Linograms

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INTRODUCTION

The several successful solutions to the problem of image reconstruction from projections have caused a rapid growth of a number of new techniques for the reconstruction of distributions and images in several scientific fields. The importance of these techniques, especially in medicine, can hardly be overestimated.

In a new algorithm for image reconstruction from projections [1, 2], a special form of the projection data is employed providing some certain advantages. This new form or map of the projection data are called linograms.

This is intended as an overview of linograms and the algorithm based on them. Thorough discussions of conventional techniques are to be found in [3, 4 and 5]. In conventional techniques for image reconstruction, a two dimensional distribution of some property is reconstructed. The property might be the x-ray attenuation in a cross-section of the body, the distribution of a radioactive substance or something else. The distribution is not directly accessible but it is possible to measure line integrals (rays) through it. The problem now is to reconstruct the distribution (the image) from its line integrals (its projections).

Let the property we are interested in be described by the function \( f(x, y) \). Projection data are estimates of line integrals of \( f \) of known location. In conventional techniques each line is specified by two parameters \( s \) and \( \theta \), where \( s \) is the (signed) distance from the origin and \( \theta \) its angle with the
y-axis. Then the line integral of f along the line specified by \((s, \theta)\) is usually called its Radon-transform and is denoted \([Rf](s, \theta)\), where \(R\) is the Radon operator defined by

\[
[Rf](s, \theta) = \int_{-\infty}^{\infty} f(sc\theta - ts\sin \theta, ss\sin \theta + tc\cos \theta)\,dt
\]  

(1)

where

\[
\begin{cases}
  s = x\cos \theta + y\sin \theta \\
  t = -x\sin \theta + y\cos \theta
\end{cases}
\]

If the projection data are arranged in a two dimensional map with the axes \(s\) and \(\theta\), rays through a fixed point in the object \(f(x, y)\) correspond to a sinusoidal curve, which is why a display of this map is called a sinogram \([6, 7]\).

It is possible to arrange the projection data in a new map in which rays through a fixed point in the object correspond to a straight line instead of a sinusoidal curve. Analogous to the term sinogram this new map is called a linogram \([1, 2]\).

In \([8]\) a novel property of the two dimensional Fourier transform of the sinogram was presented. This property is that values on a line through the origin of this transform, correspond to projection data coming from details in the object all lying at a certain distance from the detector where the projection data are registered. The slope of the line through the origin of the transform is proportional to the distance
from the detector. In conventional data acquisition the detector rotates around the object so that the distance between a detail and the detector varies.

The thought occurred that if the perpendicular distance between the detector and the object could be kept constant during the data acquisition a line of a certain slope through the origin of the two dimensional Fourier-transform of the projection data would correspond to data from a fixed line in the object parallel to the detector.

This is the basic idea in the new algorithm. It turns out that the projection data acquired in this way, with a detector having a constant perpendicular distance to the object, are in the form of a linogram.

**BACKPROJECTION**

Before we continue it is necessary to define the concept backprojection. Backprojection is the term used when an image is constructed from the projection data in such a way that each image point is the sum of all rays or line integrals going through the corresponding object point. Until the modern era of image reconstruction from projections, backprojection was the only method available to reconstruct a section through the body. This was called tomography, and nowadays conventional tomography.

In the present case the backprojection $f_B(x, y)$ is an image of $f(x, y)$ defined by
$$f_B(x, y) = \int_{\psi}^{\psi+\pi} [Rf](x \cos \theta + y \sin \theta, \theta) d\theta . \quad (2)$$

Backprojection in (2) is expressed as an integration of the values lying along the sinusoidal

$$s = x \cos \theta + y \sin \theta .$$

In practice the backprojected image is not formed point for point as suggested by (2). Imagine instead that the object, which has yielded the projection data $[Rf](s, \theta)$, is taken away and replaced by some kind of 2-dim. field or matrix with the capacity to store images. Suppose the number of projections are $N$ taken at the angles $\theta_i, i=1,2, \ldots, N$. For each projection a partial image $f_B^{[i]}(x, y)$ is formed defined by

$$f_B^{[i]}(x, y) = \frac{1}{N} [Rf](x \cos \theta_i + y \sin \theta_i, \theta_i)$$

and added to this field. The sum of all such partial images constitute the backprojected image.

$$f_B(x, y) = \sum_{i=1}^{N} f_B^{[i]}(x, y) .$$

This expression might be intuitively understood as if the one-
dimensional projection data on the right side are projected in over the field, so that each point \([Rf](s, \theta_i)\) in the projection is "smeared out" into a line occupying the same position as the line in the object whose integral is \([Rf](s, \theta_i)\). The image on the left side of the equation thus consists of parallel lines all with the angle \(\theta_i\), and all points in a line representing the same value. This is the explanation of the choice of the term "backprojection" for \((2)\). The backprojected image has a low quality due to an over-representation of low spatial frequencies.

