Bragg-Gray Dosimetry: Theory of Burch

Gudrun Alm Carlsson

Department of Medicine and Care
Radio Physics
Faculty of Health Sciences
I. Introduction

The theoretical approach to Bragg-Gray dosimetry is: a Bragg-Gray cavity is a cavity (detector) so small that, when inserted into a medium, it does not disturb the fluence of charged particles existing in the medium.

This means that the ideal Bragg-Gray cavity (detector) is one of infinitesimal dimensions, a "point" detector. In practice, such detectors do not exist but many real detectors may, in a first approximation, be treated as Bragg-Gray detectors to a high degree of accuracy. Corrections needed (so called perturbation corrections) to account for the deviation of the signal from a practical detector from that of an ideal one has been treated by, e.g., ICRU 1984, Alm Carlsson, 1985, Svensson and Brahme 1986, Alm Carlsson 1987.

Derivation of "perturbation corrections" needs careful consideration and understanding of the ideal case, i.e., that from which deviations are to be corrected for. The ideal case of a Bragg-Gray detector has been treated by Bragg 1912, Gray 1936, Laurence 1937, Spencer and Attix 1955 and Burch 1955.

The formulation of Bragg-Gray theory by Spencer and Attix has found wide practical application and has been treated in detail elsewhere (Alm Carlsson, 1978). The theory of Burch treats the same problem as did Spencer and Attix, viz., the significance of generation and slowing down of delta-particles in both medium and detector. Burch treated the problem in considerable detail but didn't find a solution for practical calculations. From a physical point of view, however, there is much to learn from Burch's approach. Also, his treatment of so called track ends, evaluated in some detail
by Burch 1957, has been adapted in later versions of the Spencer-Attix formulation of Bragg-Gray theory (Nahum 1978, ICRU 1984).

II. Short review of earlier theories

Bragg

Bragg 1912 discussed the possibility to use the ionization in a small air volume as a measure of the electron fluence (or in the terminology of Bragg "the density of β-rays") in the surrounding medium. Bragg was interested in estimating the ranges of electrons in different media. He argued that the electron fluence in a photon irradiated medium was equal to the product of the number of electrons emitted per unit volume and their ranges and the ionization in a small air volume to be proportional to this product.

Note that the last statement above requires that the contribution to the ionization from electrons liberated by photons in the air is negligible. This volume only "senses" the electrons generated in the surrounding medium which is possible provided it is sufficiently small: Let the dimensions of the air cavity be $\Delta l$ (a cavity diameter). The ionization from electrons liberated by photons in the cavity is proportional to $\Delta l^3$ (number of electrons released) times $\Delta l$ (length of travel in the cavity before escaping from it). The ionization from electrons entering the cavity from the surrounding medium is proportional to $\Delta l^2$ (the cross-section area of the cavity) times $\Delta l$ (length of travel in the cavity). Thus the quotient between the ionization caused by electrons released by photons within it and those entering from outside is proportional to $(\Delta l)^4/(\Delta l)^3 = \Delta l$ which approaches zero as $\Delta l$ approaches zero.

Gray

Gray 1929 was the first to formulate a quantitative theory for the relation between the ionization per unit volume of a small gas cavity and that in the surrounding medium.

Gray based his derivation of this relation on comparing the gas cavity with an equivalent volume in the undisturbed medium, Fig 1.
Fig 1: Gray based his theory on comparing the gas cavity (detector) with an equivalent volume in the undisturbed medium: electrons entering the two volumes at equivalent positions travel in straight lines and lose the same energy in traversing them.

The electrons were assumed to travel in straight lines through the volumes, Fig 1. The ratio of the equivalent straight lines $l_{det}$ and $l_{med}$ in the detector and the equivalent medium volume respectively could be identified with the inverse ratio of the linear stopping powers

$$l_{det} \left( \frac{dT}{dx} \right)_{det} = l_{med} \left( \frac{dT}{dx} \right)_{med}$$

$$\frac{l_{det}}{l_{med}} = \frac{(dT/dx)_{med}}{(dT/dx)_{det}}$$

$$= (1)$$

where $dT/dx$ is the linear stopping power.

