High Temperature Fatigue Crack Propagation in
INCONEL718

Babak Sharifimajd
Solid Mechanics

Examensarbete
Institutionen för ekonomisk och industriell utveckling
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1 Introduction

All mechanical structures should be designed in such way that they are safe and reliable, i.e. a guarantee that failure will not occur is required. A number of engineering system breakdowns can be attributed to preexistent flaws that caused failure when a certain critical stress was applied. In addition, such defects may grow to critical dimension prior to failure. Subcritical flaw growth is thus important for assuring the efficient functioning of the structure in a certain period of time. In some cases it is very important to guard for flaw growth. This is mainly based on the application of structure, where we note, nuclear power plants, jet engines, turbines, etc.

Crack growth in the rotating parts of turbines and jet engines has been an important topic in the field of Mechanics and Engineering Materials for decades. These components are loaded with a cyclic or constant mechanical force at an evaluated temperature, which will affect the crack initiation and crack propagation considerably. Moreover, increasing the operating temperatures in the turbines and jet engines generally raises their efficiency. On the other hand, higher temperatures increase the risk of higher crack growth rates and/or changes in the material properties. Therefore, to find an efficient operating temperature, one needs to investigate the behavior of the material (used in the discussed components) for such situations. Thus, conditions close to the real applications are to examined in the laboratories. Then, from the obtained results, useful data can be extrapolated to the real application situations.

The work presented here has been carried out during almost one year; firstly as a project course\(^1\) and then as Master Thesis at Linköping University (Spring 2010). Furthermore, it has been done as part of the research Project "Influence of hold times on the fatigue life of nickel-based superalloys" run within the Turbo Power Programme\(^2\). The latter is a research programme aiming at contributing to a sustainable and efficient energy system in Sweden in a medium and long term view, which is financed by the Swedish Energy Agency, Siemens Industrial Turbo machinery Finspång and Volvo Aero Corporation Trollhättan.

In this work, three different series of fatigue experiments on INCONEL718\(^3\) will be discussed. For each type of fatigue experiment, the method of evaluating the raw data obtained at the material laboratory\(^4\) will be mentioned. Moreover, from the obtained results, the fatigue properties of INCONEL718 will be presented in detail. Subsequently, a discussion of the modeling of the fatigue properties of the material in an isothermal condition will be performed. For that, three different type of models will be investigated and compared. Furthermore, a FEM model of a specific specimen used for TMF-testing (not to be discussed here) will be presented by which the stress intensity factor values can be evaluated.

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\(^{1}\) TMPM01 (Project Course) at Division of Solid Mechanics, Linköping University; Fall 2009.
\(^{2}\) http://www.turbokraft.se
\(^{3}\) Super alloy material which is used in Turbines and Jet engines disks.
\(^{4}\) Division of Engineering Materials, Linköping University.
2 Experimental method

2.1 Potential Drop (PD) Technique

To measure the crack size and crack growth rate in a fatigue crack propagation experiments, a large number of techniques are available. The Electrical Potential Drop technique is a widely used, accurate and efficient method for monitoring the crack initiation and crack propagation. This technique delivers estimates of the crack size based on measurement of the potential drop caused by the broken area of specimen. Not requiring visual accessibility, the method is suitable for special cases like vacuum or high temperature environments. In addition, this method can be monitored automatically by computer. The potential drop technique can be used in any conductive material. Other materials can also be tested by attaching a conductive film. There are two different technique used to measure the crack size; the direct current DC technique and the alternating current AC technique. The DC version is used more commonly since the equipment is simpler and has less parameters to be controlled. In the AC technique, the current generated is sinusoidal, with fixed amplitude [1]. In this paper only the DC-technique is discussed. For further information about the electrical potential measurement theory, the reader is referred to [2].

In the DC method, a calibration curve is needed to relate the crack size to a normalized value of the potential drop measured during the test. Since analytical calibrations, based on the solution of the Laplace equation (Equation 2.1) for specific geometry and boundary conditions, only are available for a small number of simple geometries [3], the calibration curves are generally obtained from experimental data.

\[ \nabla^2 (V) = 0 \]  \hspace{1cm} (2.1)

The technique used to measure the potential drop by means of a Wheatstone bridge [4]. Figure 2.1 shows a schematic DC potential system for measuring the crack propagation.

![Figure 2.1 Schematic Diagram of the DC potential System](image)

For measuring of the potential difference in a specimen, a pair of thin wires of the same material as the specimen is welded to the specimen as close to the notch as possible (see Figure 2.2).
Figure 2.2 Two measuring wires close to the initial notch on the specimen

By using two probes, the potential drop is measured. Another pair of reference wires, attached far from the defect is frequently used to improve accuracy (see Figure 2.3.).

Figure 2.3 Reference and notch wires on the specimen

The displacement between these two reference wires should be the same as between those close to the defect. A technique frequently used to improve the accuracy of the measurements is to read the voltage in the active and reference channel with power off or reversing the power polarity. By subtracting these measured values from the values measured with power on, the thermo-electrical effect can be reduced [4-5]. The potential drop measurements are discrete, pulsed in intervals of some seconds, to avoid heating due to the high current values that are passed through the conducting section of the specimen. Normally, the current is only powered on for some milliseconds. This is done by synchronizing the pulse with the testing machine load to ensure that
the current is only on when the crack is opened. There are some parameters affecting the results of DC crack size measurement such as material-resistivity effects, thermal effects and fracture-surface bridging effects [2]. However, none of these sources of error will be discussed in this paper.

2.2 Evaluation the crack sizes from the PD values

The general procedure in the potential drop technique for monitoring the crack propagation will be explained in this section. The electrical potential measured close to the defect and far from it, can be found in each cycle from the output data. From the PD values in each cycle, one can calculate the new crack associated to that cycle (through the Calibration Curve Function see below) and by adding the new crack area to the initial crack area (pre-crack area), the (total) crack area associated to each cycle can be obtained (see Figure 2.4.). To reduce the fluctuation of the test data and to prevent redundant data, the obtained crack areas should be normalized by a weighted mean method [6]. Since the crack areas are known one may by assuming a semi-circular crack shape, calculate the crack length for each cycle.

First of all, in order to reduce the problems associated with thermo-electrical effects, lack of stability in supplying current and changes in the instrumentation or changes of temperature, a normalized value of the potential drop is obtained through:

\[ PD = \frac{PD_{measured}}{PD_{ref}} \]  \hspace{1cm} (2.2)

where \( PD_{measured} \) is the potential drop measured close to defect and \( PD_{ref} \) is the reference potential drop measured far from the defect. \( PD \) is called normalized potential drop. The crack area is then for each cycle calculated from the PD measurements by

\[ A = A_{precrack} + A_{newcrack} \]

\[ A = A_{precrack} + f_{PD}(PD - PD_0) \]  \hspace{1cm} (2.3)

where \( A \) is total crack area in each cycle , \( A_{precrack} \) is the crack area when the thermal condition is applied (initial crack area). As mentioned previously, \( f_{PD} \) is generally an experimentally obtained calibration function and \( PD_0 \) is the corresponding ratio of potential drop when the crack area is equal to \( A_{precrack} \). In most of the cases, \( f_{PD} \) is considered to be a quadratic function.

\[ f_{PD} = b \cdot PD_{crack} + d \cdot PD_{crack}^2 \]  \hspace{1cm} (2.4)

where \( b \) and \( d \) are the calibration coefficients, and where

\[ PD_{crack} = PD - PD_0 \]  \hspace{1cm} (2.5)

In fact \( f_{PD} \) is equal to the new crack area which has been created during the testing (see Figure 2.4.).
Since the pre-crack is experimentally created from a notch by imposing a cyclic loading, it follows that $A_{precrack}$ can be calculated by the following relation:

$$A_{precrack} = A_{notch} + f_{PD}(PD_0 - PD_{notch}) \quad (2.6)$$

where

$$A_{Notch} = \frac{\pi}{2} a_n \cdot c_n \quad (2.7)$$

and where $a_n$ and $c_n$ are the notch dimensions (see Figure 2.5.).

Finally one ends up with the following formula for the crack area for each $PD$ value:

$$A = \frac{\pi}{2} a_n \cdot c_n + b \cdot (PD_0 - PD_{notch}) + d \cdot (PD_0 - PD_{notch})^2 + b \cdot (PD - PD_0) + d \cdot (PD - PD_0)^2 \quad (2.8)$$

The first 3 parts of this equation are constant during all measurements since only $PD$ is changing. To determine the crack length from the area, one should smooth the obtained $A$ values to reduce fluctuations. Figure 2.6 shows the effect of such a smoothing of the $A$ values.
According to the experimental results, one can consider the crack area as having a *semi-circular* shape with the radius of the crack length. Therefore, the crack length \(a\) can be calculated through:

\[
a = \sqrt{\frac{2A}{\pi}} \tag{2.9}
\]

However, it should be mentioned that the PD technique has the accuracy of 0.01 [mm]. Thus, one has to round the obtained crack lengths (which mostly are calculated in [mm]) to retain only two decimal. By doing so, one may achieve the same crack length for several cycles. Therefore, the mean value of all the number of cycles with the same crack length has to be calculated and then each different crack size should be associated to an identical crack size \(a_i\) for that calculated average number of cycle \(N_i\). Figure 2.7 shows an example which makes this discussion clear.
2.3 Calibration curve

To obtain the experimental Calibration Curve Function, a beach marks technique was used. In this technique, by changing the frequency, the temperature and by introducing different environments, different colors on the cracked surface of the specimen will be obtained, which makes the measurement easier. Figure 2.8 shows the beach marks on one of the samples. It is seen that the semi-circular shape assumption for the crack area is not unrealistic, especially in the middle range (the most interested range) of the crack size. Moreover, during the beach mark experiment, the PD value in each step is recorded. In Table 2.1, the calibration data is seen. By using this data in combination with Eqs. (2.3)-(2.5), we get the results found in Table 2.2.