Modern reconstruction techniques also use backprojection but the projections are first subjected to a filtration of their spatial frequencies. This has the result that each image element only represents the corresponding object element.

**DEFINITION**

A linogram is a map of projection data of the object \(f(x, y)\) in such a form that a line integral through this map represents a backprojection or partial backprojection of a point in the object. As the projection data themselves represent line integrals through the object, there is a kind of symmetry between a linogram and the object.

**LINOGRAMS DERIVED FROM A SINOGRAM**

Linograms can be derived in several ways. One way is to start with the conventional map of projection data, the so-called sinogram. We will sometimes use the expression \(p(s, \theta)\)
for the Radon transform

\[ \mathcal{Rf}(s, \theta) = p(s, \theta) \]  \hfill (3)

If \( p(s, \theta) \) is arranged as a two dimensional map with the axes \( s \) and \( \theta \), line integrals through a fixed point \( (x_p, y_p) \) in the object are to be found in this map along the sinusoidal,

\[ s = x_p \cos \theta + y_p \sin \theta \]  \hfill (4)

As

\[ p((-1)^n s, \theta + n\pi) = p(s, \theta) \] \hfill (5)

\( p(s, \theta) \) is periodic in \( \theta \) with the period \( 2\pi \). It is therefore intuitively attractive to imagine \( p(s, \theta) \) as a cylindrical surface with the \( s \)-axis, parallel to the axis of the cylinder and the \( \theta \)-axis curved so that the range \( \psi < \theta < \psi + \pi \) forms a half circle. It can now be shown that all sinusoidals (4) on this cylinder will lie in a plane in space (Fig. 1) and all such planes will intersect the central axis of the cylinder at the point where it is intersected by the plane through the \( \theta \)-axis. If, with this point as centre of projection, the cylindrical \( p(s, \theta) \) is projected on any plane, the projection of any sinusoidal (4) will be a straight line on this plane. If this new map of the projection data is combined with a suitable weighting factor it will be a linogram.
It is not possible to contain a full range of projection data in one finite linogram, at least two finite linograms are required.

Any affine transformation of a linogram will also be a linogram, provided the weighting factor is suitably changed.

Hence forth we will only consider linograms which might be obtained by projecting the cylindrical sinogram onto planes that are tangential to the cylinder. The linogram will then be tangential to the sinogram along a line parallel to the s-axis and representing a certain θ-value. Projection data on this line are common for both the linogram and the sinogram. For each linogram this specific θ-value has to be specified and it will be called θ_t. The linogram so specified will have its own coordinate system (u, v) with the u-axis coinciding with the tangential line of θ_t and the v-axis being the projection of the θ-axis in the sinogram.

As at least two linograms are needed, there are two obvious choices of θ_t, namely θ_t = 0 and θ_t = π/2. The linogram with θ_t = 0 will be called g_1(u, v) and the one with θ_t = π/2 will be called g_2(u, v) (Fig. 2).

g_1(u, v) will represent the range of projection for -π/4 < θ < π/4 and g_2(u, v) the range π/4 < θ < 3/4π. The coordinate relations between (s, θ) and (u, v) will be for g_1(u, v)
\[
\begin{align*}
\begin{cases}
    s = \frac{u}{\sqrt{1+v^2}} \\
    \theta = \arctan v
\end{cases}
\quad \text{or} \quad 
\begin{cases}
    u = \frac{s}{\cos \theta} \\
    v = \tan \theta
\end{cases}
\quad \text{(6)}
\end{align*}
\]

and for \( g_2(u, v) \)

\[
\begin{align*}
\begin{cases}
    s = \frac{u}{\sqrt{1+v^2}} \\
    \theta = -\arccot v
\end{cases}
\quad \text{or} \quad 
\begin{cases}
    u = \frac{s}{\sin \theta} \\
    v = -\cot \theta
\end{cases}
\quad \text{(7)}
\end{align*}
\]

