Centre of Rotation Determination
Using Projection Data in X-ray Micro Computed Tomography

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Centre of Rotation Determination using Projection Data in X-ray Micro Computed Tomography

INTRODUCTION

There are several methods available to determine the Centre Of Rotation, COR, and align detectors and X-ray focus to COR in X-ray computed tomography. Some methods use narrow rods/needles or specially made alignment objects or phantoms. In X-ray Micro Computed Tomography (or Computerized Micro Tomography), µCT (CMT), methods using sample projection data for COR measurements are preferred since the replacement of alignment objects with samples often displace translation stages and make the alignment obsolete. To achieve an optimal image quality, precise positioning of COR to the detector and X-ray focus is essential. In µCT this can be accomplished in an alignment procedure using sample projection data prior to scanning. This alignment procedure will add examination time and increase the dose to the sample. Therefore the alignment procedure should incorporate as few projections as possible and be insensitive to noise. Some scanning equipment cannot be modified to implement such alignment procedure but actual COR can be determined from projection data and used in reconstruction. This report introduces a new Translated Opposite Projection, TOP, technique using a pair of opposite parallel projections (180° apart). Two TOP methods are developed: TOPlin using linear interpolation in the spatial domain and TOPfft in the frequency domain. The two TOP methods are compared to two Centre Of Gravity, COG, methods. The two COG methods are: COGsin, an enhancement of a method presented by Hogan et al [Hogan93] and COGopp, a simplification of this method possible with a fixed COR. In this report all projections in one scan are assumed to have a fixed, COR, as in third (or higher) generation tomography or first (and second generation) tomography if the translation stage errors is negligible. This also means that the rotation stage errors must be negligible. The COGsin is the only method presented here capable of determine a COR for each projection angle, thus allowing for a COR moving as a function of projection angle. The TOP methods normally give better precision with non-ideal projection data compared to the COG methods. Tests using both simulated and scanned projection data indicate that the TOP methods give higher precision in presence of stochastic errors (noise) and system errors like calibration errors. A µCT scan often takes a long time and detector calibration and X-ray intensity profiles might vary with time giving non-stationary system errors during a single scan. If the system errors can be approximated with simple polynomial functions, a new Baseline Restoration, BR, technique can be used together with the TOP methods to reduce COR errors.
DEFINITIONS

All methods described in this report use parallel projection data, \( P_\theta(t) \). The parallel projection data is the Radon-transform of the object attenuation function, \( f(x,y) \)

\[
P_\theta(t) = P(\theta,t) = \int \int f(x,y) \delta(x \cos \theta + y \sin \theta - t) \, dx \, dy
\]  

where

\( \delta(x) \) is the Dirac-function.

Fan-beam projection data needs to be rebinned to parallel projection data if not \( D >> r_{\text{max}} \), where \( r_{\text{max}} \) is the radius of the smallest circle centred at the origin subscribing the object and \( D \) is the origin to source distance. In \( \mu \text{CT} \) the fan-beam geometry in figure 1 with equidistant detectors in line is the most common non-parallel geometry. The rebinning procedure includes a two-dimensional interpolation technique, normally bilinear interpolation, and uses the relationship between the co-ordinates \( (\beta, s) \) in fan-beam geometry (figure 1) and \( (\theta, t) \) in parallel geometry (figure 2).

\[
\begin{cases}
  t = s \cos \gamma, & \theta = \beta + \gamma \\
  t = \frac{s D}{\sqrt{D^2 + s^2}}, & \theta = \beta + \tan^{-1} \frac{s}{D}
\end{cases}
\]  

FIGURE 1. Fan-beam geometry.  
FIGURE 2. Parallel geometry.
The parallel data is a set of $M$ projections with projection angles $\theta_i, i = 1..M$, each projection with $N$ equidistant sampled projection data at $t = t_j, j = 1..N$. Let the angular increment be $\Delta\theta$, the sample distance $\Delta t$ and the centre data index $c$. The discrete sampled projection data is denoted by $P_{ij}, P_{\theta_i,j}, P_{\theta_i}(t_j)$ or $P(\theta_i,t_j)$.

The term position, $\nu \in \mathbb{R}$, will be used here to specify position in terms of sample index. A sample with index $k$ at $t = t_k$ is at position $\nu = k$ and position $\nu = k + \epsilon$, $0 < \epsilon < 1$ is in between index $k$ and $k + 1$ at $t = t_k + \epsilon \Delta t$.