<table>
<thead>
<tr>
<th>Number of cycles</th>
<th>Rounded crack lengths</th>
<th>Identical crack lengths</th>
<th>Average of number of cycles for each identical crack length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.32</td>
<td>0.32</td>
<td>30.14285714</td>
</tr>
<tr>
<td>10</td>
<td>0.32</td>
<td>0.33</td>
<td>210</td>
</tr>
<tr>
<td>20</td>
<td>0.32</td>
<td>0.34</td>
<td>205</td>
</tr>
<tr>
<td>30</td>
<td>0.32</td>
<td>0.35</td>
<td>310</td>
</tr>
<tr>
<td>40</td>
<td>0.32</td>
<td>0.36</td>
<td>450</td>
</tr>
<tr>
<td>50</td>
<td>0.32</td>
<td>0.37</td>
<td>525</td>
</tr>
<tr>
<td>60</td>
<td>0.32</td>
<td>0.38</td>
<td>625</td>
</tr>
<tr>
<td>70</td>
<td>0.33</td>
<td>0.39</td>
<td>745</td>
</tr>
<tr>
<td>80</td>
<td>0.33</td>
<td>0.4</td>
<td>835</td>
</tr>
<tr>
<td>90</td>
<td>0.33</td>
<td>0.41</td>
<td>945</td>
</tr>
<tr>
<td>100</td>
<td>0.33</td>
<td>0.42</td>
<td>1050</td>
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<tr>
<td>110</td>
<td>0.33</td>
<td>0.43</td>
<td>1110</td>
</tr>
<tr>
<td>120</td>
<td>0.33</td>
<td>0.44</td>
<td>1170</td>
</tr>
<tr>
<td>130</td>
<td>0.33</td>
<td>0.45</td>
<td>1240</td>
</tr>
<tr>
<td>140</td>
<td>0.33</td>
<td>0.46</td>
<td>1325</td>
</tr>
<tr>
<td>150</td>
<td>0.33</td>
<td>0.47</td>
<td>1420</td>
</tr>
<tr>
<td>160</td>
<td>0.34</td>
<td>0.48</td>
<td>1485</td>
</tr>
<tr>
<td>170</td>
<td>0.34</td>
<td>0.49</td>
<td>1550</td>
</tr>
<tr>
<td>180</td>
<td>0.34</td>
<td>0.5</td>
<td>1615</td>
</tr>
<tr>
<td>190</td>
<td>0.34</td>
<td>0.51</td>
<td>1685</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>PD (mV)</th>
<th>Crack length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>240</td>
<td>0.08325</td>
</tr>
<tr>
<td>432</td>
<td>0.95125</td>
</tr>
<tr>
<td>648</td>
<td>1.47845</td>
</tr>
<tr>
<td>814</td>
<td>1.84865</td>
</tr>
<tr>
<td>1102</td>
<td>2.46885</td>
</tr>
</tbody>
</table>

Table 2.1 Data obtained by measuring the beach marks
By making a least square fit, the calibration curve shown in figure 2.9 is obtained. 

<table>
<thead>
<tr>
<th>PD (mV)</th>
<th>Crack length (mm)</th>
<th>A (mm²)</th>
<th>A_{new crack} (mm²)</th>
<th>PD_{crack} (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>240</td>
<td>0.08325</td>
<td>0.010886181</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>432</td>
<td>0.95125</td>
<td>1.421334861</td>
<td>1.410449</td>
<td>0.192</td>
</tr>
<tr>
<td>648</td>
<td>1.47845</td>
<td>3.433367973</td>
<td>3.422482</td>
<td>0.408</td>
</tr>
<tr>
<td>814</td>
<td>1.84865</td>
<td>5.368048841</td>
<td>5.357163</td>
<td>0.574</td>
</tr>
<tr>
<td>1102</td>
<td>2.46885</td>
<td>9.574067322</td>
<td>9.563181</td>
<td>0.862</td>
</tr>
</tbody>
</table>

Table 2.2 Calibration Data
As can be seen, the required coefficients (b, d in Eq. 2.8) are given

\[ b = 5.9917 \left( \frac{mm^2}{V} \right); \quad d = 5.9044 \left( \frac{mm^2}{V^2} \right) \]

Note that the units of crack area [mm\(^2\)] and potential drop [V] should be observed carefully.

### 2.4 Evaluation of the crack growth rates

Other required quantities in the crack propagation study are the crack growth rate \( \frac{da}{dN} \) and the stress intensity factor range \( \Delta K \). From the mathematical point of view, in the case of discrete calculation one can define the former by a forward difference as:

\[ \left( \frac{da}{dN} \right)_i = \left( \frac{\Delta a}{\Delta N} \right)_i = \frac{a_{i+1} - a_i}{N_{i+1} - N_i} \]  

(2.10)

where \( N_i \) is the number of cycles associated with the identical crack length \( a_i \). Considering the stress intensity factor range \( \Delta K_i \), it depends on the applied stress, crack length and geometry of the specimen. In the case of a Semi-elliptic surface crack in a plate, the stress intensity factor can be calculated through [7], [8]:

\[ y = 5.9044x^2 + 5.9917x \]
\[ K_I = \sigma_0 \sqrt{\pi a} \cdot f \left( \frac{a}{c}, \frac{a}{t} \right) \]  \hspace{1cm} \text{(2.11)}

where,

\[ f \left( \frac{a}{c}, \frac{a}{t} \right) = \frac{1}{\sqrt{Q}} \left[ M_1 + M_2 \left( \frac{a}{t} \right)^2 + M_3 \left( \frac{a}{t} \right)^4 \right] \]

\[ Q = 1 + 1.464 \left( \frac{a}{c} \right)^{1.65}; M_1 = 1.13 - 0.09 \left( \frac{a}{c} \right); M_2 = -0.54 + \frac{0.89}{0.2 + a/c}; M_3 = 0.5 - \frac{1}{0.65 + a/c} + 14 \cdot \left( 1 - \frac{a}{c} \right)^{24} \]

where \( c \) is half the surface length of the crack area (see Figure 2.5.). As mentioned before, one can assume that \( c = a \). Thus Equation 2.11 gives:

\[ K_I = \frac{P}{tW} \cdot \sqrt{\pi a} \cdot \frac{1}{\sqrt{Q}} \cdot \left[ 1.04 + 0.202 \left( \frac{a}{t} \right)^2 - 0.106 \left( \frac{a}{t} \right)^4 \right] \]  \hspace{1cm} \text{(2.12)}

where \( t \) is the thickness of the specimen and \( W \) is the width of it (see Figure 2.5). The following table shows a typical dimensions of a used specimen in our case.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>b_n (mm)</th>
<th>t (mm)</th>
<th>( \frac{L_{max}}{2} ) (mm)</th>
<th>( L_{ref} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>08-0521-1</td>
<td>0.088</td>
<td>4.31</td>
<td>0.371/0.373</td>
<td>3.000</td>
</tr>
<tr>
<td>4.11</td>
<td>0.199</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.067</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3 INCONEL718 specimen used at LiTH

For most metals, the crack propagation behavior will take the form illustrated in Figure 2.11 below. The intermediate region (Region 2 in Figure 2.11) is in focus in the current work and can be expressed by Paris' law [7]:

\[ \frac{da}{dn} = C \cdot (\Delta K_I)^n \]  \hspace{1cm} \text{(2.13)}
where \( C \) and \( n \) are the material parameters. By using

\[
\Delta K_i = K_{i_{\text{max}}} - K_{i_{\text{min}}} \quad (2.14)
\]

in Equations 2.12 and 2.14, we get

\[
\Delta K_i = \frac{P_{\text{Max}} - P_{\text{Min}}}{tW} \cdot \sqrt{\pi a} \cdot \frac{1}{\sqrt{24a}} \left[ 1.04 + 0.202 \left( \frac{a}{t} \right)^2 - 0.106 \left( \frac{a}{t} \right)^4 \right] \quad (2.15)
\]

Finally by,

\[
a_{\text{mean}} = \frac{a_i + a_{i+1}}{2} \quad (2.16)
\]

we find the following expression for the mean stress intensity factor range in a load cycle

\[
\Delta K_i = \frac{P_{\text{Max}} - P_{\text{Min}}}{tW} \cdot \sqrt{\pi a_{\text{mean}}} \cdot \frac{1}{\sqrt{24a_{\text{mean}}}} \left[ 1.04 + 0.202 \left( \frac{a_{\text{mean}}}{t} \right)^2 - 0.106 \left( \frac{a_{\text{mean}}}{t} \right)^4 \right] \quad (2.17)
\]
3 Base line (BL) test

3.1 Introduction

Base line tests have been carried out in order to find the crack propagation behavior of the material under a cyclic load with no hold time. The maximum and minimum load levels (load ratio) and the load frequency have been kept constant during the experiments. However, it is to be noted that e.g. in [9] it has been shown that the crack propagation rate in BL tests will decrease in the vacuum which underlines the important role of environment effects (mainly oxidation) on the crack propagation in INCONEL718.

![Figure 3.1 Baseline tests](image)

Figure 3.1 Baseline tests

Six Base line tests were carried out for 3 different temperatures in the laboratory of the Division of Engineering Materials (see Table 3.1).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Applied temperature [°C]</th>
<th>Maximum Applied stress [MPa]</th>
<th>Load frequency [Hz]</th>
<th>R-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>08-0521-2</td>
<td>450</td>
<td>650</td>
<td>0,5</td>
<td>0,05</td>
</tr>
<tr>
<td>08-0521-3</td>
<td>450</td>
<td>650</td>
<td>0,5</td>
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<tr>
<td>08-0521-4</td>
<td>550</td>
<td>650</td>
<td>0,5</td>
<td>0,05</td>
</tr>
<tr>
<td>08-0521-7</td>
<td>550</td>
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<tr>
<td>08-0521-9</td>
<td>650</td>
<td>500</td>
<td>0,5</td>
<td>0,05</td>
</tr>
</tbody>
</table>

Table 3.1 BL tests’ conditions in LiTH

3.2 Crack propagation evaluation for BL tests

To evaluate the crack lengths and crack growth rates in the BL tests, we use the procedure outlined in Sections 2.2 and 2.3.