Thus

\[
g_1(u, v) = w_1(u, v) \ p\left(\frac{u}{\sqrt{1+v^2}}, \ \arctan v\right) \quad \text{(8)}
\]

and

\[
g_2(u, v) = w_2(u, v) \ p\left(\frac{u}{\sqrt{1+v^2}}, -\arccot v\right), \quad \text{(9)}
\]

where \( w(u, v) \) represents a weighting factor.
LINOGRAMS DERIVED FROM BACKPROJECTION

To find this factor we can try to derive a linogram from the definition of backprojection. The backprojected image \( f_B(x, y) \) from \( p(s, \theta) \) is defined by

\[
f_B(x, y) = \int_{-\pi/4}^{\pi/4} p(x\cos\theta + y\sin\theta, \theta) d\theta \tag{10}
\]

\[
= \int_{-\pi/4}^{\pi/4} d\theta + \int_{\pi/4}^{3\pi/4} d\theta = f_B^{[1]}(x, y) + f_B^{[2]}(x, y) .
\]

The first part of the RHS, representing a partial back projection is,

\[
f_B^{[1]}(x, y) = \int_{-\pi/4}^{\pi/4} p(x\cos\theta + y\sin\theta, \theta) d\theta . \tag{12}
\]

We now change the variable of integration from \( \theta \) to \( v \). From (6) we have

\[
\theta = \arctan v
\]

from which we define

\[
d\theta = \frac{dv}{1+v^2}, \quad \cos\theta = \frac{1}{\sqrt{1+v^2}} \quad \text{and} \quad \sin\theta = \frac{v}{\sqrt{1+v^2}} . \tag{14}
\]

Substitute these expressions in (12).
\[
\mathcal{f}^{[1]}_B(x, y) = \int_{-1}^{1} \frac{1}{1+v^2} p(\frac{x}{\sqrt{1+v^2}} + \frac{y v}{\sqrt{1+v^2}}, \arctan v) dv
\]

from which we see that

\[
w_1(u, v) = \frac{1}{1+v^2}, \quad (17)
\]

and we also note on comparison with (8) that in \(g_1(u,v)\)

\[
u_1 = x + yv \quad .
\]

Similarly we get \(w_2(u, v) = \frac{1}{1+v^2}\), and note that in \(g_2(u,v)\)

\[
u_2 = y - xv. \quad (19)
\]

Thus

\[
g_1(u, v) = \frac{1}{1+v^2} p(\frac{u}{\sqrt{1+v^2}}, \arctan v) \quad ,
\]

and
\[ g_2(u, v) = \frac{1}{1+v^2} p\left(\frac{u}{\sqrt{1+v^2}}, -\arccot v\right) \]  

(21)

A full backprojection with the two linograms is

\[ f_B(x, y) = \int_{-1}^{1} [g_1(u, v) + g_2(u, v)] dv \]  

(22)

**BACKPROJECTION BY PARALLEL PROJECTION OF THE LINOGRAM**

From (18 and 19) we see that \( u_1 \) and \( u_2 \) are straight lines. (22) thus represents backprojection by line integrals (Fig. 3).

Consider the first part of (22)

\[ f_B^{[1]}(x, y) = \int_{-1}^{1} g_1(x+yv, v) dv \]  

(23)

We see that, if \( y \) is kept at a constant value, (23) represents parallel line integrals, all with the slope \( y \) and resulting in the partial backprojection of the whole line in the image with this \( y \)-value (Fig. 4). Similarly in

\[ f_B^{[2]}(x, y) = \int_{-1}^{1} g_2(y-xv, v) dv \]  

(24)

if \( x \) is kept constant it represents the partial backprojection of the whole line in the image with this \( x \)-value.

A bundle of parallel line integrals can be regarded as a parallel projection. This leads the thought to the possibility
to express (23) and (24) as the Radon transforms of the linograms.

The Radon transform is defined as (2) and if this is applied to the linogram $g_1(u, v)$, we have

$$
[Rg_1](s', \theta') = \int_{-\infty}^{\infty} g_1(s'\cos\theta' - t'\sin\theta', s'\sin\theta' + t'\cos\theta')d\theta.
$$

(25)

We now change the variable of integration from $t$ to $v$

$$
v = s'\cos\theta' + t'\cos\theta', \quad dt = \frac{dv}{\cos\theta'}
$$

(26)

$$
[Rg_1](s', \theta') = \int_{-\infty}^{\infty} \frac{1}{\cos\theta'} g_1(s' - v\sin\theta', v)dv
$$

(27)

From Figure 5, we see that $\theta' = -\arctan y$. From the figure we can derive

$$
\frac{s'}{\cos\theta'} = x, \quad \frac{\sin\theta'}{\cos\theta'} = -y, \text{ and } \frac{1}{\cos\theta'} = \sqrt{1+y^2}
$$