We define the alignment offset $\alpha$ as the positional displacement of COR from centre data position (see figure 3).

$$\alpha = \frac{t_c}{\Delta t}$$

(3)

The Centre Of Rotation, COR, will have the position

$$\nu_{COR} = c - \alpha$$

(4)

As an example the quarter offset often used in X-ray CT corresponds to $\alpha = \pm 0.25$.

The Radon Transform from $\mathbb{R}^2$ needs to be defined for $t \in \mathbb{R}$ and $\theta \in [0,\pi]$ and it is periodic in $\theta$ with the following periodicity property

$$P(\theta + \pi, t) = P(\theta, -t)$$

(5)
Another important property is that the generalised Centre Of Gravity, COG, of \( f(x,y) \)

\[
(x_{COG}, y_{COG}, x_{COG}) = \frac{\int \int x f(x,y) \, dx \, dy}{\int \int f(x,y) \, dx \, dy}, \quad y_{COG} = \frac{\int \int y f(x,y) \, dx \, dy}{\int \int f(x,y) \, dx \, dy}
\]

with polar co-ordinates \( (r_{COG}, \phi_{COG}) \), \( x_{COG} = r_{COG} \cos \phi_{COG} \), \( y_{COG} = r_{COG} \sin \phi_{COG} \) is transformed onto the one-dimensional COG

\[
t_{COG} = \frac{\int t \, P(\theta,t) \, dt}{\int P(\theta,t) \, dt}
\]

of the projection data at angle \( \theta \). That is

\[
t_{COG} = r_{COG} \cos(\theta - \phi_{COG}) \tag{6}
\]

In the description of the alignment methods the following notations will be used:

- \( X_{\theta,j} \) is data at projection angle \( \theta \) and index \( j \), \( X_{\theta}(t_j) \)
- \( \hat{X}_{\theta,j} \) is measured data
- \( X_{\theta,j}^a \) is data translated \( \alpha \Delta t \), \( X_{\theta}(t_j - \alpha \Delta t) \)
- \( \hat{X}_{\theta,j}^a \) is estimated translation of measured data

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CENTRE OF GRAVITY METHODS

The COG position of projection data at projection angle $\theta_i$ is estimated using

$$
\dot{v}_{COG}(\theta_i) = \frac{\sum_{j=1}^{N} \hat{p}(\theta_j, t_j)}{\sum_{j=1}^{N} \hat{p}(\theta_j, t_j)}
$$

Equation (6) can be expressed in terms of position $v_{COG} = \frac{t_{COG}}{\Delta t} + v_{COR}$ with $\rho_{COG} = \frac{r_{COG}}{\Delta t}$ and for each projection angle we get

$$
v_{COG}(\theta_i) = \rho_{COG} \cos(\theta_i - \phi_{COG}) + v_{COR}
$$

THE COGsin METHOD

A Minimum Mean Square Error, MMSE, algorithm can be used to find $\rho_{\text{COG}}$ and $v_{\text{COR}}$ for a fixed $\phi_{\text{COG}}$. Then the $\phi_{\text{COG}}$ giving minimum Mean Square Error, MSE, is searched [Hogan93].

The MSE is calculated using

$$MSE = \sum_{i=1}^{M} \left( v_{\text{COR}} + \rho_{\text{COG}} \cos(\theta_i - \phi_{\text{COG}}) - \hat{v}_{\text{COG}}(\theta_i) \right)^2$$

(9)

The search of $\phi_{\text{COG}}$ giving MMSE results in computing a lot of cosine's and sums and solving one second order linear equation system for each tested $\phi_{\text{COG}}$. If the $\rho_{\text{COG}} \cos(\theta_i - \phi_{\text{COG}})$-term is split into one COS- and one SIN-term, finding MMSE can be reduced to solving a third order linear equation system and no search for $\phi_{\text{COG}}$ is necessary.

Let

$$\rho_{\text{COG}} \cos(\theta_i - \phi_{\text{COG}}) = ac_i + bs_i$$

(10)

where

$$a = \rho_{\text{COG}} \cos(\phi_{\text{COG}})$$

$$b = -\rho_{\text{COG}} \sin(\phi_{\text{COG}})$$

$$c_i = \cos(\theta_i)$$

$$s_i = \sin(\theta_i)$$

Equation (9) is rewritten as

$$MSE = \sum_{i=1}^{M} \left( v_{\text{COR}} + ac_i + bs_i - \hat{v}_{\text{COG}}(\theta_i) \right)^2$$

(11)