In the laboratory, for each BL experiment, a computer recorded time, number of cycle, maximum and minimum load and PD value during the experiment (see Figure 3.2). To decrease the number of recorded data, each 10 loading cycle, one observation was saved in the computer. As a matter of fact, the main reason to record data each 10 cycle in the BL tests is that the crack does not grow considerably during each loading cycle.
To evaluate the results, one may use a program like MATLAB or EXCEL. The work in this project has been done by MATLAB programming (Appendix 10.1.1). In Figure 3.3 the crack propagation behavior in a 650°C experiment is shown.

Moreover, it is possible to evaluate the crack growth rate for BL tests by the mentioned MATLAB codes. The result for the crack growth rate in the latter experiment has been shown in Figure 3.4. As mentioned, it is seen that at the beginning of the experiment, the crack does not grow considerably.
3.3 Comparison the crack propagation rates of the all BL tests

It has been seen that temperature has a significant role for the material properties and consequently, for the crack propagation property of INCONEL 718. Accordingly, higher crack growth rates have been observed at higher temperatures. Figure 3.5 shows this fact clearly. Notice that, apart from temperature and applied stress, other parameters and conditions have been kept constant for these BL experiments.
Concerning the applied stress it should be mentioned that at 450°C and 550°C, it was 650MPa, whereas at 650°C the applied stress was 550MPa. It is obvious that the stiffness of material at higher temperatures is decreased, thus, the crack propagation rate under the same applied load will increase dramatically which may cause a few number of loading cycles until fracture or a certain crack size.

Despite the lower applied load at the higher temperatures, it is seen from Figure 3.6 that at 650°C the number of cycles needed to create a crack length of 2,5 mm is very much lower than in the other cases. Moreover, it is noticed that all performed tests have been interrupted at the crack length of 2,5 mm.

Figure 3.6 Crack propagation in INCONEL718 at different temperatures (c.f. Table 3.1)
3.4 Discussion

As mentioned, baseline (BL) tests are performed to record the “pure” fatigue crack propagation behavior. From the experiments carried out, it has been observed that higher temperatures and also higher applied loads make the crack grow faster.

Concerning the fatigue mechanisms, it is seen that the fracture mode in the BL tests is mainly trans-granular (see Figure 3.7 and [10]).

Moreover, a lot of plastic deformations (slip bands) were observed around the crack tips in these experiments, which indicates that the dominating fracture mechanism in the BL tests is cyclic plastic deformation around the crack tip (see Figure 3.8).

![Figure 3.7 BL crack at 650°C](image)

![Figure 3.8 Slip bounds around the crack tip in a BL test](image)
4 Hold time (HT) tests

4.1 Introduction

In real applications, the rotating parts of gas turbines or jet engines, experience a continuous mechanical load (mostly at the maximum level) at an elevated temperature. Therefore, the mechanisms which control the crack propagation are slightly different from the pure fatigue situation (Baseline tests). To simulate such conditions, a hold time can in the experiments be introduced during the maximum loading on the specimen (see Figure 4.1.). Since the crack is open under the maximum load at a high temperature, it is expected to monitor a crack growth also during the hold time. The mechanisms which drive the crack propagation process during hold time may be creep and/or environment related phenomena.

![Hold-time tests configuration](image)

Figure 4.1 Hold-time tests configuration

Eight hold time tests were carried out in the laboratory of the Division of Engineering Materials (see Table 4.1).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>650</td>
<td>90</td>
<td>0,05</td>
</tr>
<tr>
<td>550</td>
<td>650</td>
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<tr>
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<td>650</td>
<td>510</td>
<td>21600</td>
<td>0,05</td>
</tr>
</tbody>
</table>

Table 4.1 Different HT tests performed at the Division of Engineering Materials

4.2 Crack propagation evaluation for HT tests
Similar to the baseline tests, in HT tests, data were saved in a text and/or Excel file. However, in HT tests *PD values* should be recorded during the period of applying maximum stress (since the crack is growing during the hold time). For that, one has to introduce a frequency of recording data which is different from the frequency of the applied load. As an example, in the 90s HT tests, if the data are recorded each 2 seconds, then 45 different values are obtained during the hold time and 1 datum is recorded during loading and unloading, i.e. in total 46 data are recorded in each loading cycle. The recording steps can be seen as the red dots in Figure 4.2.

![Figure 4.2 Data recording in a 90s HT test](image)

### 4.2.1 Evaluation of the crack sizes from the all recorded PD values

To evaluate the crack sizes for the HT tests, one may use the same procedure as discussed in Section 2.2. However, if after calculation of each crack size, a rounding process has to be done, the same crack size will be observed for many successive recording steps (within each loading cycle). Therefore, one may be interested in the raw evaluated data to conclude a general idea about the crack propagation behavior in HT tests. A special MATLAB code (Appendix 10.1.2) was written to evaluate the crack sizes in HT tests for all recorded PD values. Figure 4.3 shows the crack propagation in different HT tests at different temperatures.
Figure 4.3(a,b,c,d,e,f) a, crack propagation for 90s HT at 550°C. b, crack propagation for 90s HT at 650°C. c, crack propagation for 2160s HT at 550°C. d, crack propagation for 2160s HT at 650°C. e, crack propagation for 21600s HT at 550°C. f, crack propagation for 21600s at 650°C (note that, in this last experiment the specimen stood only one cycle!)

From these figures, many interesting aspects can be seen. For example, from a specific crack size (around 1,5 mm), a steps like behavior can be observed that represent sudden crack growth occurring during the unloading-loading. This phenomenon will be discussed more in detail subsequently. Moreover, it is seen that the size of these jumps increases with increasing temperature and hold time duration.

Concerning to the scatter in these graphs it should be mentioned that one of the main causes is the desynchronization between the test equipment and the recording computer. However, this may be solved by a smoothing method (see Figure 4.4). Notice that by smoothing (or filtering) the obtained data, the effect of unloading-loading might disappear.
Figure 4.4 The left hand figure is the smoothed crack propagation for 90s HT at 650°C while right hand figure is the unsmoothed ditto

4.2.2 Evaluating the crack growth rates in HT tests

To evaluate the crack growth rates, one cannot easily use the same procedure as for the BL tests. In HT tests there is as mentioned previously, a difference between the number of loading cycles and observations. Therefore, each row in the recorded data file does not represent one loading cycle. To solve this problem, one may calculate a mean value of the PD for each loading cycle, which thus represents an average crack size during one loading-hold time-unloading sequence (one loading cycle). This has been illustrated in Figure 4.5.

Figure 4.5 A schematic graph of a 90s HT test considering the average crack sizes in each loading cycle

Regarding to the smoothing method used to smooth the obtained crack areas (see Section 2.2 and Figure 2.6), it should be noticed that in the HT tests the difference between the crack sizes (especially between long cracks) are usually significantly large, which eliminates this step in the HT tests evaluation procedure.

As an example, Figure 4.6 shows the crack propagation behavior for 90s HT at 650°C.
4.3 Comparison of the crack propagation rates of the HT tests at the same temperature

As mentioned previously, several mechanisms drive the crack propagation in the HT tests. One of the most important factors is the hold time duration. Regarding to this factor, two important matters were observed. First, the amount of the jumps in crack size (after unloading-loading) are larger in the longer hold time experiments. This phenomenon will be discussed further below, with regard to the damaged zone. However, it is appropriate to mention that the longer the hold time is, the more time is available for the environment to attack the material at the crack tip. Thus during unloading-loading, part of this weakened region will be ruptured suddenly and the crack size will increase substantially. The second important matter is the crack growth rate \( \frac{da}{dt} \) during the hold time. However, the effect of the hold time duration is not fully clear for this part of the crack growth and is a topic for further studies.
Figure 4.7(a) shows the crack propagation with 90s HT at 650°C and (b) crack propagation with 2160s HT at the same temperature. The size of the jumps for the 2160s test is almost 5 times larger than for the 90s test.
Since the amount of crack propagation during unloading-loading is larger for a larger hold time, it follows that the overall crack propagation rate \( \frac{da}{dN} \) increases considerably by increasing the hold time duration.

**Figure 4.8 Crack propagation in hold time tests at 550°C**

4.4 **Comparison of the crack propagation rates in HT tests with the same hold time at different temperatures**

Another important factor that affects the crack propagation rates in the HT tests is temperature. It has been observed that at high temperatures the crack grows faster both during hold time and unloading-loading (see Figure 4.9 and 4.10 below).
Figure 4.9(a) Crack propagation at 550°C with 2160s HT and (b) Same experiment but at 650°C
It is easily seen that the crack propagation rate is dramatically raised at higher temperature. This acceleration in the crack growth at higher temperatures can be described by a change of material properties and by an increased environmental effect. It has been observed that at higher temperatures, INCONEL718 shows a more ductile behavior and that the fracture mode is mainly intergranular [10] (see Figure 4.11).

4.5 Discussion

In general, by introducing a hold time at the maximum load in the fatigue crack propagation experiments, the crack growth rate will increase considerably in comparison to the BL tests (see Figure 4.12).
Figure 4.12 *A comparison between the crack propagation rates in INCONEL718 in BL and 90s HT tests at 650°C*

The main reason for this increase (see also [10]) is the higher amount of environmental attack in the HT tests, which e.g. causes the change of fracture mode seen in the experiments. Moreover, to clarify the differences between the crack growth in BL and HT experiments (trans-granular in BL to intergranular in HT), one may compare the crack growth in one loading cycle as illustrated in Figures 4.13 and 4.14 below.

Figure 4.13 *Difference between crack growth in one cycle in HT and BL tests at 550°C (approximately equal $\Delta K$ value)*
Figure 4.14 Difference between crack growth in one cycle in HT and BL tests at 650°C (approximately equal ΔK value)

It is seen that by increasing the hold time duration, the crack grows to a larger size within the cycle. The difference between the crack propagation rates for the different HT experiments can be seen in Figure 4.15.
Figure 4.15 Crack propagation rates and hold times at different temperature
5 Block tests

5.1 Introduction

As mentioned in the previous section, in real applications, the rotating parts of gas turbines or jet engines, experience a continuous mechanical load (mostly at the maximum level) at an elevated temperature. In addition, during start and stop there is a mechanical load transient, and vital components are therefore subjected to a combination of cyclic and sustained loads.

![Figure 5.1 A schematic figure of a Block test](image)

As it is seen in Figure 5.1, first a simple cyclic load is applied to the pre-cracked specimen until a specific time or a particular crack size is reached. This is followed by one cycle with a specified hold time at sustained load, after which there are, again, cyclic loadings.