Electrons were assumed to lose their energies continuously such that the energy lost = imparted energy. Moreover, the stopping power ratio was assumed to be independent on energy.
The gas volume "senses" more incident electrons than the equivalent volume: the ratio of the number of electrons entering the two volumes equals the ratio of the equivalent straight lines squared: \((l_{\text{det}}/l_{\text{med}})^2\) (the area of projection is proportional to the square of the linear dimension of a volume). Since each electron entering one of the volumes (at equivalent positions) imparts the same energy to this volume, the ratio of the energies imparted per unit volume is given by:

\[
\frac{[\varepsilon/V]_{\text{det}}}{[\varepsilon/V]_{\text{med}}} = \left(\frac{l_{\text{det}}}{l_{\text{med}}}\right)^2 \left(\frac{l_{\text{med}}}{l_{\text{det}}}\right)^3 = \frac{l_{\text{med}}}{l_{\text{det}}} \quad (2)
\]

or in terms of absorbed dose

\[
\frac{D_{\text{det}}}{D_{\text{med}}} = \frac{(S/\rho)_{\text{det}}}{(S/\rho)_{\text{med}}} \quad (3)
\]

where \(S/\rho\) is the mass stopping power \(\left(\frac{1}{\rho} \frac{dT}{dx}\right)\).

Gray also discussed the contribution to the ionization in the gas cavity from electrons generated by photons in it. He demonstrated (cf the discussion above) that this could be reduced to a negligible fraction provided the dimensions of the cavity are sufficiently small. In addition, he argued that Eq(2) (and consequently Eq(3)) is valid independent of cavity size provided it acts as a Bragg-Gray detector. Inversely, if a linear relationship between the ionization in a gas cavity and the volume of this cavity is found, this is a demonstration of the validity of the theory as well as an indication that the detector behaves as a Bragg-Gray detector: one that does not disturb the fluence of electrons (charged particles) in the medium.

Note that as soon as electrons liberated by photons in the detector volume contribute a significant part of the total energy imparted to it, the fluence of electrons in the detector can no longer be identical to that in the undisturbed medium (provided it is not medium equivalent with respect to atomic composition in which case Fano’s theorem may invalidate the statement). Therefore, a prerequisite for applying Bragg-Gray theory to a detector in a photon irradiated medium is that the contribution to the absorbed dose from electrons liberated by photons in it is negligible.
Experimental findings

Gray was aware that the theory, Eq (2), could never be exactly valid. He did some experiments to test the constancy of the ionization obtained per unit volume of an air cavity. He found that the requirements for such a cavity to behave like an ideal Bragg-Gray detector depend on photon energy. With unfiltered $\gamma$-radiation (from a Ra-source), a 3 cm$^3$ air volume fulfills Eq (2) with an accuracy of about 1% while with 100 kV X-rays the corresponding volume must not exceed 0.1 cm$^3$ (air within graphite).

Gray 1937 also measured the ionization in an 0.1 cm$^3$ air volume within walls of differing atomic numbers. When irradiated in the same photon beam ($\gamma$-rays from a Ra-source) and with electronic equilibrium in the surrounding wall, the ionization per unit volume increased with increasing atomic number of the wall. Gray demonstrated that this is caused by a decreasing stopping power per electron with increasing atomic number of the stopping medium. Thus, even in cases when Compton scattering is the predominant interaction process, i.e. when the number of secondary electrons released per unit mass is proportional to the number of electrons per unit mass, the equilibrium fluence in a medium of high atomic number is larger than that in a medium of a lower atomic number.

In varying the gas pressure in the ionization chambers, the ionization increased linearly with the pressure in the graphite but not in the lead chamber (Gray 1936). Similar experiments were later carried through by Attix and De la Vergne (subsequently published by Attix et al 1958) using plane parallel chambers and varying the air volume by varying the plate separation from 1 mm to 12 mm. The results were in accordance with those of Gray: the ionization per unit volume of air was a constant with chamber walls of low atomic numbers but increased with decreasing air volume with walls of high atomic numbers. The theoretical explanation of this had to await the theories of Spencer and Attix 1955 (Alm Carlsson 1978) and Burch 1955 taking the effects of delta particle production into account.