MMSE is found by setting each partial derivative with respect to $a$, $b$ and $v_{\text{COR}}$ to zero resulting in the linear equation system

$$
\begin{bmatrix}
\sum_{i=1}^{M} c_i^2 & \sum_{i=1}^{M} c_i s_i & \sum_{i=1}^{M} c_i \\
\sum_{i=1}^{M} c_i s_i & \sum_{i=1}^{M} s_i^2 & \sum_{i=1}^{M} s_i \\
\sum_{i=1}^{M} c_i & \sum_{i=1}^{M} s_i & M
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
v_{\text{COR}}
\end{bmatrix}
= 
\begin{bmatrix}
\sum_{i=1}^{M} c_i \hat{v}_{\text{COG}}(\theta_i) \\
\sum_{i=1}^{M} s_i \hat{v}_{\text{COG}}(\theta_i) \\
\sum_{i=1}^{M} \hat{v}_{\text{COG}}(\theta_i)
\end{bmatrix}
$$

(12)

The equation system is solved using Gaussian elimination.

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This method is called the COGsin method in this report. The COGsin method needs several projection angles to be accurate but projection data from less than 180°, \((\theta_m - \theta_i) < \pi\), is sufficient. An alignment offset, \(\alpha_i\), can be estimated for each projection angle \(\theta_i\) according to [Hogan93]. The other presented methods in this report require a constant alignment offset in all projections (a fixed COR).

**THE COGopp METHOD**

The COG method can be considerably simplified if projection data from at least one pair of opposite projection angles \(\theta\) and \(\theta + \pi\) is available. Equation (5) gives \(t_{COG}(\theta + \pi) = -t_{COG}(\theta)\). In terms of position we get

\[
v_{COG}(\theta + \pi) - v_{COR} = -(v_{COG}(\theta) - v_{COR})
\]

or

\[
v_{COR} = \frac{v_{COG}(\theta) + v_{COG}(\theta + \pi)}{2}
\]  

(13)

Estimates of \(v_{COG}(\theta)\) and \(v_{COG}(\theta + \pi)\) are calculated using equation (7) and estimated \(v_{COR}\) is the mean value of the two COG positions. Finally \(\alpha\) is calculated using equation (4).

This simplified method is called the COGopp-method in this report.
TRANSLATED OPPOSITE PROJECTION METHODS

The TOP methods need projection data from at least one pair of opposite projection angles $\theta$ and $\theta + \pi$. The idea is to estimate translated projection data for the two opposite projections

$$
\begin{align*}
P_{\theta,j}^\alpha &= P(\theta, t_j - \alpha \Delta t) \\
\hat{P}_{\theta+\pi,j}^\alpha &= P(\theta + \pi, t_j - \alpha \Delta t)
\end{align*}
$$

(14)

and a fixed $\alpha$. This translation will align the centre projection data at index $c$ to COR if $\alpha$ is correct. Equation (5) gives

$$
P_{\theta,c+j}^\alpha = P_{\theta+\pi,c-j}^\alpha
$$

(15)

![Diagram of the principle of the Translated Opposite Projection methods](image)

**FIGURE 5.** The principle of the Translated Opposite Projection methods (see text for explanation).

The measured projection data can be modelled as the "true" data with an additional error term

$$
\hat{P}_{\theta,j} = P_{\theta,j} + \hat{\epsilon}_{\theta,j}
$$

(16)
The estimated Translated Opposite Projection, TOP, difference is

\[ \hat{D}_{\theta,j}^a = \hat{p}_{\theta,c+j}^a - \hat{p}_{\theta,c-j}^a = p_{\theta,c+j}^a + \hat{\varepsilon}_{\theta,c+j}^a - p_{\theta,c-j}^a - \hat{\varepsilon}_{\theta,c-j}^a \] (17)

Note that estimation (interpolation) errors are included in the \( \hat{\varepsilon}_{\theta,j}^a \)-terms.

Equation (15) gives

\[ \hat{D}_{\theta,j}^a = \hat{\varepsilon}_{\theta,c+j}^a - \hat{\varepsilon}_{\theta,c-j}^a \] (18)

The TOP methods find the alignment offset \( \alpha \) giving Minimum Mean Square Difference, MMSD, between the two estimated TOP’s. The Mean Square Difference, MSD, is calculated using

\[ \text{MSD} = \sum_{j=k}^{K} \left( \hat{D}_{\theta,j}^a \right)^2, \quad K = \max(c - k - 1, N - c + k + 1), \quad k \leq \alpha < k + 1, \quad k \in \mathbb{Z} \] (19)