By these Block tests, we want to see "what will happen if material which has experienced a hold time test is loaded with a cyclic load again and vice versa".

5.2 Crack propagation evaluation for Block tests

Regarding the Block tests, five different experiments were carried out at the Engineering Materials laboratory, Linköping University as specified in Table 5.1.

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</tr>
<tr>
<td>550</td>
<td>650</td>
<td>21600</td>
<td>0,05</td>
</tr>
</tbody>
</table>

Table 5.1 Block tests performed at the Engineering Materials laboratory, Linköping University
As mentioned previously, in Block experiments, first a simple cyclic load similar to the BL tests is applied to the sample until a specific crack length (~1.12 mm) is reached. After the BL sequence, a hold time at maximum load is applied to the sample until a certain crack length (~1.6 mm) is reached. For a second time another BL sequence is applied to the sample (~2.1 mm) and finally a second HT is applied up to the final crack length 2.5 mm. In short, one can describe a Block test as a combination of two BL tests and two HT tests.

5.2.1 Evaluating the crack sizes for all the recorded PD values

Figure 5.2 shows the crack propagation in a Block test at 550°C and with 90s HT.

![Diagram of crack propagation in INCONEL718 in Block test at 550°C with 90s HT](image)

It can be observed that the crack grows with different rates during the BL and HT sequences. Furthermore, in the Block tests, the same behavior of material at the HT tests (sudden jumps) during the load reversal can be seen (see Figure 5.3).
5.2.2 Evaluation of the crack growth rates in the Block tests

It is always important to evaluate the crack growth rates in the fatigue experiments to be able to compare the obtained results under different conditions and make some conclusions. In the block tests, as mentioned, there are 4 sequences which can be thought simply as two BL and two HT experiments separately. Therefore, one can use the same procedure to evaluate the crack propagation rates as in the BL and HT tests. Particularly, for the first and the second BL sequences, the same MATLAB code to evaluate the BL tests (Appendix 10.1.1) will be used. Notice that for the two HT sequences, one has to determine an average for the PD values in each loading cycle and correlate it to the mean time value of the corresponding cycle (see section 4.2.2).

By this procedure it is possible to evaluate the crack propagation rates for Block tests like Figure 5.4.
In Figure 5.4 the change of the crack growth rate is easily seen during the different sequences.

5.3 Comparison of the crack propagation rates in the Block tests and in the BL and HT tests

In the Block tests, the same mechanical and thermal data have been used as in the BL and HT experiments previously. Therefore, we can expect to see the same behavior (crack propagation) as previously. To investigate this thought, the obtained results of the Block test at 650°C with 90s HT were compared with those of the BL test at the same temperature (also with the same loading frequency and applied stress).
In Figure 5.5 it is clearly seen that there is a good agreement between the BL sequences in the Block test and the BL test performed previously. This shows that, regardless of the Transient Region between each consecutive sequence, material behaves equally under the constant mechanical, thermal and environmental conditions which is very important in the crack propagation study.

In addition, it is interesting to see whether the crack propagation rates obtained from the hold time sections of the Block tests agree with those of the HT test or not. Figure 5.6 shows the results from such an investigation, which it is again seen that there is a good agreement between them.

Figure 5.5 Comparison between the crack propagation rates in INCONEL718 in Block and BL tests at the same conditions.
5.4 Comparison of the crack propagation rates of the Block tests at the same temperature

In HT tests, we discussed the effect of the hold time duration on the crack propagation rate in Inconel718. To investigate this effect in the Block tests, one may compare the crack growth rates obtained from the different Block experiments at the same temperature. Figure 5.7 illustrates this investigation. It is seen that concerning to the first and second BL sequences, all the Block tests have the same crack propagation rate. However, in the Block tests with longer hold time duration sequences, the crack propagation rate during the first and second HT sequences, increases significantly.
Figure 5.7 Crack propagation in the Block tests at the same temperature with the different hold time durations

Under “normal” circumstances, the same crack growth rate in the BL sequences for the Block tests with different HT durations (but at the same temperature) would have been reasonable. The main reason for the crack propagation rate increase for the longer hold time in this case can probably be explained by the damaged zone phenomenon (see 4.3).

5.5 Comparison of the crack propagation rates of the Block tests with the same hold time duration at different temperatures

To investigate the effect of temperature on the crack propagation rate in the Block test, we have compared the two Block tests with the same hold time duration but at different temperatures. It is observed that higher temperature makes the crack growth rate improve dramatically.
Figure 5.8 Comparison between the crack propagation rates obtained from two Block tests with the same HT duration at different temperatures

The reason of this was explained in Section 4.4 where the effect of temperature on the crack propagation rate in HT experiments was discussed.

5.6 Transient Regions

One of the most interesting concepts in the block tests is the transient region between two consecutive sequences. For example, after the first BL sequence, it takes a while to see a stabilized crack growth rate in the first HT sequence. Moreover, after this sequence, the crack growth rate does not drop instantly to the BL level; instead, this decrease takes place over a certain number of BL cycles. In total, there are three different transient regions which have been numbered in Figure 5.9. Each of these regions is discussed in details here.
In this region one can see a developing damaged zone, which takes a while to be fully developed. Therefore, the crack propagation rate increases gradually until reaching the same level as in the HT test.

II. The most interesting region is this region where there is a fully developed damaged zone. When the BL sequence starts the crack grows suddenly at a significantly higher rate than in the “normal” BL test. However, after some cycles the damaged zone will be completely passed by the crack and the crack propagation rate returns to the expected value in BL tests.

III. This region is like the first region, where it takes a while to develop the damaged zone. In the most cases we do not obtain a fully developed damaged zone in the second HT sequence.

5.7 Discussion

As mentioned, Block tests have been designed mainly to investigate the history memory of the material. It was seen that the crack propagation rates in the different sequences of a Block test agree with the crack propagation rates in the corresponding BL or HT at the same conditions. However, there are some regions between them which indicate the existence of a memory in the Inconel718 material, which can be described based on the damaged zone phenomenon. The time it takes to pass the transient regions depends on different parameters such as temperature, hold time duration and applied load. Figures 5.10 and 5.11 show a comparison of the transient region II between two Block tests with 90s HT at 550°C and 650°C. It is easily seen that in the 90s Block test at 550°C there is no transient region at all while at the 650°C this region exists and it takes a time for the crack to pass this zone. Thus, one can conclude that there is a larger damaged zone at higher temperature which agrees with our previous discussion.
Figure 5.10 region II in the 90s Block test at 650°C.

Figure 5.11 region II in the 90s block test at 550°C.

In addition, Table 5.2 summarizes roughly measurements from the different transient regions in different Block tests.
In Table 5.2, $\Delta a$ is the crack size difference at the beginning and end of the region, and $\Delta t$ is the time it takes to pass the transient region. It is generally seen from Table 5.2 that the size of transient regions ($\Delta t$) and ($\Delta a$) are increased by increasing the $t_{HT}$. Moreover, $\Delta t$ is reduced by increasing temperature.
6 FCP Model for the Hold Time (HT) tests

6.1 Background

In the real application of a gas turbine components, there will be more complicated situation which cannot exactly be simulated in laboratories. Moreover, these machineries are usually **Supersensitive** (also expensive); thus, service times should be very restricted and any **flaw** should be detected before re-run. One of the most common flaws in these machines is existence of cracks. Regarding to model the crack propagation behaviour, a Paris law type of equation is commonly used.

In this Section we will try to model the material behavior in HT tests. Figure 6.1 shows a schematic HT test. To investigate more in detail, the loading sequence can be split into two parts.

![Figure 6.1 An HT test](image)

- **Part I**: The hold time part which the sample is under the maximum load for a specific interval of time. During the hold time crack grows due to time dependent mechanisms.
- **Part II**: The unloading-loading (load reversal) part. During this part, crack grows due to fatigue crack propagation.

Regarding the modeling of the material in the HT tests, one can consider the effects of the different mechanisms in Part I and Part II on the crack growth separately or all together. In the case of considering them separately, then one can think of adding these effects to each other to obtain the total crack growth. This type of models can be called an **“additive”** model. On the other hand, if one considers Part I and Part II both together as a “one Part”, then all mechanisms which governs the crack propagation are considered at the same time. Accordingly, many works have been done which based on the additive type of description.

One of the elder studies in this field belongs to J. Gayda [11]. Their work is based on two assumptions:

1. **Part I** in the HT test can be considered as the pure **creep crack propagation**.
2. **Part II** can be represented by a **pure fatigue crack propagation**.

This model will be discussed in detail in 6.2.
T. Nicholas [12] introduced a model also based on the same assumptions in the previous model. This model is very similar to the Gayda’s model.

F.V. Antunes [13] also has introduced a model which in that the crack propagation in a HT tests have been considered as a combination of three mechanisms; Cyclic, Time dependent and a mixture of them. In each step (similar to loading cycle) the maximum value of crack propagation obtained from these three mechanisms will drive the crack propagation process. However, the same assumptions (as Gayda) have also been used here.

A. J. Baker [14] has used specific procedure to evaluate the crack growth in 316L(N). One interesting point of this work is that the crack length has been correlated directly to the time and the interactions of Part I and Part II in the HT experiments have been considered. However, this model needs a lot of material properties to be determined experimentally which makes it difficult to use it in our case.

A. Piard [15] has created a model for evaluating the crack growth rates in INCONEL718 based on the "damaged zone" concept which is not the same with what will be discussed in this work. It should be mentioned that they have performed their experiments in vacuum to eliminate the environmental effect on the crack propagation process.

One of the most interesting works in this field belongs to S. Kruch [16]. They have introduced a model to represent the crack propagation in some metals at 650℃ based on fatigue-creep-environment crack growth. The unique point in this model is introducing several material parameters in order to describe the history of the various processes that operate close to the crack tip during hold time. That is, the “Memory” of the material is taken in account through three parameters:

- A threshold (opening) stress intensity factor for fatigue crack growth.
- A threshold (opening) stress intensity factor to describe the creep crack growth.
- A damaged zone (embrittled by oxidation).