(28)

These expressions are introduced in (27).

$$
[Rg_1]\left(\frac{x}{\sqrt{1+y^2}}, \ -\arctan y\right) = \int_{-\infty}^{\infty} \sqrt{1+y^2} g_1(x+vy, v)dv \ .
$$

(29)

Comparing (29) with (23) we see that (23) may be written
BACKPROJECTION BY THE USE OF THE PROJECTION THEOREM

The advantage of this is that we can apply the so-called projection theorem. Let $f^{[1]}$ be an operator that performs a Fourier transform of the first variable of a 2-dimensional distribution, $F^{[2]}$ similarly a transform of the second variable and $F_2$ a 2-dimensional Fourier transform of the distribution. The projection theorem says that

$$[F^{[1]}Rg](S', \theta') = [F_2 g](S' \cos \theta', S' \sin \theta'). \quad (31)$$

We start by doing a Fourier transform of the first variable on both sides of (30). On the right side we then have to use the similarity theorem which says that if

$$f(x, y) = g(ax, y),$$

then

$$[F^{[1]}f](x, y) = \frac{1}{|a|} [F^{[1]}g](x/a, y). \quad (32)$$

Using (32) on (30) we get
We now apply (31) on the right side

\[
[F^{[1]} f^{[1]}_B](x, y) = [F^{[1]} g_1](x \sqrt{1+y^2} - \frac{y}{\sqrt{1+y^2}}, y \sqrt{1+y^2} + \frac{1}{\sqrt{1+y^2}})
\]

and we have that

\[
[F^{[1]} f^{[1]}_B](x, y) = [F^{[2]} g_1](x, -y) \quad . \tag{34}
\]

In a similar way for the second part of (22) we get that

\[
[F^{[2]} f^{[2]}_B](x, y) = [F^{[2]} g_2](y, xy) \quad . \tag{35}
\]

(34) and (35) say that the Fourier transform of rows and columns in the backprojected image respectively are to be found as oblique lines through the origin of the 2-dimensional Fourier transforms of the two linograms.

We see also that there is no need to do interpolations of the Fourier-transformed variables.

**Filtration**

So far we have only discussed pure backprojection. In an algorithm for reconstructing the distribution \(f(x, y)\), the projection data has to be filtered before they are back-
projected. Let us first see how this is done in the conventional algorithm using projection data $p(s, \theta)$.

Let

$$
\hat{p}_w(S, \theta) = |S|W(S)[F[1]p](S, \theta),
$$

(36)

then

$$
[(F[1])^{-1} \hat{p}_w](s, \theta)
$$

(37)

are the properly filtered data to be used in the back-projection. In (36) the projection data are first Fourier transformed in the first variable and then multiplied by the absolute of the Fourier variable and a bandlimiting window $W(S)$.

Let $\hat{g}_{1W}(u, v)$ be the properly filtered linogram data that corresponds to $\hat{p}_w(S, \theta)$ in (36).

This entails that

$$
[(F[1])^{-1} \hat{g}_{1W}](u, v) = \frac{1}{1+v^2} [(F[1])^{-1} \hat{p}_w](\frac{u}{\sqrt{1+v^2}}, \text{arctan}(v))
$$

(38)

according to (20). Do a Fourier transform of the first variable on both sides and use the similarity theorem in (32),

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\[ \hat{g}_{1W}(U, v) = \frac{1}{\sqrt{1+v^2}} \hat{P}_W(U \sqrt{1+v^2}, \arctan v) \]  \hspace{1cm} (39)

Use the definition (36) on the right side.

\[ \hat{g}_{1W}(U, v) = |U| W(U \sqrt{1+v^2})[F[1] P](U \sqrt{1+v^2}, \arctan v) \]  \hspace{1cm} (40)

Now let's go back to (20) and do a Fourier-transform on both sides of the first variable and using (32).

\[ [F[1] g_1](U, v) = \frac{1}{\sqrt{1+v^2}} [F[1] p](U \sqrt{1+v^2}, \arctan v) \]  \hspace{1cm} (41)

\[ \sqrt{1+v^2} [F[1] g_1](U, v) = [F[1] p](U \sqrt{1+v^2}, \arctan v) \]  \hspace{1cm} (42)

Insert the left side of (42) in (40) and we have

\[ \hat{g}_{1W}(U, v) = |U| W(U \sqrt{1+v^2})[F[1] g_1](U, v) \]  \hspace{1cm} (43)

The same filtration applies to \( \hat{g}_{2W} \). Let \( f_W(x, y) \) be the properly reconstructed image, then

\[ f_W(x, y) = f_W^{[1]}(x, y) + f_W^{[2]}(x, y) \]  \hspace{1cm} (44)

The complete algorithm is then to use (43) in (34) and (35).
THE ALGORITHM DESCRIBED STEP BY STEP

**Step 1** Produce 2 linograms of the object either by the use of a special scanner or by rebinning from a conventional scanner.