The estimation of the translated projection data can be implemented in both spatial domain and in Fourier domain. In spatial domain translated data is interpolated from original projection data. The TOPlin method uses linear interpolation. Higher order interpolation filters can be used but the linear interpolation has one major advantage besides simple implementation and a moderate number of calculations; the MSD is a second order polynomial of \( \alpha \) in each interval \( k \leq \alpha < k + 1 \). Thus the minimum MSD in each such interval is easily calculated, no search for minimum MSD is needed. The TOPfft method calculates the Discrete Fourier-transform of the projection data using FFT. The translation is implemented using a phase-shift proportional to \( \alpha \) in the Fourier domain. The MSD is also calculated in the Fourier domain using Parseval’s relation, no inverse Fourier-transform is needed. The algorithm calculates the \( \alpha \) giving minimum MSD. The TOPfft method has two major advantages compared to the TOPlin method: better precision because no interpolation is needed and the ability to filter data prior to alignment without any extra convolution or FFT and IFFT (inverse FFT) calculations. The major drawback is a higher computational complexity compared to the TOPlin method.

The alignment offset \( \alpha \) is searched in the alignment offset interval \( \alpha_{\text{min}} \leq \alpha \leq \alpha_{\text{max}} \). This interval must be wide enough to include the correct \( \alpha \). A good way to get high precision with a moderate computational effort is to make a coarse alignment using the TOPlin method and a wide interval followed by a TOPfft alignment with a narrow interval.

**THE TOPlin METHOD**

For \( \alpha \) in the range \( k \leq \alpha < k + 1 \), the linear interpolation is calculated as

\[
\begin{align*}
\hat{p}_{\theta,j}^a &= \alpha_k \hat{p}_{\theta,j-k-1}^a + (1 - \alpha_k) \hat{p}_{\theta,j-k}^a, \quad \alpha_k = \alpha - k, k \leq \alpha < k + 1, k \in \mathbb{Z} \\
\hat{p}_{\theta,j} &= 0 \text{ if } j < 0 \text{ or } j > N
\end{align*}
\] (20)
The estimated translated opposite projection difference is calculated according to equation (17) and the MSD will be

\[
MSD_k = \sum_{j=-k}^{k} \left( a_k \tilde{P}_{\theta,j+k} + (1-a_k) \tilde{P}_{\theta,j-k} - a_k \tilde{P}_{\theta+\pi,j-k} - (1-a_k) \tilde{P}_{\theta+\pi,j+k} \right)^2
\]

\[K = \max(c-k-1, N-c+k+1)\], \quad a_k = \alpha - k, \quad k \leq \alpha < k + 1, \quad k \in \mathbb{Z}

This MSD calculation results in a polynomial of the form \( MSD_k = c_2 a_k^2 + c_1 a_k + c_0 \) with a minimum or maximum for \( a_k = \frac{-c_1}{2c_2} \). If it is a minimum for \( a_k \) in the range \( 0 \leq a_k < 1 \) this minimum \( MSD_k \) is stored. The stored minima from all \( k \) in the alignment offset interval \( \alpha_{\min} \leq k \leq (\alpha_{\max} - 1) \) are compared and the \( a_k \) resulting in the lowest minimum \( MSD_k \) gives the alignment offset \( \alpha = k + a_k \).

**THE TOPfft METHOD**

The projection data is initially zero-padded and then shifted (circular shift) to get the centre projection data index, \( c \), at origin (the origin is index 0 in the FFT data array).

\[
\hat{P}_{\theta,j} = \begin{cases} 
\hat{P}_{\theta,j}, & j = 1...N \\
0, & j = N + 1...L 
\end{cases} \quad \text{Zero padding to } L \text{ length}
\]

\[
\hat{q}_{\theta,j} = \begin{cases} 
\hat{P}_{\theta,j+c} , & j = 0...L-c \\
\hat{P}_{\theta,j-L}, & j = L-c+1...L-1 
\end{cases} \quad \text{Circular shift}
\]

\( L \) must be chosen according to

\[L \geq \max(L_k), \quad L_k = 2 \cdot \max(c-k-1, N-c+k+1) + 1, \quad k \leq \alpha < k + 1\]

for all \( k \) in the alignment offset interval \( \alpha_{\min} \leq k \leq (\alpha_{\max} - 1) \) to avoid circular convolution of the translated projection data.