It was observed that after Part I (or Part II), some of the material parameters are changed and need to be updated in order to move on to the next step (other part)of FCP calculation. Thus, this model can represent a complex loading condition. Very similar to the mentioned work, F. Gallerneau, S. Kruch [17] have developed the latter model to a Non-Isothermal loading condition (TMF).

As a matter of fact, all of these studies are based on classical Linear Fracture Mechanics assumptions. Below follows a discussion concerning a Paris law like model and two additive models.

### 6.2 A Paris law like model for the crack growth rates in the HT tests

#### 6.2.1 Background

As mentioned in Section 4.2.2, one can evaluate the crack growth rates in the HT tests by considering a hold time section and a load reversal (Part I & part II) as a “one cycle” (see Figure 6.2).
As seen in Section 4.2.2, by evaluating the crack propagation rate based on this method for an HT test, one can easily obtain a graph of $da/dN$ versus $\Delta K$ for each test. As mentioned before, it has been assumed that Paris law can describe the material behavior in the intermediate (linear) region of the crack propagation. Therefore, based on this assumption and also, from a regression method, one can easily evaluate the material parameters in a Paris law context for each of the HT. In Figure 6.3 it is seen that from a regression technique the equation of the trend line through the experimental data has been obtained which gives the material parameters in this specific test.
By considering a Paris law expression [7], Table 6.1 summarizes the obtained data from the HT experiments concerning to the material parameters.

\[
\frac{da}{dN} = C \Delta K^m \tag{6.1}
\]

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<th>Applied stress [MPa]</th>
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<td>5,2223</td>
</tr>
</tbody>
</table>

Table 6.1 Material parameters in Paris law obtained for the HT tests

Moreover, one can compare the material parameters at each temperature against the hold time duration (see Figures 6.4 and 6.5)

Figure 6.4 Material parameters “m” in Paris law at 550°C obtained for different HT tests
6.3 The Additive model I

6.3.1 Introduction

In the fatigue study, if the influence of other mechanisms such as creep and environment should be considered in the crack propagation, then it is important to recognize how these influences have to be accounted. As mentioned, in HT experiments beside the loading-unloading, the crack can also be driven by creep, environment effect, etc. Furthermore, since these mechanisms affect crack propagation mainly during the hold time (at the maximum load level), any crack growth based on them, altogether, is called time dependent crack propagation (TDCP). On the other hand, crack growth based on the unloading-loading is called fatigue crack propagation (FCP).

In conclusion, crack propagation in the HT tests can be considered as a combination of time-dependent crack propagation and fatigue crack propagation; where the fatigue crack propagation is here thought as the simple cyclic load crack propagation (BL) and the time-dependent part can be considered as a creep process where a constant continuous load is applied to the sample at the elevated temperature. In conclusion, a Hold Time experiment is a combination of a Baseline and a Creep experiment.

The additive model I is based on this assumption, and to determine the total crack propagation, one superimposes the FCP and TDCP linearly as follows [11]

\[
\frac{da}{dn} = \left( \frac{da}{dn} \right)_f + \left( \frac{da}{dn} \right)_{TD} = \left( \frac{da}{dn} \right)_f + \int_{t_1}^{t_2} \left( \frac{da}{dn} \right) dt
\]

(6.2)

where the first term is FCP and can be described by Paris' law

\[
\left( \frac{da}{dn} \right)_f = B \Delta K^m
\]

(6.3)
where B and m are the material constants and should be determined from the experimental data.

The second term of Equations 6.2 is TDPCP. The integral is evaluated over one cycle from \( t_1 = 0 \) to \( t_2 = (1/f) + t_{\text{hold}} \) where \( f \) is the frequency of the applied load and \( t_{\text{hold}} \) is the duration of any dwell. \( \left( \frac{da}{dt} \right) \) is the creep crack propagation rate, in which the effect of environment crack propagation is also included. An expression for \( \left( \frac{da}{dt} \right) \) can be formulated as follows:

\[
\frac{da}{dt} = AK^n \tag{6.4}
\]

where \( A \) and \( n \) should be determined from the experiments and \( K \) is the stress intensity factor during the hold time. By considering \( \Delta K_f \) as a linear function of time, then after integration one obtains (for integration details see Appendix 10.2)

\[
\left( \frac{da}{dN} \right)_{TD} = A\Delta K^n \left[ \frac{Z}{f(n+1)} + \frac{t_{\text{hold}}}{(1-R)^n} \right] \tag{6.5}
\]

where \( Z = \frac{(1-R^{n+1})}{(1-R)^{n+1}} \). Finally the total crack growth rate can be obtained through:

\[
\left( \frac{da}{dN} \right)_{HT} = B\Delta K^m + A\Delta K^n \left[ \frac{Z}{f(n+1)} + \frac{t_{\text{hold}}}{(1-R)^n} \right] \tag{6.6}
\]

6.3.2 Evaluation of the constants

In this model there are four material constants which have to be determined from the experimental data. Regarding the FCP term, one can easily use the crack growth rates evaluated for the BL tests and determine \( B \) and \( m \) as follows.
this curve (which is a line in a log-log diagram), the two material parameters of Paris' law can be found. For instance at 650℃ one obtains 
\[ \frac{da}{dN} = 1,286 \cdot 10^{-7} (\Delta K)^{2.825} \]  
\( \frac{da}{dN} \) in [mm/cyc] and \( \Delta K \) in [MPa√m])

Concerning the TDCP term, \( A \) and \( n \) also need to be determined from the experimental data. However, as mentioned, a creep experiment at the same environment and temperature has to be performed, which we did not have in our case. As a fortune, the 21600s (6hr) HT at 650℃ stood only one loading cycle which makes the test a creep-like experiment. Thus, to evaluate \( A \) and \( n \) the results of crack propagation in this test were used (through the same process as previously). This can be seen in Figure 6.7.

Accordingly, the creep equation (for this specific experiment) can be written as 
\[ \frac{da}{dt} = 6,3354 \cdot 10^{-11} K_{Max}^{5.1722} \]

6.3.3 Comparison between the model and the experimental data

From the obtained constants based on the previous procedure, one can obtain the final equation for the additive model I. As an example, the model for a 90s HT test at 650℃ will be
\[ \frac{da}{dN} = 1,286 \cdot 10^{-7} (\Delta K)^{2.825} + 7,4614 \cdot 10^{-7} (\Delta K)^{5.1722} \]  

By comparing the model with the experimental data for 90 s hold time, it is seen that the agreement is not too bad. However, since questions can be raised against the physical background of the model, it is difficult to draw any deeper conclusions about this result (see Figure 6.8).
6.4 Additive model II

6.4.1 Introduction

In the previous model any interactions between the FCP and the TDCP was not considered. However, recent researches have shown that it might be an interaction between those two parts of crack propagation; e.g. S. Kruch in [16] considered such an interaction through some parameters which govern the crack growth in the following steps of a HT test. However, the procedure of calculating the crack propagation in [16] needs many material properties which must be determined from experiments and/or numerical calculation (FEM). In our case, it was not easy to perform such experiments in order to determine these constants. Thus, it was tried to establish a method which can easily be used and in addition be able to represent the crack propagation behavior in INCONEL718 in HT tests perfectly.

To clarify the basic idea in this model, one may describe the crack growth in each loading cycle in a HT test as follow

\[ \Delta a_{total} = \Delta a_{part II} + \Delta a_{part I} \quad (6.7) \]

where \( \Delta a_{total} \) is the total crack increment during one loading cycle, \( \Delta a_{part II} \) is the crack increment during unloading-loading part (which can be calculated from the previous or next cycle) and \( \Delta a_{part I} \) is the crack increment during the hold time. It is easily seen that the crack length after each loading cycle can be obtained by adding \( \Delta a_{total} \) to the previous crack size. Moreover, the crack growth rate \( (da/dN) \) between each two successive cycles is equal to \( \Delta a_{total} \) (since \( N_2 - N_1 = 1 \)). Therefore, one can conclude

![Graph showing comparison between additive model I with the crack propagation in 90s HT test at 650°C](image)

Figure 6.8 *Comparison between additive model I with the crack propagation in 90s HT test at 650°C*
\[
\left( \frac{\Delta a}{\Delta N} \right)_{HT} = \left( \frac{\Delta a}{\Delta N} \right)_{Part II} + \left( \frac{\Delta a}{\Delta N} \right)_{Part I} \cdot t_{HT} \quad (6.8)
\]

As mentioned, \(\left( \frac{\Delta a}{\Delta N} \right)_{Part II}\) is the crack growth rate during the unloading-loading part which can be considered as fatigue crack propagation in HT tests. To the sake of simplicity, this term can be expressed by a Paris law like expression as follow

\[
\left( \frac{\Delta a}{\Delta N} \right)_{Part II} = B K_{Max}^m \quad (6.9)
\]

where \(B\) and \(m\) are material properties and \(K_{Max}\) is the maximum stress intensity factor obtained from an average crack length (average of the two different crack lengths during Part II). To determine this term, HT experiments were used through the following procedure. As mentioned in 4.2.1., in HT tests (after a certain crack size) crack grows dramatically during Part II (see Figure 4.3). It was observed that these jumps happen during unloading-loading (see Figure 6.5). If one can measure these jumps, \(\Delta a_{Part II}\) has been determined and consequently, \(\left( \frac{\Delta a}{\Delta N} \right)_{Part II}\) between two successive cycles has been determined. However, as mentioned in 4.2.1., these jumps can be observed from a specific crack size which makes the situation below this crack size unclear.

**Figure 6.9**

2160s HT at 550°C. The jumps in the crack propagation happens at the unloading-loading

It was discussed in 4.2.1 that the crack growth during Part II increases by increasing temperature and hold time duration. Therefore, it can be concluded that this increments depends on both Temperature and \(t_{HT}\). Basing on this idea, it can be assumed that the material properties in Equation 6.9 depend on Temperature and \(t_{HT}\) as well. Thus,

\[
B = f(T, t_{HT})
\]

\[
m = f(T, t_{HT})
\]
Regarding the crack growth during Part I, it was discussed in 6.2.4 that in HT test \( \frac{da}{dt} \) is not equivalent to the \( \frac{da}{dt} \) in a pure creep experiment. Thus, it was tried to evaluate this term directly from the HT experiments.

