4.8.2 Case 2

When $\underline{\rho} = \rho^n = 0.001 = \epsilon$, the results are shown in the figures 4.8.5 to 4.8.10.

4.8.3 Case 3

When $\rho^n = 0.7$, the results are shown in the figures 4.8.11 to 4.8.18.

4.8.4 Case 4

When $\rho^n = 0.5$, the results are shown in the figures 4.8.17 to 4.8.22.

Remark:- From figures 4.8.5 to 4.8.10, it is clear that for $\rho^n = \underline{\rho}$ the results obtained are not good in engineering point of view. So it is required to take care during selection of initial guess.

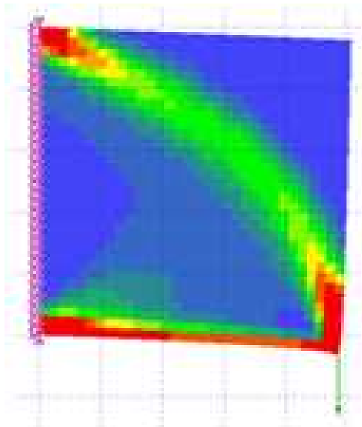


Figure 4.8.1: $q = 1$, $d_- = 0$, $d_+ \geq 0$, $\rho^n = 1$

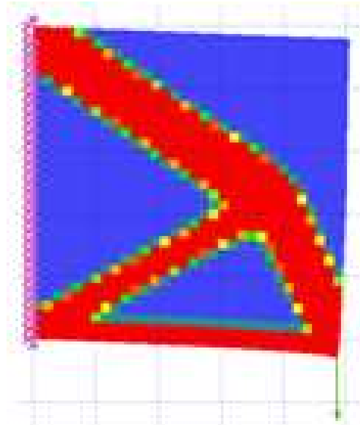


Figure 4.8.2: $q = 3$, $d_- = 0$, $d_+ \geq 0$, $\rho^n = 1$

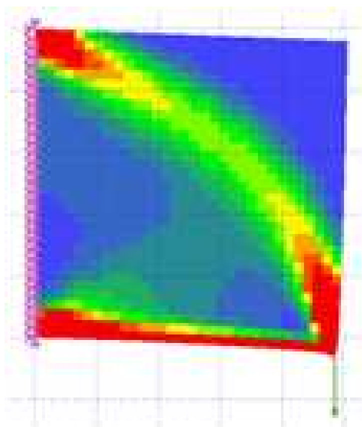


Figure 4.8.3: $q = 1$, $d_- = 0.1E - 10$, $d_+ \geq 0$, $\rho^n = 1$

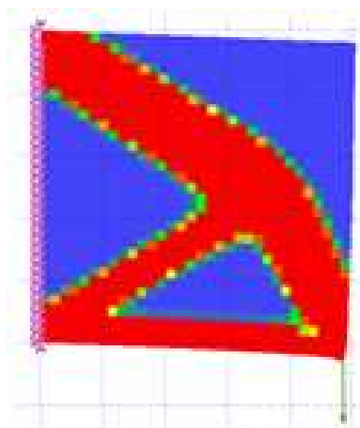


Figure 4.8.4: $q = 3$, $d_- = 0.1E - 10$, $d_+ \geq 0$, $\rho^n = 1$

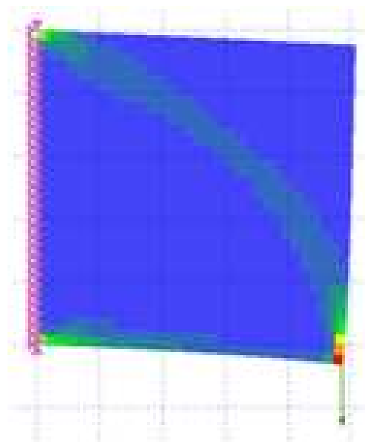


Figure 4.8.5: $q = 1$, $0 \leq d_- < 0.1E-10$, $d_+ \geq 0$, $\rho^n = 0.001$

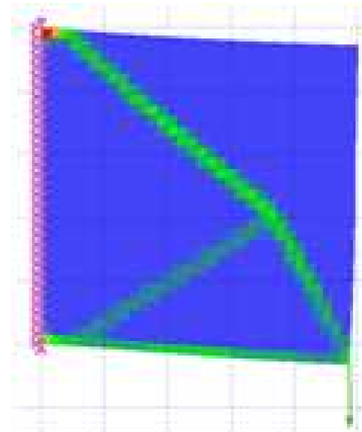


Figure 4.8.6: $q = 3$, $0 \leq d_- < 0.1E-10$, $d_+ \geq 0$, $\rho^n = 0.001$

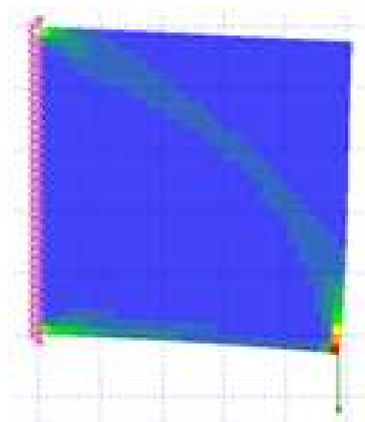


Figure 4.8.7: $q = 1$, $d_- = 0.1E-10$, $d_+ = 0.1E-09$, $\rho^n = 0.001$

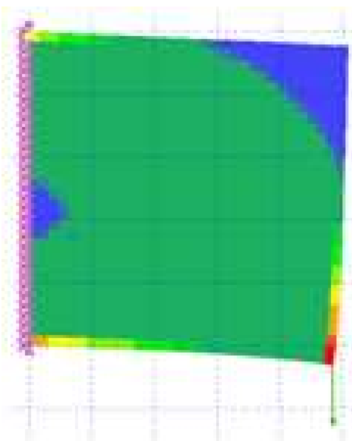


Figure 4.8.8: $q = 3$, $d_- = 0.1E-10$, $d_+ = 0.1E-09$, $\rho^n = 0.001$

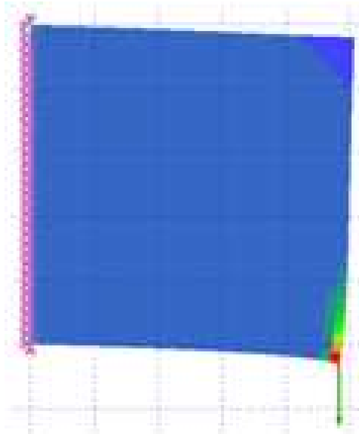


Figure 4.8.9: $q = 1$, $d_{-} = 0.1E-10$, $d_{+} = 0.1E-07$, $\rho^n = 0.001$

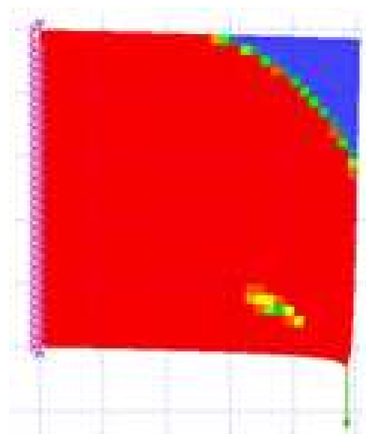