**Step 2** Make a one dimensional fast Fourier transform of the linograms in the $u$-direction.

**Step 3** Multiply the linograms by $|U+1+v^2| W(U+1+v^2, \tan^{-1} v)$.

**Step 4** Make a one dimensional discrete Fourier transform in the $v$-direction so that we get the exact values we need on the right-hand-side in (45). Using the Chirp-z-transform this can be done by a sequence of three fast Fourier transforms [2].

**Step 5** Make an inverse Fourier transform in the $x$-direction so that we get the partial images from the left-hand-side of (45).

**Step 6** Add the two partial images.

A LINOGRAM SCANNER

In the preceding discussion linograms were derived as a re-mapping of sinograms. Linograms can be produced directly in
a special scanner of a very simple construction.

Let a fixed x-ray source emit a fan beam downward to a horizontal fixed detector array and let the object move with a constant velocity above and parallel to the array as in Figure 7. Each detector will then receive a parallel projection of the object with a v-coordinate equal to the tangent for the projection angle $\theta$.

Insert a coordinate-system $(x', y')$ and let the detector array coincide with the $x'$-axis and with its mid-point at the origin (Fig. 8). Let the x-ray source be at $(0, y'_s)$. The v-coordinate for a detector at $(x'_d, 0)$ then is

$$v = \frac{x'_d}{y'_s}.$$  

(47)

Insert a coordinate system $(x, y)$ in the object and parallel to $(x', y')$, and let the object move together with its coordinate system as a translation so that the origin follows the line

$$y' = y'_0,$$

with the constant velocity, $h$. Let the time coordinate, $t$, be zero when the origin of the object is projected on the origin of the scanner, i.e., on the detector at $(0, 0)$ in the $(x', y')$ system.

As before, we define the u-coordinate for each ray to be the x-coordinate on the x-axis of the object intersected by
that ray. For each detector the u-coordinate will be a function of t. For the detector at \( x' = 0 \), the u-coordinate will evidently be

\[
u = -ht , \tag{48}\]

and for an arbitrary detector at \((x_d', 0)\) there will be an offset in time so that

\[
u = -h(t + t_d) \tag{49}\]

\(t_d\) is the point in time when the center of the object is projected on the detector at \((x_d', 0)\). From Figure 8

\[
t_d = \frac{t_d}{h} (1 - \frac{y'_0}{y'_s}) . \tag{50}\]

Then

\[
u = -ht + x'_d(1 - \frac{y'_0}{y'_s}) . \tag{51}\]

The data collected in this simple scanner will have the proper coordinates to be a linogram. Each ray through the object \( f(x, y) \) represents the line integral
Can the collected data directly be used as a linogram or do we have to multiply with a weighting factor?

DIRECT DERIVATION OF THE LINOGRAm

Let us go back to (2) and try to derive the linogram directly as line integrals through \( f(x, y) \). From (2)

\[
[Rf](s, \theta) = \int_\infty^\infty f(s s \cos \theta - t s \sin \theta, s s \sin \theta + t \cos \theta) dt . \tag{53}
\]

Similar to what we did in (27), we now change integration variable from \( t \) to \( y \)

\[
[Rf](s, \theta) = \int_\infty^\infty \frac{1}{\cos \theta} f(s - y s \sin \theta, \cos \theta, y) dy . \tag{54}
\]

Define \( v = \tan \theta \) and \( u = \frac{s}{\cos \theta} \). Then \( \frac{1}{\cos \theta} = \frac{1}{\sqrt{1 + v^2}} \), \( s = \frac{u}{\sqrt{1 + v^2}} \) and \( \theta = \arctan v \). Use these expression in (54) and we get

\[
[Rf](\frac{u}{\sqrt{1 + v^2}}, \arctan v) = \int_\infty^\infty \sqrt{1 + v^2} f(u - y v, y) dy \tag{55}
\]
\[ \int_{-\infty}^{\infty} f(u-yv, y) \, dy = \frac{1}{\sqrt{1+v^2}} \left[ Rf\left( \frac{u}{\sqrt{1+v^2}}, \arctan v \right) \right] \quad (56) \]