In this work Additive model II will be investigated only at 650°C. Moreover, determination of material parameters in both terms of Equation 6.8 will be discussed.

### 6.4.2 Determination of fatigue crack propagation and time-dependent crack propagation

To evaluate the fatigue crack propagation based on the latter discussion, one has to measure the amount of the increments after each reloading. To automate this process there were a lot of difficulties. First of all, to evaluate the latter increments, one has to use the crack evaluation process described in 4.2.1. It was seen that in this evaluation since every PD recorded is used to evaluate the corresponding crack size, there is a huge scatter (noise) in the evaluated data which makes the automation process very hard. Secondly, as it was seen in Figure 4.4., by a general smoothing, these increments (jumps) will disappear, and the evaluated data will be useless in that case. Finally, it was observed that in the recorded data for HT tests these jumps do not happen exactly at the unloading-loading instant sometimes (according to the recorded time during the test). However, it is believed that there is a problem in the synchronization between the controller of the loading machine and the recorder computer. Overall, an algorithm was defined to distinguish those points which belong to the same cycle. From now the points which belong to the same cycle are called “Group”. After identifying each group, to decrease the fluctuation a trend line based on the regression method is fitted to them and by introducing the start and end time of each group two crack sizes will be determined which represent the crack lengths at the beginning of each HT cycle and at the end of HT cycle, respectively. The results obtained based on this technique can be seen in Figure 6.10.

![Figure 6.10 The crack propagation in INCONEL718 in 90s HT at 650°C obtained from the regression technique](image-url)

---

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From Figure 6.10 the step-like behavior of HT tests is clearly seen. It should be mentioned that these obtained points are just connected linearly to clarify these steps; whereas, it is not clear that how the crack behaves during the hold time. Since the crack sizes are available in format that has been shown in Figure 6.10, one can easily determine $\Delta a_{\text{Part I}}$ and $\Delta a_{\text{Part II}}$ as follows.

From Figure 6.11 a schematic crack propagation in HT tests is shown. Accordingly the increments in Part I and Part II can be determined through the following equations

$$\Delta a_{\text{Part I}} = (a_2 - a_1) \quad (6.10)$$

Consequently

$$\frac{da}{dt} = \frac{\Delta a_{\text{Part I}}}{\Delta t} = \frac{\Delta a_{\text{Part I}}}{T_{\text{HT}}} \quad (6.11)$$

Moreover,

$$\Delta a_{\text{Part II}} = (a_3 - a_2) \quad (6.12)$$

$$\frac{da}{dN/\gamma} = \frac{\Delta a_{\text{Part II}}}{N_2 - N_1} = \Delta a_{\text{Part II}} \quad (6.13)$$

This procedure is called "Crack Increments Technique" and has been implemented in a MATLAB code. This code has been used for evaluating the FCP and TDCP from the HT tests. The results obtained are shown in Figures 6.12 and 6.13.
Figure 6.12 FCP obtained from the crack increments technique for 90s HT at 650°C

Figure 6.13 TDCP obtained from the crack increments technique for 90s HT at 650°C

6.4.3 Evaluation of Paris’ law parameters as a function of hold time duration

As mentioned in 6.3.2., it has been seen that FCP and HTCP in the HT tests are strongly dependent on the temperature and $t_{HT}$. In this section we will try to evaluate the parameters of FCP $B$ and $m$ as functions of $t_{HT}$. It should be noticed that since the HT tests only at 650°C are studied in this work, the temperature dependence cannot be evaluated.

To evaluate functions which correlate $B$ and $m$ (material parameters) to the $t_{HT}$, we need to establish the fatigue crack propagation from the increments technique for all available HT experiments which in our case are only two (90s, 2160s). Since, the crack propagation rates are available, one can (through a regression technique) fit an exponential curve to them and, accordingly, find out the material parameters in that case (see Figures 6.14, 6.15)
According to the Figures 6.14 and 6.15, Table 6.2 can be obtained:

<table>
<thead>
<tr>
<th>$t_{HT}$</th>
<th>$B$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90s</td>
<td>$9,573\times10^{-6}$</td>
<td>2.704</td>
</tr>
<tr>
<td>2160s</td>
<td>$7,3785\times10^{-9}$</td>
<td>5.079</td>
</tr>
</tbody>
</table>

Table 6.2 Material parameters in Paris law, obtained from increment technique
The same procedure has been performed to evaluated time dependent crack propagation \((da/dt)\) in HT tests as follow.
Accordingly, Table 6.3 shows the results for $A$ and $n$ as follow:

<table>
<thead>
<tr>
<th>$I_{HT}$</th>
<th>$A$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90s</td>
<td>5.226E-11</td>
<td>4.62706</td>
</tr>
<tr>
<td>2160s</td>
<td>5.295E-14</td>
<td>6.49286</td>
</tr>
<tr>
<td>21600s</td>
<td>6.331E-11</td>
<td>5.17220</td>
</tr>
</tbody>
</table>

Table 6.3 *Time dependent crack propagation constants obtained from the HT tests*

However, for the sake of simplicity, $A$ and $n$ will be considered as constants. After several investigations, it was concluded that the constants obtained in 90s HT give the best results for TDCP term.

### 6.4.4 Comparison between the model and experimental data

In this part the obtained results of Additive model II will be compared with the HT experiments and the comparisons will be discussed. Before comparison, once more the basic idea in this model is mentioned as follow.
Based on the relations mentioned above, the following results were obtained for 90s HT at 650°C

\[
\left( \frac{da}{dN} \right)_{HT} = \left( \frac{da}{dN} \right)_{F} + \left( \frac{da}{dt} \right)_T \cdot t_{HT} \quad (6.14)
\]

\[
\left( \frac{da}{dN} \right)_{HT} = B K_{Max}^m + A K_{Max}^n \cdot t_{HT} \quad (6.15)
\]

Figure 6.20 Comparison the crack propagation rate at 650°C between 90s HT and Additive model II

It is seen that there is a good agreement only for a specific region \( (K_{Max} \geq 20[MPa\sqrt{m}] ) \) where the FCP has been evaluated (based on crack increment technique). In fact, this model cannot represent the material behavior for the crack propagation close to the threshold stress intensity factor region (where the crack grows rapidly). However, as mentioned at the beginning of this work, we are more interested in the linear part of the crack propagation than close to the threshold \( (K_{Ith}) \) or \( K_{IC} \) regimes.
As well, it is seen from Figure 6.21 that in 2160s HT the model does not agree with the experiment in the primary regime.

One possible explanation could be that the parameters of $\frac{da}{dN}$ were evaluated from test results for cracks with $a>1$ mm. It may well be that these parameters are therefore not valid for the early part of the process, when the crack length is considerably shorter.

**Figure 6.21 Comparison the crack propagation rate at 650°C between 2160s HT and Additive model II**

**Figure 6.22 Crack propagation behavior of metallic materials**
From Figure 6.22 one can define a stress intensity factor range above which the behavior is linear to a good approximation ($\Delta K_{SL}$ starting the linear region). From the tests, it is also seen that $\Delta K_{SL}$ varies with temperature and hold time duration, see Table 6.4.

<table>
<thead>
<tr>
<th>Temp.</th>
<th>Test</th>
<th>$\Delta K_{SL}$ [MPa$\sqrt{m}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>550°C</td>
<td>BL</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>90s</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>2160s</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>21600s</td>
<td>21</td>
</tr>
<tr>
<td>650°C</td>
<td>BL</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>90s</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2160s</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 6.4 Start stress intensity factor range of the intermediate region in different tests

In our case, some constant values for $\Delta K_{th}$ and $\Delta K_{SL}$ have been assumed for all HT tests as follow

$$\Delta K_{th} = 14 \ [MPa\sqrt{m}]$$

$$\Delta K_{SL} = 18 \ [MPa\sqrt{m}]$$

It has been observed that at the beginning of HT tests separation of the crack propagation into FCP and TDCP is difficult. Therefore, for the sake of simplicity, we assume that only one mechanism governs the crack propagation in this region. Accordingly, one can consider one complete loading cycle as a baseline cycle which contains the effect of cyclic loading and hold time at the same time. Then, evaluate $\frac{dN}{da}$ for this region based on this idea and from that, evaluate $B$ and $m$ specifically for this region only. By that, it is seen that these material parameters change with the hold time duration (which is promising). Notice that, since we considered the effects of Part I and Part II at the same time, thus, $\left(\frac{da}{dt}\right)_{TD}$ in this region should be neglected. The results obtained from this method are better than the previous model (see figure 6.23, 6.24.).
As can be seen, the modified version of Additive model II gives a perfect agreement with the experiments at 650°C.
6.4.5 Summary of the model

In summary, this model is based on the interaction of the processes which drive the crack during different parts in a HT test. The material parameters in the fatigue crack propagation were correlated to the hold time duration directly and the material parameters in the time dependent crack propagation were considered constant and were evaluated from 90s HT at 650℃. For short cracks (K<K_{SL}), where the separation of crack growth into fatigue and time-dependent growth is difficult, the crack growth has, instead, been assumed to be fatigue growth with hold time-dependent Paris’ law parameters.

On the other hand, this model has some limitations. The most important problem with this model is having difficulties in evaluating \( \frac{da}{dN} \) from the crack increment technique. For shorter hold time duration and/or at lower temperatures, it is not possible to identify the jumps in Part II. Moreover, this model is a curve fitting base model which will be more reliable if there are proper data available to evaluate the material parameters’ functions.

6.5 Comparison of the 3 discussed models

In total, three models have been evaluated and discussed in this work. Each of these models has been based on different assumptions.

In additive model I, FCP can be taken from the BL experiments and TDCP from the pure creep experiment. It was seen that such assumption could not represent the INCONEL718 crack propagation properties in HT tests. On the other hand, in Additive model II, FCP was evaluated from the crack growth during unloading-loading part (Part II) in HT experiments by crack increment technique and TDCP was obtained from the crack growth rates during hold time period. Besides many limitations which have to be considered, this model could represent the crack propagation behavior of INCONEL718 in HT tests at 650℃.