Laurence

In his theory, Gray assumed the stopping power ratio to be independent of electron energy. Gray was himself aware of this being an approximation. Laurence 1937
improved the theory by taking into account the energy dependence of the stopping power ratio (continuous slowing down was still assumed). This in turn requires derivation of the energy distribution of the electrons in the medium. In cases with photon irradiated media and with electronic equilibrium existing at the site of the detector, calculation of the energy distribution of the electron fluence is manageable. This was a common presumption in the early theories (Bragg 1912, Gray 1929, 1936, Laurence 1937, Spencer and Attix 1955, Burch 1955) before use of high energy photon and electron beams started. For the latter cases, calculations of electron fluence energy distributions at various points in a medium are now performed using Monte Carlo methods (Berger 1963, Berger and Seltzer 1969, Nahum 1978).

Assuming (with Laurence) continuous slowing down and electronic equilibrium, the differential fluence $\Phi_T$ can be identified with the differential track length $y(T)$ of the emitted electrons (see, e.g. Alm Carlsson 1985, Eq(63) on p 49), Fig 2

![Diagram of an electron slowing down](image)

$$dL = \frac{dT}{(dT/dx)}$$

Fig 2: An electron with initial kinetic energy $T_0$ (released from volume element $dV$) slows down continuously losing energy $dT$ while passing the track length $dL = dT/(dT/dx)$; $dT/dx$ is the linear stopping power for an electron of kinetic energy $T$.

When $dS/dV$ electrons of kinetic energy $T_0$ are emitted per unit volume and continuously slowed down, one has

$$\Phi_T dT = (dS / dV) y(T) dT = \left(\frac{dS}{dV}\right) \frac{dT}{(dT/dx)}$$

(4)
where \( y(T) \, dT = dL \) in Fig 2.

Assuming continuous slowing down, the absorbed dose is the product of charged particle fluence and mass collision stopping power. Thus, for \( D_{\text{det}} \) and \( D_{\text{med}} \) one has in Bragg-Gray conditions (the same charged particle fluence in detector and medium)

\[
D_{\text{det}} = \int_0^{T_{\text{max}}} \Phi_T \left( \frac{S_{\text{col}}}{\rho} \right)_{\text{det}} \, dT \tag{5a}
\]

\[
D_{\text{med}} = \int_0^{T_{\text{max}}} \Phi_T \left( \frac{S_{\text{col}}}{\rho} \right)_{\text{med}} \, dT \tag{5b}
\]

The quotient \( D_{\text{det}}/D_{\text{med}} \) is the quotient between the integrals in Eqs (5a) and (5b). This is the way the so called Bragg-Gray-Laurence theory is depicted (ICRU 1984) as extended also to cases with high energy photon (lacking electronic equilibrium) and electron beams.

Going back to the more specific conditions presupposed by Laurence: electronic equilibrium, negligible bremsstrahlung energy losses and monoenergetic electrons of kinetic energy \( T_0 \) emitted in the medium, Eqs (5a) and (5b) can be written (with \( \Phi_T \) from Eq(4))

\[
D_{\text{det}} = \frac{dS}{dV} \frac{T_0}{(dT/dx)_{\text{med}}} \left( \frac{S}{\rho} \right)_{\text{det}} \tag{6a}
\]

\[
D_{\text{med}} = \frac{dS}{dV} \frac{T_0}{(dT/dx)_{\text{med}}} \left( \frac{S}{\rho} \right)_{\text{med}} = \frac{dS}{dV} \frac{T_0}{\rho_{\text{med}}} \tag{6b}
\]

The quotient \( D_{\text{det}}/D_{\text{med}} \) finally is

\[
\frac{D_{\text{det}}}{D_{\text{med}}} = \frac{1}{T_0} \int_0^{T_0} \left( \frac{S}{\rho} \right)_{\text{det}} \, dT = \left( \frac{S}{\rho} \right)_{\text{med}} \tag{7}
\]
where \( \left( \frac{S}{\rho} \right)_{med}^{det} \) is a weighted mean (weighting factor is the function \( 1/T_o \)) of the mass stopping power ratio \( \left( \frac{S}{\rho} \right)_{med}^{det} \) for detector and medium. Since photons liberate electrons with varying initial kinetic energies, the conversion factor \( D_{det}/D_{med} \) is a suitably weighted mean of that in Eq (7). Values of weighted means of mass stopping power ratios with air as detector material are given by, e.g., Burlin 1968, for various media and monoenergetic electrons, Eq (7), as well as for the energy distributions of electrons liberated by monoenergetic photons.