Figure 4.8.10: $q = 3$, $d_{-} = 0.1E-10$, $d_{+} = 0.1E-07$, $\rho^n = 0.001$

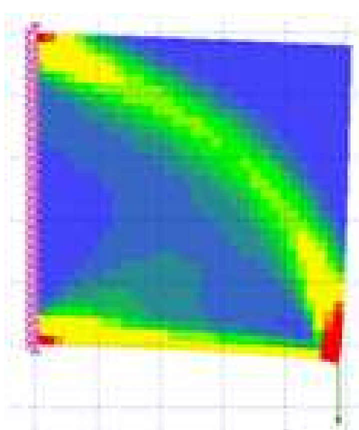


Figure 4.8.11: $q = 1$, $d_{-} = 0.1E-10$, $d_{+} \geq 0.1E-10$, $\rho^n = 0.7$

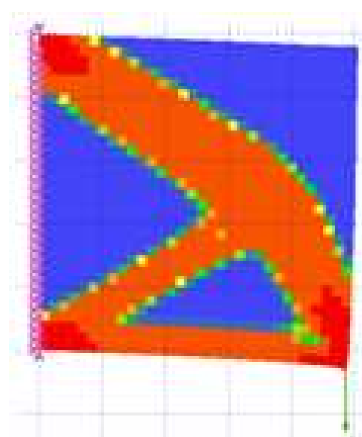


Figure 4.8.12: $q = 3$, $d_{-} = 0.1E-10$, $d_{+} \geq 0.1E-10$, $\rho^n = 0.7$

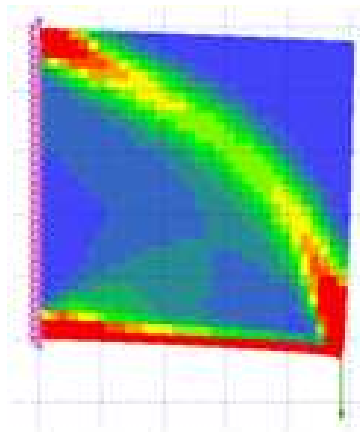


Figure 4.8.13: $q = 1$, $d_- = 0$, $d_+ > 0.1E-08$, $\rho^n = 0.7$

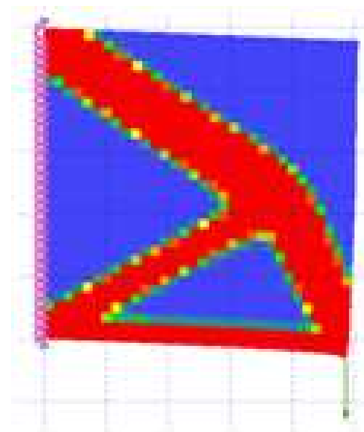


Figure 4.8.14: $q = 3$, $d_- = 0$, $d_+ > 0.1E-08$, $\rho^n = 0.7$

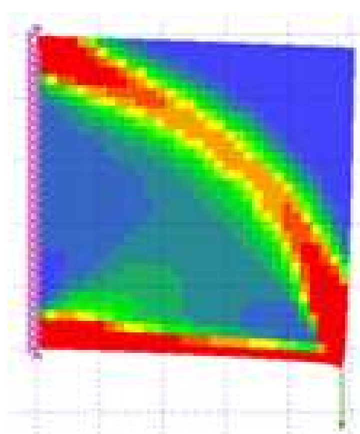


Figure 4.8.15: $q = 1$, $d_- = 0.1E-10$, $d_+ > 0.1E-08$, $\rho^n = 0.7$

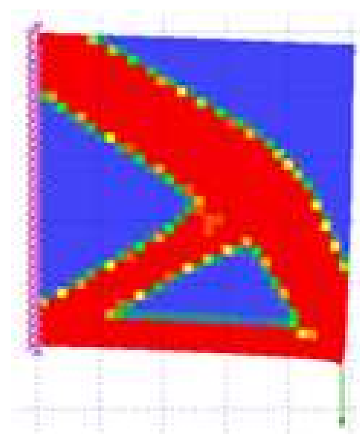


Figure 4.8.16: $q = 3$, $d_- = 0.1E-10$, $d_+ > 0.1E-08$, $\rho^n = 0.7$

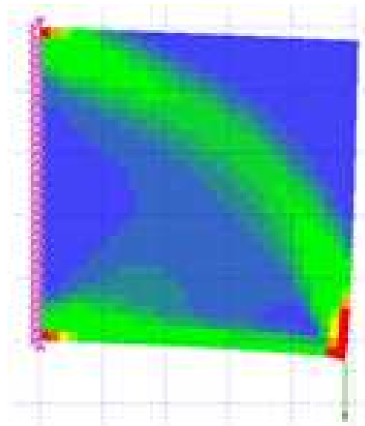


Figure 4.8.17: $q = 1$, $d_- = 0.1E - 10$, $d_+ = 0.1E - 10$, $\rho^n = 0.5$

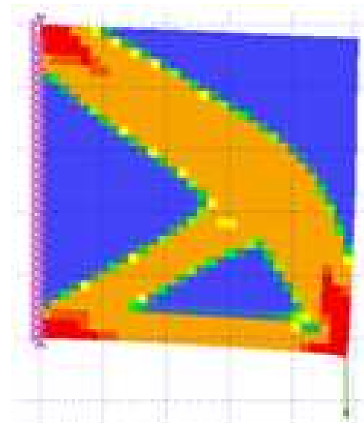


Figure 4.8.18: $q = 3$, $d_- = 0.1E - 10$, $d_+ = 0.1E - 10$, $\rho^n = 0.5$

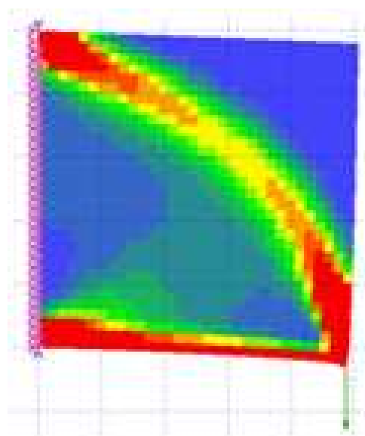


Figure 4.8.19: $q = 1$, $d_- = 0$, $d_+ > 0.1E - 08$, $\rho^n = 0.5$

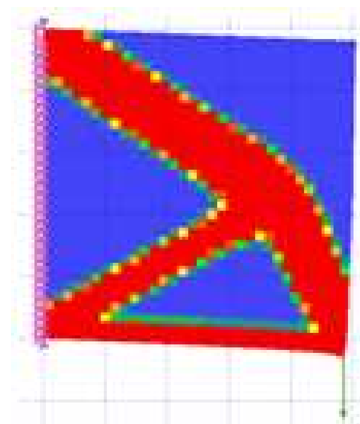


Figure 4.8.20: $q = 3$, $d_- = 0$, $d_+ > 0.1E - 08$, $\rho^n = 0.5$

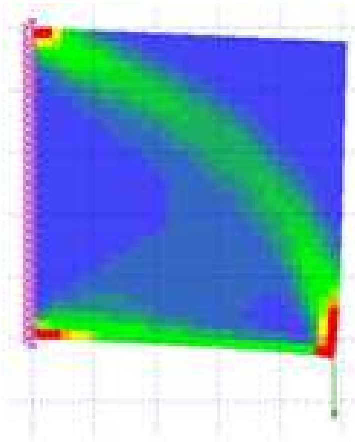


Figure 4.8.21: $q = 1$, $d_- = 0.1E-10$, $d_+ > 0.1E-08$, $\rho^n = 0.5$

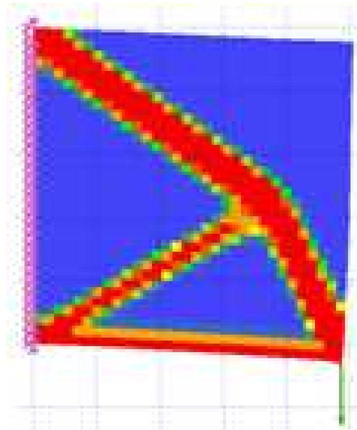


Figure 4.8.22: $q = 3$, $d_- = 0.1E-10$, $d_+ > 0.1E-08$, $\rho^n = 0.5$

4.9 Example 2

4.9.1 Geometry

We are using now a rectangular design domain ($L = 0.6$ and $W = 0.4$) to analyze these parameters.

4.9.2 Material properties