This should be noticed that these models were studied only based on the available experiments. In the case of having more HT test results, one can conclude more precisely which one of them gives the most reliable results. Figure 6.27 shows a comparison between the obtained results from the models and the experimental data in the case of 90s HT at 650℃.
It is seen that Additive model II gives the best agreement with the experiment in both primary and intermediate regions of the crack propagation.

6.6 Damaged Zone (DZ)

In the HT tests, it has been observed that during the hold time, different mechanisms influence the crack propagation rate; where we note oxidation, creep, etc. Based on the temperature and $t_{HT}$, the effect of these mechanisms are changed. As mentioned in 5.6., during the hold time some phenomena happen in the region around the crack tip which decrease the stiffness and strength of the material in this region (see Figure 6.28). This region is called Damaged zone (DZ). It should be noticed that the damaged zone is not equivalent to the plastic region around the crack tip in the material. However, the damaged zone may contain some plastically deformed grains within.

One of the most important mechanisms in the study of crack propagation in INCONEL718 is environmental crack propagation related. Many studies have shown a great influence of environment on the crack propagation process, compare to the same experiments in the vacuum [9].
The main environmental effect is the material oxidation at the crack tip which leads to a reduction of the mechanical properties in this oxide region. By applying a tensile load, the crack is opened, the oxidation develops and the material becomes more brittle in at this region which is likely to increase the crack growth rate.

If we assume that the damaged zone is completely created by the environment (which is fairly reasonable), the damaged zone is equivalent to the oxide region; then, one can estimate the size of damaged zone (DZS) from a Forman expression as well [18].

\[
l = at^{1/4} \quad (6.16)
\]

where \(l\) is the penetration length of oxide inside the material and \(a\) is a material parameter. Here, the penetration length of oxide is correlated to the "time" available for environmental attack.

As mentioned, within the damaged zone the toughness is reduced. This reduction of material toughness can influence both the Creep and Fatigue crack propagation rates during a HT experiment. Through the following equation one can correlate the toughness of material to the size of damaged zone at specific time (t) [16].

\[
K_c = K_{c0} \left[ 1 - u + u \exp \left( \frac{mx}{l} \right) \right] \quad (6.17)
\]

where \(mx\) and \(u\) are material parameters and \(K_{c0}\) is the minimum value of \(K_c\) at the crack tip, i.e. a completely embrittled material.

Therefore, from the damaged zone size (DZS) one can predict the crack propagation rate in the HT or Block tests even in very complex cases. This is the procedure which has been performed in [16] where the damaged zone size is the most important parameter to evaluate the crack propagation rates in their model.

### 6.7 Final Discussion

This thesis is the final report of almost one year work on studying “High temperature fatigue crack propagation in INCONEL718”. At the beginning, baseline tests were evaluated by a developed MATLAB code. After that, hold time tests were carried out as well, but through a different strategy.
As the next step, block tests were performed which as well were processed. Finally, modeling of the crack propagation behavior has been studied. As a summary here, the head lines will be reviewed briefly.

To study the fatigue crack propagation in INCONEL718 some different experiment were carried out based on the Potential Drop technique. The results were recorded raw data in laboratory and then evaluated by some different MATLAB codes (see Appendix 10.1).

In the Baseline (BL) tests, it was seen that in the case of constant load range and load frequency, the crack growth rates were higher at higher temperature.

In the hold Time (HT) tests, the crack propagation rates were increased dramatically compared to the BL experiments. This was seen especially at the longer hold time durations. There is the same situation in the case of applying higher temperature. Moreover, it was observed that, in HT tests, different mechanisms influence the crack propagation properties in INCONEL718. These mechanisms are mainly cyclic loading, creep and environmental effect (which is mostly oxidation). In addition there is an interaction between the effects of these mechanisms which actually all together make the crack propagation rate to accelerate.

Block tests which are a combination of BL and HT tests, gave very interesting results. Particularly, it was seen that after each block (sequence) there is a transient region which proves the existence of a damaged zone around the crack tip and the manner of it.

Through the modeling work, three different models were discussed. From the mathematical point of view, the evaluations of all those models were based on the curve fitting technique. It was seen that Additive model II gives the best result compared to the experimental data. However, as discussed, one needs to build an iterative model, where in each step the influence from the previous steps is to be taken in account. Finally, in order to set up a reliable mechanism-based crack growth model, it is believed that the introduction of the damaged zone concept is central.
7 The FE model of TMF specimen

7.1 Background

Besides the experiments performed at the Division Engineering of Materials at Linköping University, some Thermo Mechanical Fatigue (TMF) experiments have been carried out at Materials technology in Siemens Industrial Turbomachinery AB, Finspång. To perform these tests, another specimen has been used (see Appendix 10.3). Concerning to the test conditions, here beside the applied load, also temperature varies. The crack growth has been monitored by some optical devices. Therefore, the crack propagation rates can be evaluated easily by dividing the crack size difference to the number of cycles taken for each one. Moreover, these experiments have been carried out in a displacement control manner. Therefore, the applied displacement, crack opening displacement (COD) and the reaction forces at the clamps are known during the experiment. Regarding to the reaction forces, it is noticed that for applying a specific displacement, one needs to introduce a certain load to the machine which can be considered as the reaction forces in the clamp regions. The only variable which has to be evaluated is Stress Intensity Factor Range ($\Delta K_I$).

To evaluate $\Delta K_I$, an analytical solution has to be determined. In this section, the procedure to obtain an analytical solution for evaluating $\Delta K_I$ for this specific geometry will be discussed.

7.2 Evaluation of $K_I$ values

To determine the stress intensity factor $K_I$, one may generally suggest following equation

$$K_I = \sigma_N \cdot \sqrt{\pi \cdot a \cdot f(a)} \ (7.1)$$

where $\sigma_N$ is the nominal stress, $a$ is the crack length and $f(a)$ is a geometrical function. The nominal stress can be considered as $\sigma_N$ far from the crack tip (see figure 7.1).

![Figure 7.1 TMF specimen. Nominal stress region](image)

Notice that in the cases of high applied load and long crack lengths, the latter assumption is not proper any more.

Thus, one can easily determine the nominal stress through

$$\sigma_{yy} = \frac{P_{\text{applied}}}{A} = \frac{P_{\text{applied}}}{W \cdot t} = \frac{P_{\text{applied}}}{0.012 \cdot 0.003} \ (7.2)$$

where $P_{\text{applied}}$ is the applied load to the specimen, $W$ and $t$ are width and thickness of the specimen respectively. Since the experiments are mostly carried out at the displacement control condition, this load can be assumed as the reaction force at the Grips boundaries. However, in the real test
performances the pre-described displacement cannot be directly introduced to the equipments; instead, a load (equivalent to $P_{\text{applied}}$) will be applied to create the necessary displacement.

Another factor to be determined is the geometrical function in the Equation 7.1. This function depends only on the geometry of the sample. As a matter of fact, this function does not exist for this specific TMF specimen discussed here. Form a FE model we will try to evaluate this function which will be discussed in further steps.

### 7.3 FE Model

To evaluate the geometrical function, first we need to evaluate the stress intensity factor values for different crack sizes and applied stress level. For that matter, a 3D FE model of the specimen was created in TRINITAS\textsuperscript{5}. It should be mentioned that one-quarter of the specimen was modeled (since there are two symmetric planes); moreover, the half-length of the specimen in the model was considered from the crack path (tip) to the grips boundary (42 mm). The following figures show the latter model in TRINITAS.

\textbf{Figure 7.2 3D FE model of the one-quarter of the TMF specimen in TRINITAS}

At the crack tip, a finer mesh and special structure have been used to improve the accuracy and also to avoid the mesh dependency of calculations (see Figure 7.3).

\textsuperscript{5} A FEM software developed by Bo Torstenfelt at Soli Mechanics Division, Linköping University.
In addition, a *semi-circular* crack shape was created by using Bézier Splines which can be controlled by 4 points and gives more reasonable results compared to a *straight* crack tip.

Figure 7.4 shows the analysis results obtained in TRINITAS in the case of having a 1\text{mm} crack while a fixed displacement ($7.3 \times 10^{-5}[m]$) has been applied to the top surface of the specimen. This fixed displacement gives $\sigma_{yy} = 317.62$ MPa at the discussed region (as the nominal stress).
It is seen that, TRINITAS gives the stress intensity factor all around the crack tip. Therefore, one can calculate the stress intensity factor at the crack tip by averaging the obtained values across the crack tip. Table 7.1 shows an example of the latter procedure.

<table>
<thead>
<tr>
<th>crack length</th>
<th>1,00</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{I_1} )</td>
<td>56,73</td>
<td>MPa√m</td>
</tr>
<tr>
<td>( K_{I_2} )</td>
<td>49,14</td>
<td>MPa√m</td>
</tr>
<tr>
<td>( K_{I_3} )</td>
<td>46,11</td>
<td>MPa√m</td>
</tr>
<tr>
<td>( K_{I_4} )</td>
<td>45,32</td>
<td>MPa√m</td>
</tr>
<tr>
<td>( K_{I_5} )</td>
<td>45,47</td>
<td>MPa√m</td>
</tr>
<tr>
<td>( K_{I_{\text{mean}}} )</td>
<td>48,55</td>
<td>MPa√m</td>
</tr>
</tbody>
</table>

Table 7.1 Obtained \( K_I \) for the 1mm crack

Notice that, since the model has been created in a millimetric scale, \( K_I \) values should be divided by \( \sqrt{0,001} \) to obtain \( Pa\sqrt{m} \).

### 7.4 Evaluating the geometrical function

In the previous example \( K_I = 48,55 \text{ MPa}\sqrt{m} \), the applied stress was \( \sigma_N = 317,62 \text{ MPa} \), and the crack length was \( a = 1 \text{ mm} \) (which is measured from the end of the notch). From the Equation 7.1, one can easily calculate the geometrical function value as follow

\[
f(a) = \frac{48,55}{317,62 \cdot \sqrt{\pi \cdot 0,001}} = 2,72
\]

This step was repeated for different crack lengths and \( f(a) \) values were calculated. It is reasonable to consider this variable only as a function of crack length (since all specimens have the same geometry). After calculating geometrical function for different crack lengths, one can plot them against crack lengths, then by fitting a curve with a regression method, the mathematical function for \( f(a) \) will be obtained.
Overall, the analytical solution for calculating the stress intensity factor in the TMF specimen will be

$$K_I = \frac{P_{\text{applied}}}{0.012 - 0.003} \sqrt{\pi \cdot a \cdot f(a)} \quad (7.3)$$

where

$$f(a) = 0.0094a^4 - 0.1195a^3 + 0.6177a^2 - 1.4864a + 3.7059 \quad (7.4)$$

It should be noticed that the crack size in Equation 7.4 should be introduced in **millimeter**.