The discrete Fourier-transform of the zero-padded and shifted projection data is calculated using FFT according to

\[
\hat{Q}_{\theta,j} = \sum_{j=0}^{L-1} e^{-i2\pi j/L} \hat{q}_{\theta,j}, \quad J = 0...L-1
\]
Let the discrete spatial angular frequency be defined as

\[
\omega_j = \begin{cases} 
\frac{2\pi J}{L}, & J = 0, \ldots, \frac{L}{2} - 1 \\
\frac{2\pi (J - L)}{L}, & J = \frac{L}{2}, \ldots, L - 1 
\end{cases}
\] (24)

If a function \( f(x) \) has Fourier-transform \( F(X) = \int_{-\infty}^{\infty} e^{-2\pi i X x} f(x) \, dx \) the translation \( g(x) = f(x + a) \) in the spatial domain corresponds to a phase shift \( G(X) = e^{i2\pi \alpha a} F(X) \) in the Fourier domain. In the discrete case a translation of \(-\alpha, \hat{q}^{\alpha}_j\) corresponds to the phase shift \( \hat{Q}^{\alpha}_\theta, j = e^{-i\omega_j} \hat{Q}_{\theta, j} \). Together with Parseval’s theorem \( \sum_{j=0}^{L-1} |q_j|^2 = \frac{L}{L} \sum_{j=0}^{L-1} |\hat{Q}_j|^2 \) and the knowledge that the Fourier-transform of a Real function is Hermitian we can write equation (19) as

\[
MSD = \frac{1}{L} \sum_{j=0}^{L-1} \left| e^{-i\omega_j} \hat{Q}_{\theta, j} - e^{i\omega_j} \overline{\hat{Q}}_{\theta+\pi, j} \right|^2
\] (25)

where \( \overline{Q} \) is the complex conjugate of \( \hat{Q} \). The TOPfft method calculates \( \alpha \) giving minimum MSD. This is done using the Newton-Raphson, NR, method to solve \( MSD = \frac{\partial MSD}{\partial \alpha} = 0 \).

There might be several local minima or maxima of MSD where the derivative is zero. In order to find the global minimum and a starting point to the NR method we initially calculate \( MSD_k \) for all \( \alpha_k = \Delta \alpha k \) in the alignment offset interval \( \alpha_{\text{min}} \leq \alpha_k \leq \alpha_{\text{max}} \) where \( \Delta \alpha \) is sufficiently small to ensure that the NR method converges to the global minimum. The \( \alpha_k \) giving minimum \( MSD_k \) is used as starting point in the \( MSD = 0 \) calculations. The first and second order derivative are calculated according to

\[
\begin{align*}
MSD' = \frac{\partial MSD}{\partial \alpha} &= 4 \frac{L}{L} \sum_{j=0}^{L-1} \omega_j \left( a_j \cos(2\alpha \omega_j) + b_j \sin(2\alpha \omega_j) \right) \\
MSD'' = \frac{\partial^2 MSD}{\partial \alpha^2} &= 8 \frac{L}{L} \sum_{j=0}^{L-1} \omega_j^2 \left( -a_j \sin(2\alpha \omega_j) + b_j \cos(2\alpha \omega_j) \right)
\end{align*}
\] (26)

where

\[
\hat{Q}_{\theta, j} \hat{Q}_{\theta+\pi, j} = a_j + i b_j , \quad a_j, b_j \in \mathbb{R}
\]

The NR algorithm finds the \( MSD' \) root iteratively using \( \alpha_{n+1} = \alpha_n - \frac{MSD'}{MSD''} \) repeated until \( |\alpha_{n+1} - \alpha_n| \) is sufficiently small.
BASELINE RESTORATION

In the X-ray CT applications projection data is calculated using a measured signal, $I$ (absorbed X-ray energy in detector or number of detected photons), and a calibration signal, $I_0$ (air scan), for each detector.

The projection data is calculated as $\hat{P} = -\log\left(\frac{I}{I_0}\right)$ for each detector and projection.

If the detector sensitivity is changed by a factor $\gamma_s$ or X-ray intensity is changed by a factor $\gamma_0$ since the last calibration the projection data is biased by a system error

$$R_{\theta,j} = -\log\left(\frac{\gamma_s}{\gamma_0}\right) = -\log\gamma_s + \log\gamma_0$$

Detector sensitivity, $\gamma_s$, fluctuations might originate from in temperature variation or X-ray spectral variation. The X-ray intensity fluctuations, $\gamma_0$, are mainly a function of projection angle $\theta$ due to intensity variations with time. These fluctuations can be compensated using the edge detectors as reference ($I = I_0$) or using the property that the integrated projection data for any projection angle is constant. Beside the intensity variations with $\theta$ the X-ray intensity as a function of $t$ (or detector position) tends to vary with time. The $\gamma_s$ and $\gamma_0$ fluctuations as a function of $t$ are normally small but they might cause significant COR determination errors.