The conversion factor \( D_{det}/D_{med} \) derived from Bragg-Gray theory is commonly called the "stopping power ratio". The significance of this is demonstrated in Eqs (3) and (7) under two specific conditions. Taking the quotient of the integrals in Eqs (5a) and (5b), the generalized Bragg-Gray-Laurence relation is obtained as a quotient of weighted means of stopping powers: \( \left( \frac{S}{\rho} \right)_{det}/ \left( \frac{S}{\rho} \right)_{med} \). The weighting factor for both averages is the relative energy distribution of the charged particle fluence in the medium at the site of the detector. It may be of some interest to note that the generalized Bragg-Gray-Laurence relation may also be derived as a weighted mean of the stopping power ratio as in Eq (7):

\[
D_{T, det} \, dT = \Phi_T \left( \frac{S_{col}}{\rho} \right)_{det} \, dT \tag{8a}
\]

\[
D_{T, med} \, dT = \Phi_T \left( \frac{S_{col}}{\rho} \right)_{med} \, dT \tag{8b}
\]

Here, \( D_T \, dT \) is the absorbed dose from electrons with kinetic energies in the interval \( dT \) around \( T \).

From Eq (8b), \( \Phi_T \) can be solved as \( \frac{D_{T, med}}{\left( \frac{S_{col}}{\rho} \right)_{med}} \). Substituted for \( \Phi_T \) in Eq (8a), one has for \( D_{det} = \int D_{T, det} \, dT \)

\[
D_{det} = \int_0^{T_{max}} \left( \frac{S_{col}}{\rho} \right)_{med}^{det} D_{T, med} \, dT \tag{9}
\]

and finally
\[ D_{\text{det}} / D_{\text{med}} = \frac{\tau_{\text{max}}}{0} \left( S_{\text{col}} / \rho \right)_{\text{med}} \frac{D_{T, \text{med}}}{D_{\text{med}}} dT = \left( S_{\text{col}} / \rho \right)_{\text{med}} \]  

Eq (10) has the same form as Eq (7). Weighting factor in averaging the mass collision stopping power ratio is (in both cases) the relative absorbed dose to the medium from electrons with kinetic energies in the interval dT around T at the site of the detector.

III. Theory of Burch

Electrons do not lose their energies continuously in slowing down but can occasionally produce \( \delta \)-particles of high kinetic energies. The Bragg-Gray-Laurence relation, Eq (7), can be interpreted as being valid in cases when \( \delta \)-particle equilibrium (Alm Carlsson 1985) exists in both medium and detector. This is a bad approximation when detector and medium differ considerably in atomic composition. Burch 1955 like Spencer and Attix 1955 suggested that the production of \( \delta \)-particles in both medium and detector must be considered in the theory of Bragg-Gray detectors.

Definition of "infinitesimal" cavity

Burch starts with a careful description of an "infinitesimal" cavity yielding the initial assumptions of the theory:

1) the cavity is so small that the number of electrons passing into it with a range less than the cavity dimensions is a negligible fraction of the total number traversing it

2) the energy imparted ("ionization" in Burch's paper) to the cavity by electrons liberated by photons in it is a negligible proportion of the total

3) the energy imparted per unit mass (absorbed dose) in the medium in the immediate neighborhood of the cavity is assumed to be sensibly constant

Note that assumption 3) means that the cavity dimensions are small compared to any absorbed dose gradients in the medium. This is equivalent to saying that the dimensions of the cavity are small with respect to the ranges of the charged particles in the medium: As seen from assumption 3), electronic equilibrium in a photon irradiated medium is not a prerequisite for Bragg-Gray theory. Such assumption as used by Laurence 1937 in
deriving Eq (7) and in the numerical calculations by Spencer and Attix 1955 only serve (as pointed out above) the purpose to make calculations of differential fluences manageable.