Material symmetry: Isotropic, Youngs Modulus = $0.20E+12$, Poisson Ratio = 0.3

4.9.3 Boundary conditions

Fixed left end line in both x and y direction, Point Load =-10 unit applied on mid point of right side.

4.9.4 Mesh properties

We are using here 2048 2D 4-node quadrilateral membrane elements as shown in the figure 4.9.1.

4.10 Case study

Let $FR = 0.02$ and $\underline{\rho} = 0.001$ are taken to be fixed for all analysis.

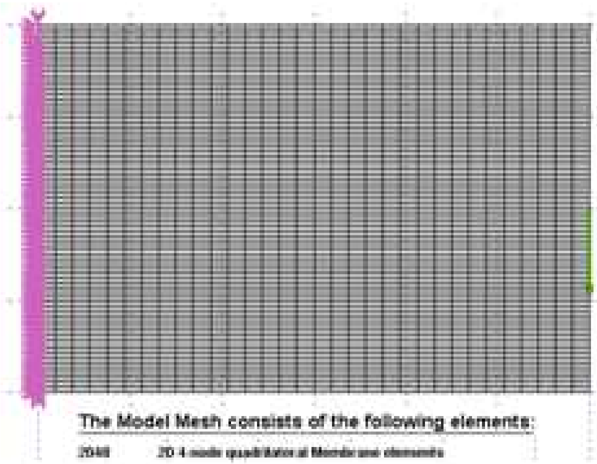


Figure 4.9.1: Design Domain

4.10.1 Case 1

Let $\rho^n = 0.3333$, the results are shown in the figures 4.10.1 to 4.10.14.

Remark:- It is clear from figure 4.10.2 and 4.10.4 that by introducing $d_+ > 0$, we got thicker topology elements as compared to $d_+ = 0$. When $q = 3$ the results obtained for any value of $d_+ \geq 0.1E - 08$ and $d_- = 0$ are same, so just one result is presented in the figure 4.10.8.

Remark:- When $\mu > 0.1E - 06$, $\underline{\rho} < \rho^n < \overline{\rho}$, $d_+ \geq 0$ and $d_- > 0$ with inequality (4.2.1), results obtained are similar and presented in the figures 4.10.9 and 4.10.10.

Remark:- Since $\mu = 0.1E-06$ so it is required to choose $d_- = 0.1E-10$. When $\mu < 0.1E - 06$ then to satisfies inequality (4.2.1), d_- tends to zero because the exponential difference between μ and d_- value is at least 04. So we are using $\mu = 0.1E - 06$ for further analysis.

4.11 Behaviour of ρ^n

Let $\mu = 0.1E - 06$, $FR = 0.02$ and $\underline{\rho} = 0.001$ are taken to be fixed.

4.11.1 Case 1

When $\overline{\rho} = \rho^n = 1$, the results are shown in the figures 4.11.1 to 4.11.4.