The best way to get small system errors in the alignment procedure is to make a separate alignment procedure using sample projection data prior to scanning. If a method is used which rely on few projections this alignment scan has a low cost in time and dose to the object and the system errors are minimised. It is also possible to align the COR to the X-ray focus and detector in an optimal way. Some reconstruction software also requires projection data to be aligned with a specific alignment offset, usually $\alpha = 0$. If the scanning equipment cannot be modified to implement a separate alignment procedure the actual COR can be determined from scanned projection data and used later in the reconstruction procedure (if supported by the reconstruction software).

Another source of system errors in µCT is detector non-linearity. We can correct for detector non-linearity if each individual detector’s characteristics are known. These corrections can be dead-time corrections for photon counting systems and lookup tables for semiconductor detectors. These corrections are hard to get correct and often contribute significantly to the system error.

The TOP methods tend to be less sensitive for these system errors than the COG methods. To further improve alignment accuracy in presence of system errors an iterative Baseline Restoration, BR, technique can be combined with the TOP methods.

Assume that an opposite projection pair has system errors $R_{\theta}(t)$ and $R_{\theta+\pi}(t)$. A stationary system implies $R_{\theta}(t) = R_{\theta+\pi}(t)$. Let the projection data error term consist of
the system error and a noise term \( \varepsilon_{\theta,j} = R_{\theta,j} + \eta_{\theta,j} = R_{\theta}(t_j) + \eta_{\theta}(t_j) \) (quantum, electronic and other types of random noise).

If \( \alpha \) is known we can write the estimated TOP difference in equation (18) as

\[
\hat{D}_{\theta,j} = R_{\theta,c+j}^\alpha - R_{\theta+c-1}^\alpha + \hat{\eta}_{\theta,c+j}^\alpha - \hat{\eta}_{\theta+c-1}^\alpha
\]

(27)

The estimation errors are included in the \( \hat{\eta}_{\theta,j}^\alpha \)-terms.

**Figure 6.** Baseline Restoration principle (see text for explanation).

The BR-technique approximates the TOP system error difference, \( D_\theta = R_\theta(t) - R_{\theta+\pi}(-t) \), with an \( l \) order polynomial \( \beta(t) = c_l t^l + c_{l-1} t^{l-1} + \ldots + c_0 \). \( \beta(t)/2 \) is the baseline.

For a given \( \alpha \) the baseline is determined using a Minimum Mean Square Error, MMSE, fit to \( \hat{D}_{\theta,j}^\alpha \)

\[
MSE = \sum_{j=-k}^{k} \hat{D}^\alpha_{\theta,j} - c_i (\Delta t)^j - c_{i-1} (\Delta t)^{j-1} \ldots - c_0,
\]

(28)

\[
K = \min(c - k - 2, N - c + k), k \leq \alpha < k + 1, k \in \mathbb{Z}
\]
where \( \hat{D}_{\theta,j}^\alpha = \hat{P}_{\theta,c+j}^\alpha - \hat{P}_{\theta,c-j}^\alpha \) is calculated using linear interpolation according to equation (20).

The MMSE calculations lead to an \( l + 1 \) order linear equation system solved using Gaussian elimination. If the system error is stationary and odd it can be determined using \( \hat{R}_0(t) = \hat{R}_{\theta+c}(t) = \hat{\beta}(t)/2 \). Even if the system error is varying with \( \theta \) or not odd, \( \hat{\beta}(t)/2 \) can be used to eliminate COR errors for each pair opposite projections.

Projection data is modified prior to COR calculation according to

\[
\begin{align}
\bar{\hat{P}}_{\theta,j} &= \hat{P}_{\theta,j} - \hat{\beta}([j - c + \alpha]\Delta t)/2 \\
\bar{\hat{P}}_{\theta+\pi,j} &= \hat{P}_{\theta+\pi,j} + \hat{\beta}([c - j - \alpha]\Delta t)/2
\end{align}
\]

(29)

The BR and alignment algorithm's are repeated iteratively until the changes in \( \alpha \) from one iteration to another is sufficiently small \( |\alpha_n - \alpha_{n-1}| \leq \alpha_{\text{stop}} \) or a maximum number of iterations is performed \( n = n_{\text{stop}} \).

The BR alignment procedure is:

0. Set \( \alpha \) to a initial value \( \alpha_0 \) and iteration number, \( n \), to 1. \( \{\hat{P}_{\theta,j}, \hat{P}_{\theta+\pi,j}\}_0 \) is the original set of measured opposite projection data.

1. Calculate baseline restoration using \( \alpha_{n-1} \) and \( \{\hat{P}_{\theta,j}, \hat{P}_{\theta+\pi,j}\}_{n-1} \), The result is \( \{\bar{\hat{P}}_{\theta,j}, \bar{\hat{P}}_{\theta+\pi,j}\}_{n-1} \).