Analysis

Burch discusses the case with a gas detector in a solid medium: His arguments are here generalized to an arbitrary cavity (detector). The following quantities are used in the analysis:

\( n_{T,c}dT = \text{number of electrons crossing the cavity with kinetic energies in the interval } dT \text{ around } T \)

\( l_c(T) = \text{average path length traversed in the cavity by electrons entering with kinetic energy } T \)

\( (dT/dx)'_c = \text{average energy imparted to the cavity per unit pathlength traversed by an electron with kinetic energy } T \)

Note that \((dT/dx)'_c\) is not the same as the electron (charged particle) stopping power: it does not include that part of the energy lost which is subsequently carried out of the cavity via photons (bremsstrahlung, characteristic roentgen rays) or energetic secondary electrons (\(\delta\)-particles). Its value depends on the cavity size and shape.

The mean absorbed dose in the cavity \(\overline{D}_{T,c}dT\) from electrons with kinetic energies in the interval \(dT\) around \(T\) is:

\[
\overline{D}_{T,c}dT = \frac{1}{\Delta M_c} \overline{\varepsilon}_{T,c}dT = \frac{1}{\Delta M_c} n_{T,c} l_c(T)(dT/dx)'_c dT \tag{11}
\]

where \(\Delta M_c\) is the mass of the cavity and \(\overline{\varepsilon}_c\) is the mean energy imparted to the cavity.

In the following, Burch takes the same approach as Gray comparing the cavity with an equivalent medium volume (Fig 2) such that

\[
l_c(T)(dT/dx)'_c = l_m(T)(dT/dx)'_m \tag{12}
\]
i.e., the mean energy imparted to the imaginary medium volume when traversed by an electron of kinetic energy $T$ equals that to the cavity when traversed by the corresponding electron ($l_m$ and $\frac{dT}{dx}m$ has the same significance as $l_c$ and $\frac{dT}{dx}c$ but refer to the medium volume).

Assuming the electrons to travel in straight lines through the cavity and medium volumes, the mass $\Delta M_m$ of the latter can be derived

$$\Delta M_m = \rho_m V_m = \rho_m \left[ \frac{l_m(T)}{l_c(T)} \right]^3 V_c = \frac{\rho_m}{\rho_c} \left[ \frac{l_m(T)}{l_c(T)} \right]^3 \Delta M_c$$

(13)

Moreover, $n_{T,c}$ and $n_{T,m}$ are related through

$$n_{T,m} = \left[ \frac{l_m(T)}{l_c(T)} \right]^2 n_{T,c}$$

(14)

The mean absorbed dose to the medium $D_{T,m}dT$ from electrons with kinetic energies in the interval $dT$ around $T$ is given by Eq (11) substituting index $m$ for index $c$. Utilizing Eqs (12) - (14), one has

$$\overline{D}_{T,m}dT = \frac{n_{T,c}}{\Delta M_c} \frac{l_c(T)}{l_m(T)} \frac{\rho_c}{\rho_m} l_m(T) \left( \frac{dT}{dx} \right)_m dT =$$

$$= \frac{n_{T,c}}{\Delta M_c} \frac{(dT/dx)_m}{(dT/dx)_c} \frac{\rho_c}{\rho_m} l_c(T) \left( \frac{dT}{dx} \right)_c dT =$$

$$\frac{1}{\rho m} \left( \frac{1}{\rho dx} \right)_m dT \left( \frac{1}{\rho dx} \right)_c$$

(15)

Integrating over $T$ and taking the ratio $\overline{D}_c / \overline{D}_m$ one has
Burch argues that the mass stopping power ratio in the earlier theories (Gray 1929, 1936 and Laurence 1937) should be replaced by a "mass energy dissipation ratio" $R(T)$ in a theory taking the incontinuous energy losses into account.

Note, that it was assumed that any absorbed dose gradient in the medium in the immediate neighborhood of the cavity is negligible: $D_m$ in Eq (16) can be replaced by $D_m$ for a point at the center of the cavity in the undisturbed medium.

**Difficulties in determining $R(T)$**

Burch discusses in some detail a method to calculate the electron fluence energy distribution (related to $n_{T,c}$ and $n_{T,m}$) in electronic equilibrium taking into account the discontinuous energy losses (generation of $\delta$-particles).

The main difficulty in evaluating the theory quantitatively is determination of the quantities $(dT/dx)'_c$ and $(dT/dx)'_m$, i.e., determination of the mass energy impartation ratio $R(T)$ ("mass energy dissipation ratio" in the terminology by Burch).