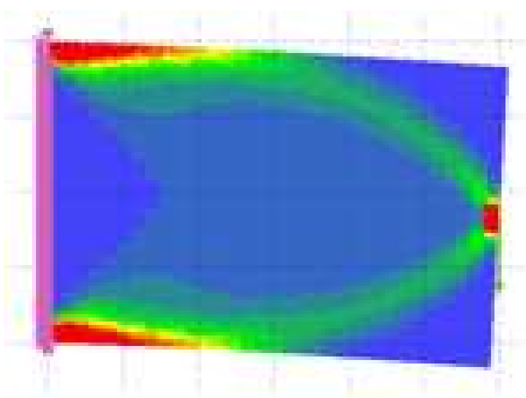


Figure 4.10.1: $q = 1$, $d_- = 0$, $d_+ = 0$, $\mu = 0.1E - 06$

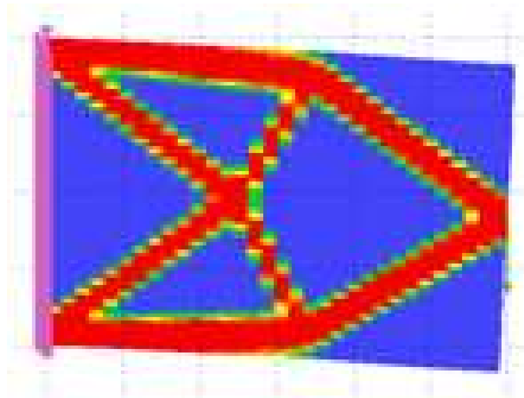


Figure 4.10.2: $q = 3$, $d_- = 0$, $d_+ = 0$, $\mu = 0.1E - 06$

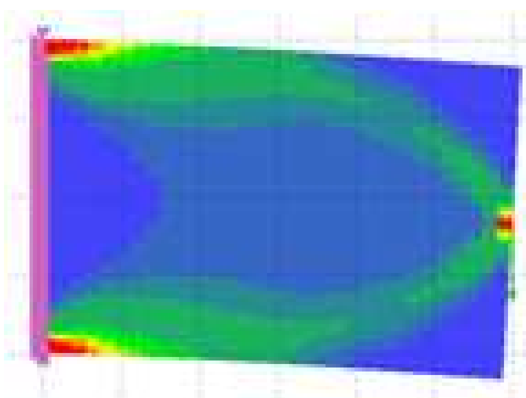


Figure 4.10.3: $q = 1$, $d_- = 0$, $d_+ = 0.1E - 10$, $\mu = 0.1E - 06$

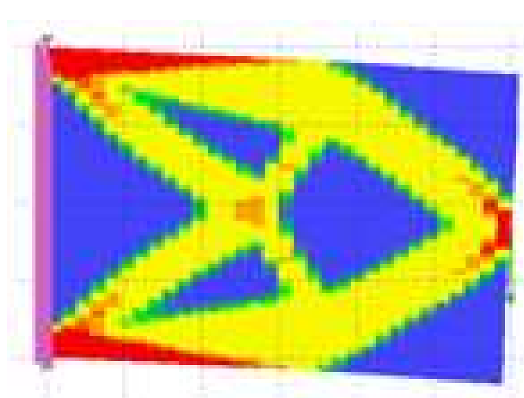


Figure 4.10.4: $q = 3$, $d_- = 0$, $d_+ = 0.1E - 10$, $\mu = 0.1E - 06$

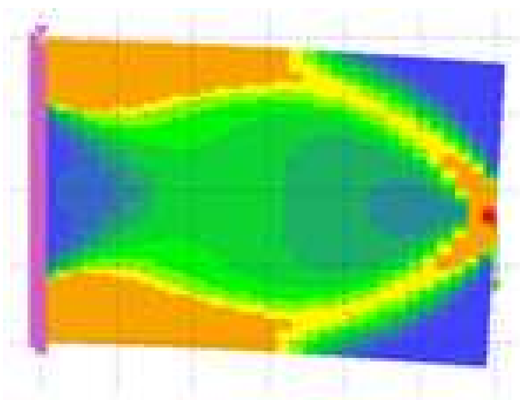


Figure 4.10.5: $q = 1$, $d_- = 0$, $d_+ = 0.1E-09$, $\mu = 0.1E-06$

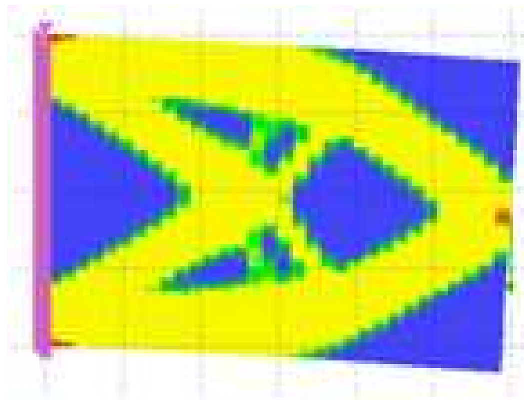


Figure 4.10.6: $q = 3$, $d_- = 0$, $d_+ = 0.1E-09$, $\mu = 0.1E-06$

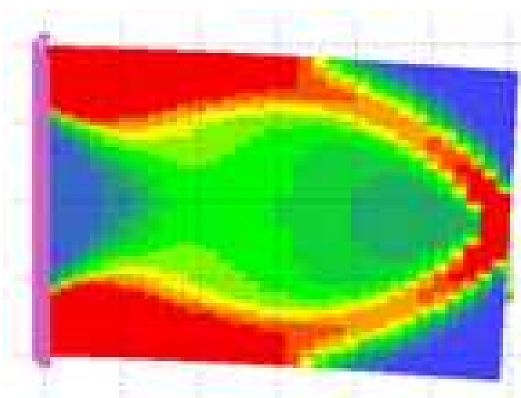


Figure 4.10.7: $q = 1$, $d_- = 0$, $d_+ \geq 0.1E-08$, $\mu = 0.1E-06$

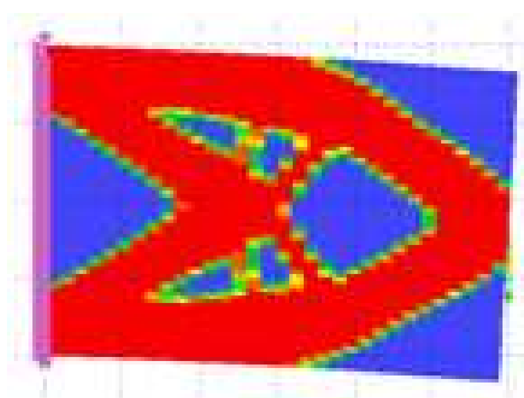
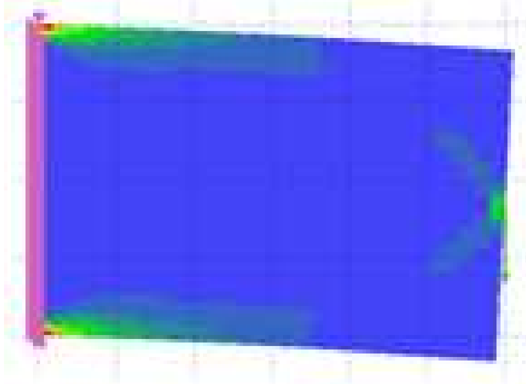