2. Calculate alignment using modified data, \( \{\hat{P}_{\theta,j}, \hat{P}_{\theta+\pi,j}\}_n = \{\bar{\hat{P}}_{\theta,j}, \bar{\hat{P}}_{\theta+\pi,j}\}_{n-1} \), The result is \( \alpha_n \).

3. If \( (|\alpha_n - \alpha_{n-1}| > \alpha_{\text{stop}} \) and \( n < n_{\text{stop}} \) then increment iteration number, \( n = n + 1 \) and repeat from 1, otherwise alignment is finished.

This iterative technique using BR and a TOP alignment method normally converges rapidly with a reliable alignment offset \( \alpha_n \) if the baseline polynomial is a good approximation to the opposite projection system error difference. The COG alignment methods are considerably more sensitive to system errors and the BR stage often has so much impact on the COG calculations that \( \alpha_n \) don't converge at all or to a false value.
RESULTS

The four presented methods have been tested and compared using both simulated projection data (with known COR and noise-level) and scanned projection data. Figures 7 and 8 illustrate one example using simulated projection data. The simulated data consists of sets with parallel projection data of a filled tube with some low and high density objects of different size and shape. Each projection data set has \( N = 255 \) samples per projection and \( M = 128 \) projection angles with \( \Delta \theta = \frac{2 \pi}{128} \) (64 opposite projection data pairs). To examine the sensitivity to noise of the different methods, projection data with different quantum noise levels are simulated - without noise, with quantum noise levels \( I_{\text{air}} = 10^6 \) and \( I_{\text{air}} = 10^4 \) photons per detector in air, respectively. Figure 7A, B and C show the first projections, \( \theta = 0 \), of the projection data using the three different quantum noise levels. Figure 7D shows a reconstruction of the projection data (no noise). To estimate the alignment error \( \sigma_{\alpha} \), the standard deviation of \( \alpha \), COR is estimated using all the available 64 opposite projection data pairs with the COGopp, TOPlin and TOPfft methods. The COGsin method uses 64 projections (\( \Delta \theta \) spacing) for each COR estimation (32 times more data than the other methods) and \( \sigma_{\alpha} \) is calculated using COR estimates from 9 overlapping projection data subsets; \( i = 1..64 \), \( i = 9..72 \) up to \( i = 65..128 \). Figure 8 shows the alignment results with simulated alignment offset \( \alpha_{\text{sim}} = 0 \) and \( \alpha_{\text{sim}} = 0.25 \) (quarter offset) respectively.

![Figure 7A](image1.png)  
**A)** First projection, no noise.  

![Figure 7B](image2.png)  
**B)** First projection, \( I_{\text{air}} = 10^6 \).  

![Figure 7C](image3.png)  
**C)** First projection, \( I_{\text{air}} = 10^4 \).  

![Figure 7D](image4.png)  
**D)** Reconstructed image from data in A.

FIGURE 7. Simulated projection data.
A) no noise, $\alpha_{sim} = 0$.

B) no noise, $\alpha_{sim} = 0.25$.

C) $I_{air} = 10^6$, $\alpha_{sim} = 0$.

D) $I_{air} = 10^6$, $\alpha_{sim} = 0.25$.

E) $I_{air} = 10^4$, $\alpha_{sim} = 0$.

F) $I_{air} = 10^4$, $\alpha_{sim} = 0.25$.

FIGURE 8. Alignment results with simulated projection data, $\alpha_{sim} = 0$ and $\alpha_{sim} = 0.25$. Vertical axis is calculated alignment offset, $\alpha$, with minimum ($\wedge$), maximum ($\vee$), mean ($\times$) values and ± one standard deviation, $\sigma_{\alpha}$.

It is clear that the methods shouldn’t be compared to each other with simulated data using the lowest noise-levels since the alignment errors are small and more due to simulation errors, partial volume effect etc. than method errors. With $\alpha = 0$ and without noise all the opposite projection methods (COGsin, TOPlin and TOPfft) have zero error because all simulated opposite projection pairs are identical but reversed. With $\alpha = 0.25$ we have a small error even without noise using the opposite projection methods. The COGsin method has a small ($\sigma \sim 0.01$) error even without noise or with low noise levels but this error is approximately the same for all $\alpha$. The COG methods seems to be more sensitive to noise then the TOP methods.