Burch discussed the possibility to approximate $(dT/dx)'$ with a restricted stopping power: $(dT/dx)'_c = (dT/dx) \eta_c$. But how should the energy restriction $\eta$ be chosen? One difficulty is that $\eta$ will probably not be the same for cavity and medium. This is elucidated in Fig 3.
Fig 3: A fast electron enters the cavity and passes along the straight line CD. An electron of the same energy enters the equivalent medium volume along the equivalent straight line AB. In both volumes a $\delta$-particle is produced at equivalent positions. Elastic scattering is larger in the cavity and causes the $\delta$-particle to be completely absorbed in it. In the medium volume, the $\delta$-particle escapes carrying some energy out of the volume.

In Fig 3, CD and AB are chosen such that the "local" energy impartation along the tracks is identical. In both volumes, a $\delta$-particle is generated. While it is completely absorbed in the cavity, it is not in the medium volume (due to, e.g., larger elastic scattering in the cavity material). Consequently, the total energy impartation will be larger in the cavity than in the medium volume. However, the prerequisite for choosing the equivalent medium volume was to make the total energy impartation the same as in the cavity. The dimensions of the medium volume must be increased. They must, however, not be increased so much that the $\delta$-particle gets completely absorbed since increasing AB means that the "local" energy impartation along the high energy particle track in the volume is also increased. The cut off energy $\eta_m$ for the restricted medium stopping power should in this case be $\eta_m < \eta_c$. Burch could not solve the problem of finding the right values for $\eta_m$ and $\eta_c$. His theory could not be evaluated numerically but adds to the theoretical understanding of the problems of Bragg-Gray dosimetry.
Track ends

No real detector is infinitesimal. In particular, the assumption that the energy imparted to the cavity from electrons which enter it with ranges which are small or comparable to the cavity dimensions is negligible may be invalidated. Burch 1957 call these electrons "track ends". He argues that in the theory of Spencer and Attix 1955 the track end contribution is neglected. (Is this true?) Burch derives the following approximate expression for $D_{det}$

$$D_{det} = \int_{\eta_m}^{\tau_{max}} \frac{\rho_m}{\rho_c} \left( \frac{dT}{dx} \right)_c N_{T,m} dT + \int_0^{\eta_m} N_{T,m} dT$$

(17)

where $(dT/dx)_m$ is the total medium stopping power and $N_{T,m}$ is the number of electrons per unit mass in the medium which pass through the energy interval from $T$ to $T - dT$. The energy limit $\eta'_m$ is not the same as $\eta_m$ discussed above: $\eta'_m > \eta_m$ since $\eta'_m$ is the kinetic energy of an electron entering from outside the cavity which can just pass the cavity while $\eta_m$ equals the kinetic energy of a $\delta$-particle released within the hypothetical medium volume which terminates at the volume boundary.

The track end problem was recently revived in the calculations of stopping power ratios by Nahum 1978. Spencer and Attix 1955 do take the track end problem into account but the concept of a track end is not quite the same as that discussed by Burch. Nahum combined calculations of mass stopping power ratios according to the Spencer-Attix formulation with the track end contribution according to Burch 1957. His expression for $D_{det}$ is (cf ICRU 1984).

$$D_{det} = \int_{\Delta} \Phi_{T,m} \left( \frac{L_{\Delta}}{\rho} \right)_{det} dT + \Phi_{T,m} \left( \Delta \langle S(\Delta) / \rho \rangle_{det} \right)_{det} \Delta$$

(18)
where \( \frac{L_\Delta}{\rho} \) is a restricted mass collision stopping power, \( \Phi_{T,m}(\Delta) \) is the differential electron fluence in the medium evaluated at \( T = \Delta \) and \( S(\Delta)/\rho \) is the mass collision stopping power for \( T = \Delta \).

**Problem (Exercise)**

Discuss the relation between Eqs (17) and (18). How can the track end term in Eq (18) be derived from that in Eq (17) ?

Guidance: In Nahum 1978 the track end term is given as \( N_\Delta \Delta \) where \( N_\Delta \) is the number of electrons per unit mass in the medium which drop below the energy limit \( \Delta \). Discuss the conditions to be fulfilled for equating

\[
N_\Delta \Delta = \int_0^{\eta_{u}} N_{T,m} \, dT
\]

Derive the relation between \( N_\Delta \Delta \) and the track end term in Eq (18).
REFERENCES