Figure 4.10.8: $q = 3$, $d_- = 0$, $d_+ \geq 0.1E-08$, $\mu = 0.1E-06$



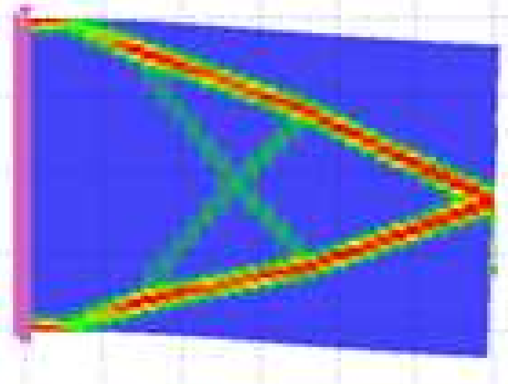
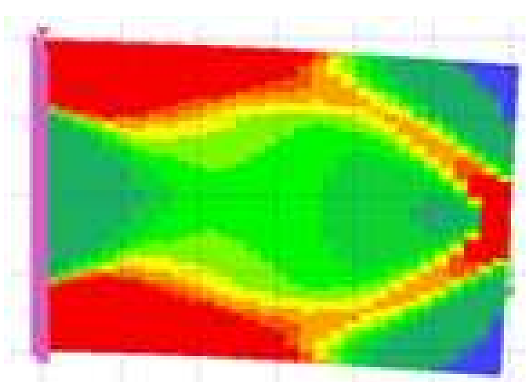


Figure 4.10.9: $q = 1$, $d_- > 0$, $d_+ \geq 0$, $\mu > 0.1E-06$, $\underline{\rho} < \rho^n < \overline{\rho}$

Figure 4.10.10: $q = 3$, $d_- > 0$, $d_+ \geq 0$, $\mu > 0.1E-06$, $\underline{\rho} < \rho^n < \overline{\rho}$