Comparing the right side of (56) with the right side of (20), which is the correct expression for \( g_1(u,v) \), we see that the left side of (56) has to be multiplied by the factor \( \frac{1}{\sqrt{1+v^2}} \) in order to be the linogram \( g_1(u,v) \). Thus

\[ g_1(u, v) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{1+v^2}} f(u-yv, y) \, dy \quad (57) \]

The data collected in the proposed scanner will thus be a proper linogram after being multiplied with this factor. If the detector array covers the range

\[-1 < v < 1,\]

one passage of the object will suffice to produce data for one of the two necessary linograms.

If the object is a patient then it would be better to have the patient fixed and move the scanner. The second necessary linogram can be obtained by rotating the scanner 90° and then move it vertically, or by another scanner.
The proposed scanner, however, is particularly suited for industrial applications where it is necessary to examine machine parts or other rigid objects by CT.

After one passage through the scanner, producing the first linogram, the object is rotated $90^\circ$ and the second linogram is produced by a second passage through the same scanner or through a second scanner.

If there should be difficulties in constructing a detector array covering the range $-1 < v < 1$, the scanner could have an array for the range $-1/\sqrt{3} < v < 1/\sqrt{3}$ covering an angular range of $60^\circ$. It will then be necessary to produce three linograms by three passages of the object. Before each new passage the object is rotated $60^\circ$. The tree partial images reconstructed from the three linograms should be in a form with hexagonal grid points so that the grid points will coincide when the images are added together.
REFERENCES


[6] This term was introduced in a poster presentation at the 1975 meeting on Image Processing for 2-D and 3-D Reconstructions from Projections at Stanford, CA. The material appeared in a collection of postdeadline papers for that meeting). PD5 - Tomogram Construction by Photographic Techniques. Paul Edholm and Bertil Jacobson.


[8] Edholm, P.R., Lewitt, R.M. and Lindholm, B., "Novel properties of the Fourier decomposition of the sinogram". 

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A sinogram with the angular range of \(2\pi\) rolled to form a cylinder in space. Any sinusoidal \(s = x_p \cos \theta + y_p \sin \theta\) on the sinogram will then form a curve in space with the equations

\[
\begin{cases}
    s = x_p \cos \theta + y_p \sin \theta \\
    z_1 = \cos \theta \\
    z_2 = \sin \theta,
\end{cases}
\]

from which it is seen that it is an ellips lying in the plane \(s = x_p z_1 + y_p z_2\).
The cylindrical sinogram in Figure 1 is projected from the origin onto two planes to form the two linograms \( g_1(u, v) \) and \( g_2(u, v) \). The former is tangential to the cylinder at \( \theta = 0 \), the latter at \( \theta = \pi/2 \). The sinusoidal from Figure 1 is projected as two straight lines, \( u = x_p + y_p v \) in \( g_1 \) and \( u = y_p - x_p v \) in \( g_2 \).
Above the object $f(x, y)$ with a point (circle) and 5 line integrals through it (1-5). Below the linogram $g_1(u, v)$ in which the line integrals are registered at points (1'-5') lying in a line. A line integral along this line constitutes a backprojection for the point in the object.
Above the object $f(x, y)$ with a line of $x$-values, all with the same $y$-value. Below the linogram $g_1(u, v)$ in which parallel line integrals represent the backprojection for the line in the object.
Above the object with a point. Below the linogram \( g_1(u, v) \) with the line \( u = x + yv \). Its line integral represents the backprojection for the point. This may be expressed as the Radon transform of the linogram. The figure shows the relations between \( s' \) and \( \theta' \) and the line.
A pictorial description of how the algorithm may be used for pure backprojection. **Left:** The backprojection for a line in the object is expressed as a projection of the linogram. The picture illustrates equation (30) for a fixed $y$-value. **Right:** In a two dimensional Fourier transform of the linogram, the components along a line with slope $(-y)$ are the components for the Fourier transform for the line in the backprojected image with this $y$-value. The whole figure may also be regarded as a description of (34) for a fixed $y$-value. The lower half describes the right-hand-side of the equation, and, if the direction of the Fourier transform is reversed, the upper half describes the left-hand-side of the equation.
FIGURE 7

Proposed scanner with fixed ray source and detector array and with moving object.
Coordinate system for scanner \((x', y')\) and object \((x, y)\).