Comparing the TOPlin and TOPfft methods with $\alpha = 0.25$ (worst case) and higher noise-levels clearly shows the improvement of using the FFT-method instead of linear interpolation.
The second example uses a set of projection data scanned with the µCT equipment at the department of Radiation Physics. The scanned object is a pencil and the rebinned projection data is a set of data with \( N = 231 \) samples per projection, \( \Delta t = 50\mu m \) and \( M = 360 \) projection angles with \( \Delta \theta = \frac{2\pi}{360} \). 60 opposite projection data pairs (COGopp, TOPlin and TOPfft methods) or 10 set of 60 projections (\( \frac{\pi}{60} \) spacing, COGsin method) are used to estimate alignment offset error. Correct alignment offset is \( \alpha \approx 0.6 \).

![Figure 9](image)

**A)** First projection.  **B)** Reconstructed image.

**FIGURE 9.** Scanned projection data - pencil.

In this example using real data with rather high noise level (mainly quantum noise), small but not negligible detector calibration and non-linearity errors clearly show the improved precision achieved using the TOP methods compared to the COG methods. Using the TOP methods with this data we can obtain the COR with a standard deviation of approximately 2µm (1/25 of the ray width).

![Figure 10](image)

**FIGURE 10.** Alignment result, scanned projection data - pencil. \( \alpha \approx 0.6 \). Vertical axis is calculated alignment offset with minimum (\( \leftarrow \)), maximum (\( \rightarrow \)), mean (\( \leftarrow \rightarrow \)) values and ± one standard deviation, \( \sigma_\alpha \).

The third example is a scan from an industrial µCT equipment with bad calibration mainly due to variations in X-ray intensity profile after several hours of scanning. The object is a resolution (both spatial and contrast) phantom made of steel with both high and low density contrast objects of different size.

The rebinned projection data set have \( N = 940 \) samples per projection, \( \Delta t = 34\mu m \) and \( M = 258 \) projection angles with \( \Delta \theta = \frac{\pi}{258} \) (from 264 fanbeam projections with \( \Delta \beta = 0.7^\circ \)).

Figure 11 shows the first projection and a reconstruction of the projection data.
Figure 12 illustrates the baseline restoration using this projection data and $\alpha = 0.185$. This example clearly shows the robustness of the TOP methods together with baseline restoration.

**FIGURE 11.** Scanned projection data - resolution phantom.

**FIGURE 12.** Scanned projection data - resolution phantom. Alignment using baseline restoration.
Table 1 lists the alignment results using the presented methods (no alignment errors are calculated since we have only data from 180°). We cannot use the COG methods with this data since the estimated COR error is > 5. The TOP methods without baseline restoration works better with a COR error < 1 but the big improvement comes with baseline restoration (TOP methods only). The BR parameters used in this example is baseline order \( l = 3 \), stop condition \( \alpha_{stop} = 0.001 \) and max. number of iterations \( n_{stop} = 10 \). Correct alignment offset is in the range \( \alpha = 0.1...0.2 \).

<table>
<thead>
<tr>
<th>Baseline restoration</th>
<th>COGopp</th>
<th>TOPlin</th>
<th>TOPfit</th>
<th>COGsin</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>5.542</td>
<td>0.779</td>
<td>1.080</td>
<td>6.995</td>
</tr>
<tr>
<td>yes</td>
<td>-</td>
<td>0.073</td>
<td>0.185</td>
<td>-</td>
</tr>
</tbody>
</table>

**TABLE 1.** Alignment results, scanned projection data - resolution phantom.
CONCLUSIONS

All the presented methods can be used to determine the Centre Of Rotation, COR, from projection data.

The Centre Of Gravity, COG, method presented in [Hogan93] is improved by a new way to obtain the $\phi_{COG}$ giving Minimum Mean Square Error, the COGsin method. Using the COGsin method an alignment offset, $\alpha_i$, can be estimated for each projection angle $\theta_i$ and parallel projection data from less than 180° is sufficient to determine COR. The other presented methods need at least one pair of parallel opposite projections, separated exactly 180° and a fixed alignment offset for all projection angles.

A simplified COG method, the COGopp method is also presented giving an easily implemented and simple way to determine COR using projection data.

The COG methods are sensitive to detector calibration, detector non-linearity, X-ray intensity profile changes and other system-errors.

The introduced Translated Opposite Projection, TOP, methods improve alignment precision in presence of system errors. The TOP-methods can also be used together with a Baseline Restoration, BR, technique to further improve the alignment of data with system errors.

The TOPlin method using linear interpolation to estimate the translated opposite projection is easily implemented and normally achieves sufficient alignment precision with moderate computing efforts.

The TOPfft method improves alignment precision but is more complex to implement, need more experience to tune (select proper parameters) and need more computing power.

These methods and especially the TOP methods have been tested thoroughly and are used successfully on the µCT equipment at the Department of Radiation Physics and a few other sites working with µCT.
REFERENCES