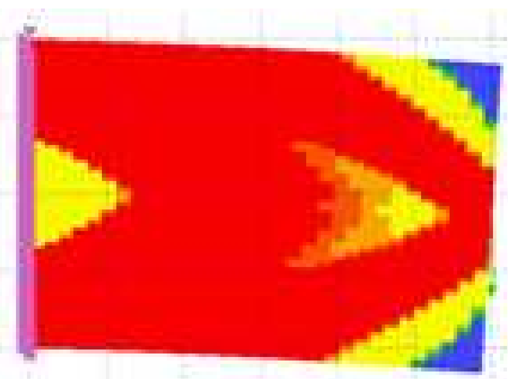


Figure 4.10.11: $q = 1$, $d_- = 0.1E-11$, $d_+ = 0$, $\mu = 0.1E-07$

Figure 4.10.12: $q = 3$, $d_- = 0.1E-11$, $d_+ = 0$, $\mu = 0.1E-07$

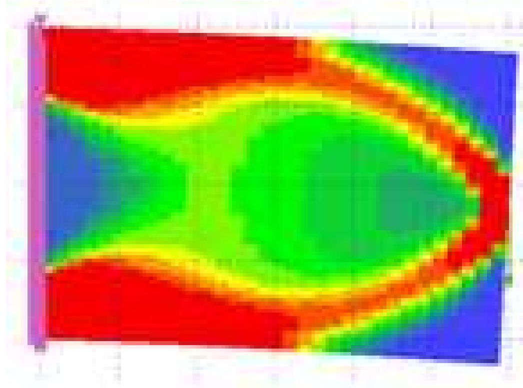


Figure 4.10.13: $q = 1$, $d_- = 0$, $d_+ = 0$, $\mu = 0.1E - 07$

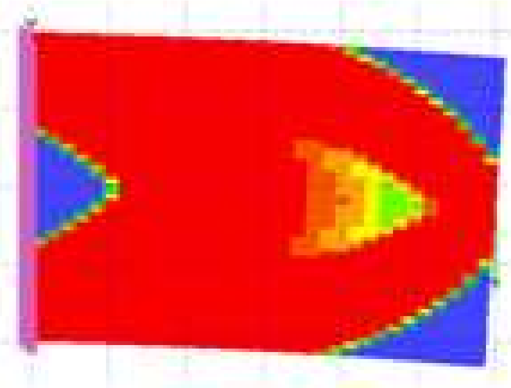


Figure 4.10.14: $q = 3$, $d_- = 0$, $d_+ = 0$, $\mu = 0.1E - 07$

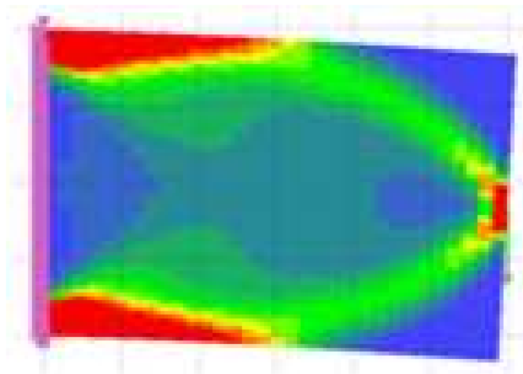


Figure 4.11.1: $q = 1$, $d_- = 0$, $d_+ \geq 0$, $\rho^n = 1$

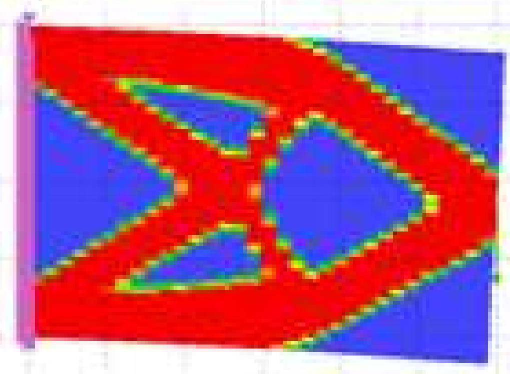


Figure 4.11.2: $q = 3$, $d_- = 0$, $d_+ \geq 0$, $\rho^n = 1$

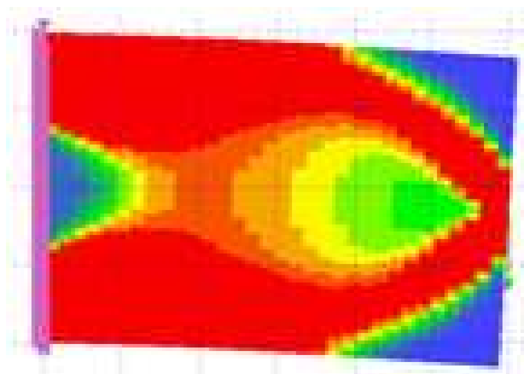


Figure 4.11.3: $q = 1$, $d_- = 0.1E - 10$, $d_+ \geq 0$, $\rho^n = 1$

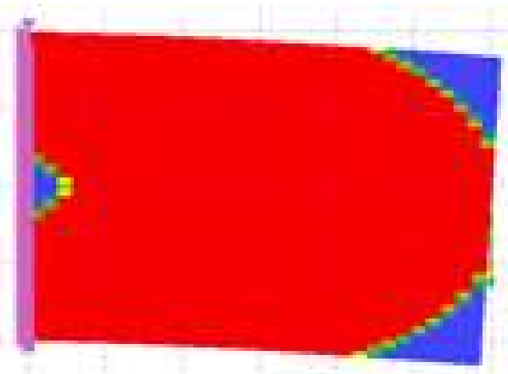


Figure 4.11.4: $q = 3$, $d_- = 0.1E - 10$, $d_+ \geq 0$, $\rho^n = 1$

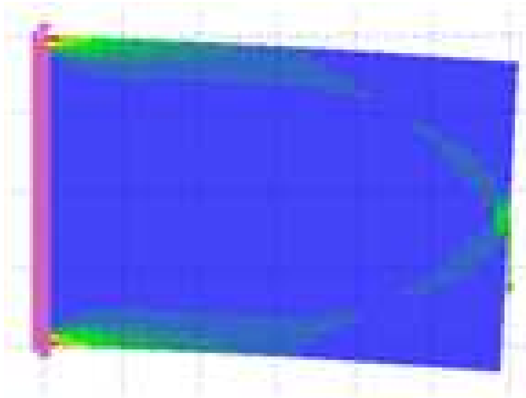


Figure 4.11.5: $q = 1$, $d_- = 0$, $d_+ = 0$, $\rho^n = 0.001$

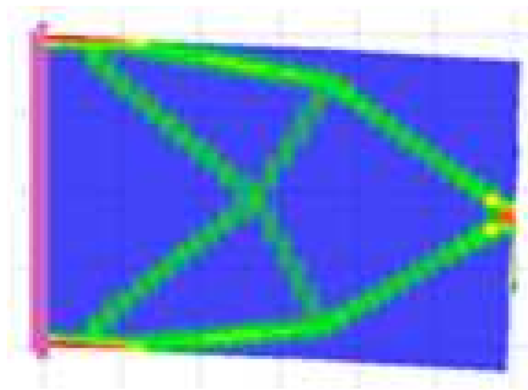


Figure 4.11.6: $q = 3$, $d_- = 0$, $d_+ = 0$, $\rho^n = 0.001$

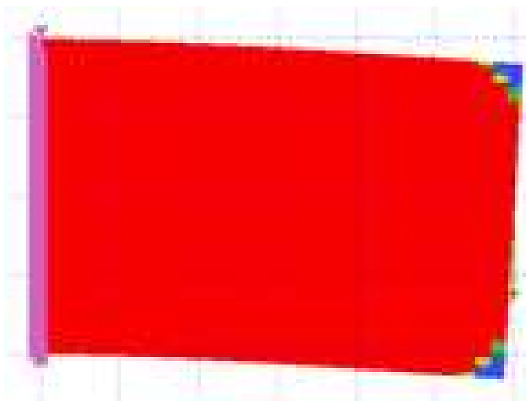


Figure 4.11.7: $q = 1$, $0 \leq d_- \leq 0.1E-10$, $d_+ \geq 0.1E-06$, $\rho^n = 0.001$

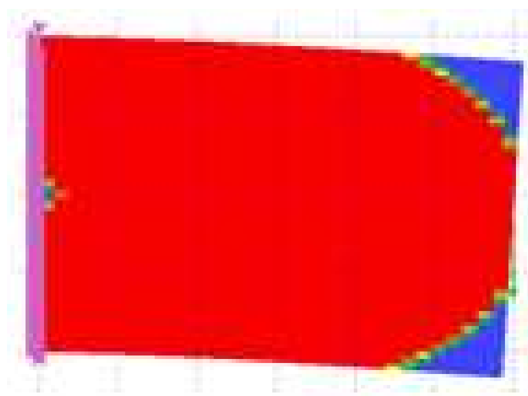


Figure 4.11.8: $q = 3$, $0 \leq d_- \leq 0.1E-10$, $d_+ \geq 0.1E-06$, $\rho^n = 0.001$

Remark:- When $\rho^n = \overline{\rho}$ then there is no difference between the results obtained by using $d_+ = 0$ and $d_+ > 0$, thats why we are presented just one result for each case as shown in the figures 4.11.1 to 4.11.4.

4.11.2 Case 2

When $\underline{\rho} = \rho^n = 0.001$, the results are shown in the figures 4.11.5 to 4.11.8.

4.11.3 Case 3

When $\rho^n = 0.7$, the results are shown in the figures 4.11.9 to 4.11.14.

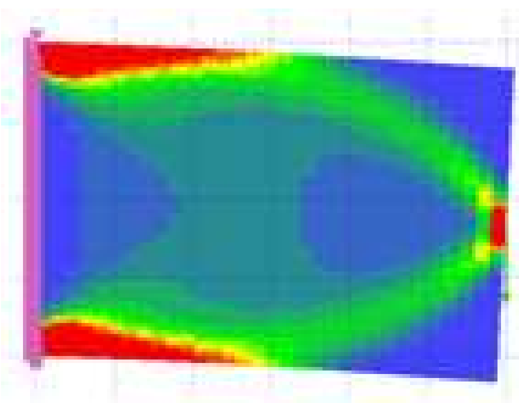


Figure 4.11.9: $q = 1$, $d_- = 0$, $d_+ = 0$, $\rho^n = 0.7$

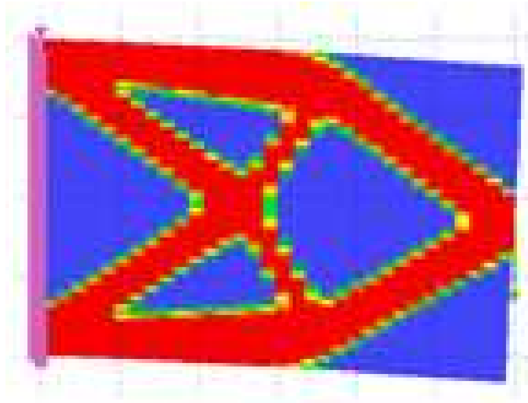


Figure 4.11.10: $q = 3$, $d_- = 0$, $d_+ = 0$, $\rho^n = 0.7$

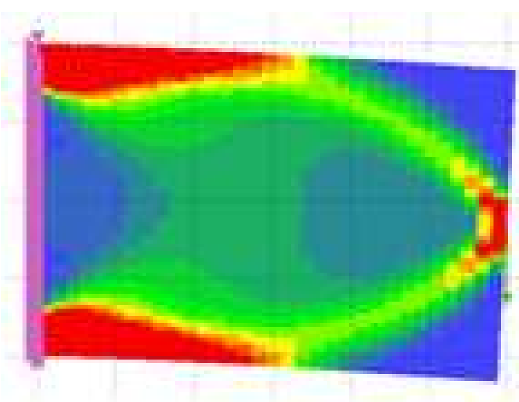


Figure 4.11.11: $q = 1$, $d_- = 0$, $d_+ > 0$, $\rho^n = 0.7$

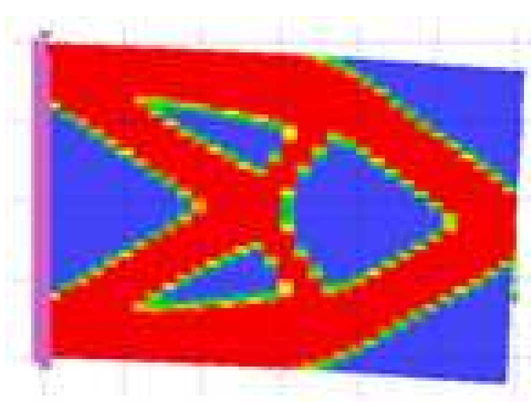


Figure 4.11.12: $q = 3$, $d_- = 0$, $d_+ > 0$, $\rho^n = 0.7$

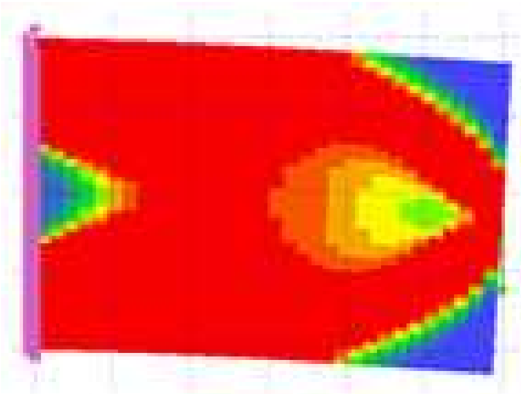


Figure 4.11.13: $q = 1$, $d_- = 0.1E-10$, $d_+ = 0$, $\rho^n = 0.7$

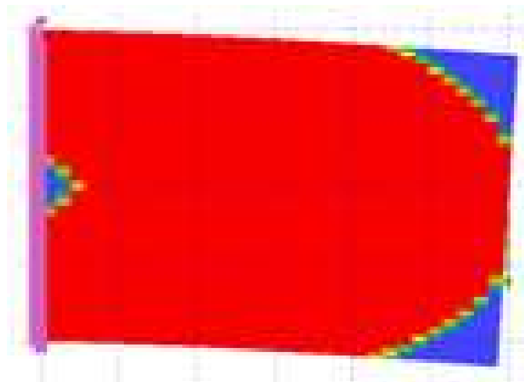


Figure 4.11.14: $q = 3$, $d_- = 0.1E-10$, $d_+ = 0$, $\rho^n = 0.7$

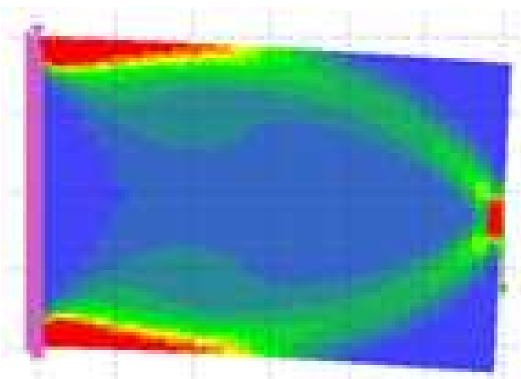


Figure 4.11.15: $q = 1$, $d_- = 0$, $d_+ = 0$, $\rho^n = 0.5$

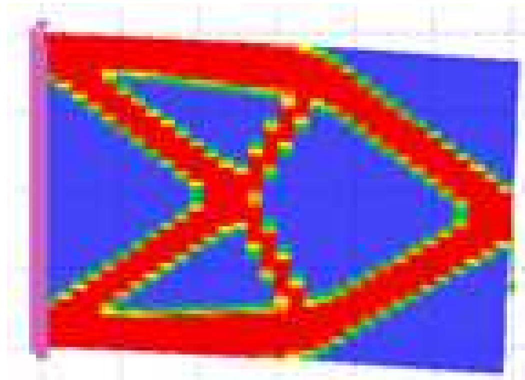


Figure 4.11.16: $q = 3$, $d_- = 0$, $d_+ = 0$, $\rho^n = 0.5$

Remark:- It observed that structural topology for $d_+ > 0$ is thicker than for $d_+ = 0$ as shown in the figure 4.11.10 and 4.11.12. So it is consider that by introducing $d_+ > 0$, material can be added where structural topology feel the need for it.

4.11.4 Case 4

When $\rho^n = 0.5$, the results are shown in the figures 4.11.15 to 4.11.20.
Remark:- For $q = 3$, $d_+ > 0$, $d_- = 0$ and $\mu = 0.1E-06$ we got same result for any initial guess $\underline{\rho} < \rho^n < \overline{\rho}$ as shown in the figures 4.11.2, 4.11.12 and 4.11.18.

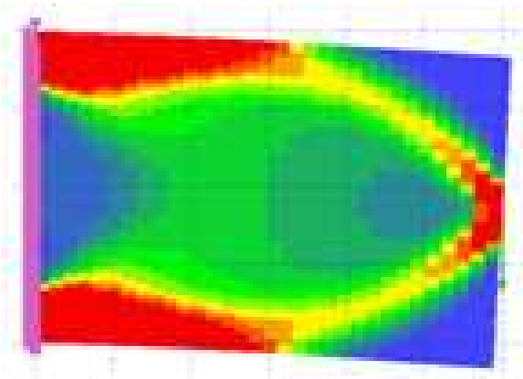


Figure 4.11.17: $q = 1$, $d_- = 0$, $d_+ > 0$, $\rho^n = 0.5$

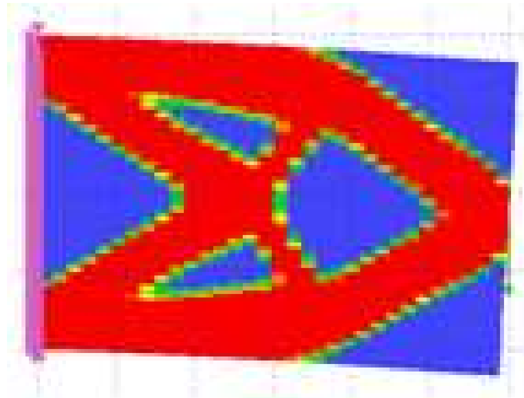


Figure 4.11.18: $q = 3$, $d_- = 0$, $d_+ > 0$, $\rho^n = 0.5$

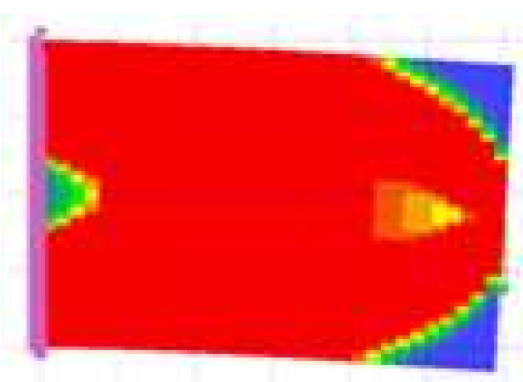


Figure 4.11.19: $q = 1$, $d_- = 0.1E - 10$, $d_+ > 0$, $\rho^n = 0.5$

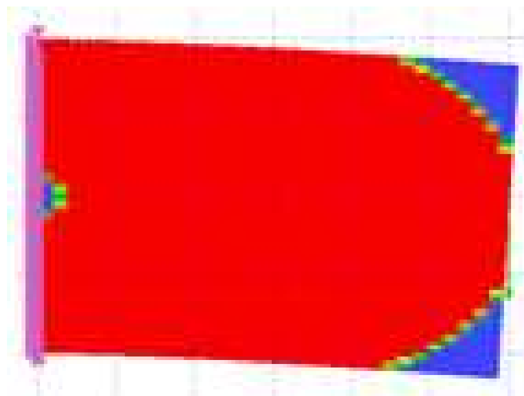


Figure 4.11.20: $q = 3$, $d_- = 0.1E - 10$, $d_+ > 0$, $\rho^n = 0.5$

Remark:- For $q = 3$, $d_+ \geq 0$, $d_- = 0.1E - 10$ and $\mu = 0.1E - 06$ we got same result for any initial guess $\underline{\rho} < \rho^n < \overline{\rho}$ as shown in the figures 4.11.4,4.11.14 and 4.11.20. But for $q = 1$ the results are different as shown in the figures 4.11.3,4.11.13 and 4.11.19.

4.12 Overall conclusion

In this thesis work a new evolutionary structural topology optimization technique is introduced, in which c and $d_\pm$ represent viscous and plastic behaviour respectively in the response of applied load applied on given design domain. Here we suppose that there does not exist any viscous affect (i.e. $c = 0$), but there exist plastic affect (i.e. $d_\pm \neq 0$) in the reaction of constant applied load.

We studied the behaviour of these two parameters d_+ and d_- with respect to others parameters such as ρ^n and μ. There are some findings during analysis

- $FR = 0.02$ is a most suitable value to overcome checkerboards problem.
- $d_- = 0.1E - 10$ is a most suitable value correspond to $\mu = 0.1E - 06$
- $\mu = 0.1E-06$ and $d_- = 0$ correspond to $d_+ > 0$ gives reasonable results for any initial guess except for $\rho^n = \underline{\rho}$
- $\rho^n = \underline{\rho}$ is not a good initial guess

Since two examples are taken to analyze this new ESTO approach. We observed that given parameters have almost same affect with respect to others parameters in both examples. So we can conclude that this new ESTO approach is applicable for all problems where topology optimization is required.

4.13 Future work

In our complete evolutionary structural topology optimization the following parameters are involved

- Lagrange Multiplier (μ)
- Forward Plastic Constant (d_+)
- Backward Plastic Constant (d_-)

- Initial Guess (ρ^n)
- Viscosity constant (c)
- Time increment (Δt)
- Load ($F(t)$)

We analyzed just first four parameters. Since $\Delta t \neq 0$ involves only if $c \neq 0$, but we supposed $c = 0$ through out our analysis, also we considered F=constant load. But to reach on an exact ESTO, it requires to study both plastic and viscous behaviour with time dependent load (i.e. $c \neq 0$, $\Delta t \neq 0$ and $F = F(t)$) at the same time.

Bibliography

[1] Peter W.Christensen, Anders Klarbring (2008). *An Introduction to Structural Optimization*, Springer.

[2] Sang-Rak Kim, Jea-Yong Park, Won-Goo Lee, Jin-Shik Yu, and Seog-Young Han (2007).*Reliability-Based Topology Optimization Based on Evolutionary Structural Optimization* , World Academy of Science, Engineering and Technology.

[3] O. Müller, A. Albers, J. Sauter, P. Allinger. *Topology Optimization of Large Real World Structures*, Institute of Machine Design, University of Karlsruhe Kaiserstrabe, Germany.

[4] Marion Meijboom (2003).*Topology optimization of dynamic problems*, Reportnr: DCT.2003.108, Id. Nr: 448220, University of Pretoria, South Africa.

[5] M. S. Bazaraa, Hanif D. Sherali, C. M. Shetty (1993). *Nonlinear Programming : Theory and Algorithms*, John Wiley and Sons, Inc.

[6] Mikhail J. Atallah (2009). *Algorithms and theory of computation handbook*, CRC Press, Taylor and Francis Group.

[7] M.P.Bendsoe, O.Sigmund (2003). *Topology Optimization:Theory Methods and Applications*, Springer, Berlin.

[8] Richard M. Golden (1996). *Mathematical methods for neural network analysis and design*, The MIT Press.

[9] Kathleen T. Alligood, Tim Sauer, James A. Yorke(1997). *Chaos: An Introduction to Dynamical Systems*, Springer.

[10] Anders Klarbring, Bo Torstentfelt (2009). *Dynamical systems and Topology Optimization*, Structural Multidisciplinary Optimization.