### 7.5 Comparison FEM with the experimental data

To investigate the reliability of the FE model, one can compare the stiffness of specimen with the FE model. The stiffness of the specimen can be determined from the measured displacement and applied load. Table 7.2 shows the obtained result during one test regarding to the stiffness of the TMF specimen.
<table>
<thead>
<tr>
<th>Crack length [mm]</th>
<th>Stiffness [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>84858</td>
</tr>
<tr>
<td>0,5</td>
<td>76284</td>
</tr>
<tr>
<td>1</td>
<td>67711</td>
</tr>
<tr>
<td>1,5</td>
<td>59137</td>
</tr>
<tr>
<td>2</td>
<td>50563</td>
</tr>
<tr>
<td>2,5</td>
<td>41990</td>
</tr>
<tr>
<td>3</td>
<td>33416</td>
</tr>
</tbody>
</table>

Table 7.2 Experimental obtained stiffness in different crack lengths for TMF specimen

It should be noticed that the presented data in Table 7.2 have been determined from

\[
\text{stiffness} = \frac{\text{stress}}{\text{strain}}
\]

where the stress is the reaction force divided by the nominal area \((12 \cdot 3 \text{ mm}^2)\) and strain is measured by a 12-mm extensometer attached over the notch.

The comparison between Table 7.2 and obtained stiffness from the FE model, gave the error less than 10% which is rather acceptable. Thus, one can conclude the calculated \(K_t\) values in the previous section are acceptable as well.
Acknowledgement

The author wishes to acknowledge the valuable discussions with Professor K. Simonsson and Professor S. Sjöström from Linköping University, Solid Mechanics division, Dr. J. Moverare and Professor S. Johansson from Linköping University, Material division and Dr. M. Hörnqvist from Volvo Aero; also Bo Skoog who has contributed to this work by performing the experiments at Engineering Materials laboratory, Linköping University. In addition, special thanks to M.Sc. D. Gustafsson for acting as the main advisor in this project work and Dr. B. Torstenfelt who helped the author extremely in FEM progress.

References


10 Appendices

10.1 MATLAB Codes

10.1.1 BL tests evaluation codes

```matlab
%% INPUT
clear
clc
b = 5.992; % CALIBRATION CURVE DATA
d = 5.904; % CALIBRATION CURVE DATA
T = 4.30; % THICKNESS OF SPECIMEN
W = 10.21; % WIDTH OF SPECIMEN
an = 0.074; % Notch dimension
cn = 0.171/2; % Notch dimension
PD0_pre = 0.231; % input('Enter the average of PDs of pre-crack measurement [v] = ');
PD_pre = 0.319; % input('Enter the last PD of pre-crack measurement [v] = ');

%% PRE-CRACK
PD_precrack = PD_pre - PD0_pre;
A_precrack = b.*PD_precrack + d.*PD_precrack.^2;
Area_pre = A_precrack + pi/2*an*cn;
pre_crack_length = sqrt(2.*Area_pre./pi);

%% CRACK
% IMPORT DATA FROM EXCEL FILE CALLED 'DATA' TO MATLAB
% OBS: IF EXCEL FILE NAME IS DIFFERENT, THE NAME IN THE () SHOULD BE CHANGED
importdata('08-0521-8.xls')

% DEFINE DIFFERENT VECTORS ACCORDING TO DATA COLUMNS IN EXCEL FILE
time = ans.data(:,1);
antal_cycle = ans.data(:,2);
max_force = ans.data(:,3);
min_force = ans.data(:,4);
PD_mat = ans.data(:,5);
PD_ref = ans.data(:,6);
P0 = zeros(1,length(PD_mat-1));

% CALCULATION OF NORMALIZED POTENTIAL DROP PD
for i=1:length(PD_mat)
    PD(i) = PD_mat(i)/PD_ref(i);
end
PD_0 = mean(PD(1:100));

% CALCULATION OF POTENTIAL DROP OF CRACK, AREA OF NEW CRACK
for i=1:length(PD)
    PD_crack(i) = PD(i) - PD_0;
    A_crack(i) = b*PD_crack(i) + d*(PD_crack(i))^2; % A_crack [mm^2] AND PD_crack [v]
    area(i) = A_crack(i) + (pi/2)*pre_crack_length^2;
end

% NORMALIZE AREA AND CYCLES
run_av = 11;
s=0;
run_A = zeros((length(area)-run_av),2);
for i = 6:(length(PD)-5)
s = s + 1;
run_A(s,:) = [ mean( antal_cycle((i - 5):(i+5) ) ) mean( area((i - 5):(i + 5)) ) ];
crack = sqrt(2.*run_A(:,2)./pi);
end

% ROUNDING UP THE CRACK LENGTHS
round_crack = round(100*crack(i))/100;
end

% FINDING THE IDENTICAL VALUES OF CRACK LENGTH AND PUT IN crack_size VECTOR
crack_size = unique(round_crack);
crack_size = sort(crack_size);

% CALCULATING THE NUMBER OF CYCLES FOR EACH IDENTICAL CRACK LENGTH
for i=1:length(crack_size)
cycle_number=[];
time_crack=[];
positions = find(round_crack == crack_size(i));
```

for j=1:length(positions)
    cycle_number(j) = antal_cycle(positions(j));
    time_crack(j) = time(positions(j));
end
cyc_mv(i) = mean(cycle_number);
time_mv(i) = mean(time_crack);
end
% CALCULATING THE AVERAGE OF MAXIMUM AND MINIMUM APPLIED FORCES IN [N]
med_force_max = mean(max_force).*1000;
med_force_min = mean(min_force).*1000;
% CALCULATION OF d_a/dN AND DELTA_K
% End

10.1.2 HT tests evaluation codes based on 4.2.1. method

%% INPUT %%%%%%%%%%%%%%%%%%%%%%%%%
%clear
clc
b = 5.992; % CALIBRATION CURVE DATA
d = 5.904; % CALIBRATION CURVE DATA
T = 4.30; % THICKNESS OF SPECIMEN
W = 10.21; % WIDTH OF SPECIMEN
an = 0.074; % Notch dimension
cn = 0.171/2; % Notch dimension
PD0_pre = 0.250; %input('Enter the average of PDs of pre-crack measurement [v] = ');
PD_pre = 0.29; %input('Enter the last PD of pre-crack measurement [v] = ');
%% PRE-CRACK %%%%%%%%%%%%%%%%%%%%%
PD_precrack = PD_pre - PD0_pre;
A_pre = d*(b.*PD_precrack + d.*PD_precrack.^2);
Area_pre = A_pre + pi/2*an*cn;
pre_crack_length = sqrt(2.*Area_pre./pi);
%% CRACK %%%%%%%%%%%%%%%%%%%%%%%%%

% END
%importdata('mean-08-0521-12.xls')
%DEFINE DIFFERENT VECTORS ACCORDING TO DATA COLUMNS IN EXCEL FILE
time = data(:,1);
number_of_cycle = data(:,8);
%PD_mat = data(:,5);
%PD_ref = data(:,6);
%PD=zeros(1,length(PD_mat));
%CALCULATION OF NORMALIZED POTENTIAL DROP PD
for i=1:length(PD_mat)
    % PD(i) = PD_mat(i)/PD_ref(i);
end
PD=data(:,7);
PD_0 = PD_pre;
%CALCULATION OF POTENTIAL DROP OF CRACK, AREA OF NEW CRACK
for i=1:length(PD)
    PD_crack(i) = PD(i)-PD_0;
    area(i) = A_crack(i) + (pi/2)* (pre_crack_length)^2;
    crack(i) = sqrt(2.*area(i)./pi);
end
time=time(:);
crack=crack(:);
number_of_cycle=number_of_cycle(:);

%EXPORTING DATA TO EXCEL FILE
xlswrite('evaluation_of_all_crack_lengths_HT_tests.xlsx',time,'90s-550C','A2');
xlswrite('evaluation_of_all_crack_lengths_HT_tests.xlsx',number_of_cycle,'90s-550C','B2');
xlswrite('evaluation_of_all_crack_lengths_HT_tests.xlsx',crack,'90s-550C','C2');

---

10.1.3 HT tests evaluation codes based on 4.2.2. method

%% INPUT %%%%%%%%%%%%%%%%%%%%%%
clear
clc
b = 5.992; %CALIBRATION CURVE DATA
d = 5.904; %CALIBRATION CURVE DATA
T = 4.30; %THICKNESS OF SPECIMEN
W = 10.21; %WIDTH OF SPECIMEN
an = 0.074; %Notch dimension
cn = 0.171/2; %Notch dimension
PD0_pre = 0.250;
PD_pre = 0.290;

%% PRE-CRACK %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
PD_precrack = PD_pre - PD0_pre;
A_precrack = b.*PD_precrack + d.*PD_precrack.^2;
pre_crack_length = sqrt(2.*A_precrack/pi);

%% CRACK %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
importdata('mean_2_08-0521-17.xls')
%DEFINE DIFFERENT VECTORS ACCORDING TO DATA COLUMNS IN EXCEL FILE
time = ans(:,1);
antal_cycle = ans(:,2);
max_force = ans(:,3);
min_force = ans(:,4);
PD_mat = ans(:,5);
PD_ref = ans(:,6);
PD=zeros(1,length(PD_mat-1));
%CALCULATION OF NORMALIZED POTENTIAL DROP PD
for i=1:length(PD_mat)
    PD(i) = PD_mat(i)/PD_ref(i);
end
PD_0 = 0.290;
%CALCULATION OF POTENTIAL DROP OF CRACK, AREA OF NEW CRACK
for i=1:length(PD)
```matlab
PD_crack(i) = PD(i) - PD_0;
A_crack(i) = b*PD_crack(i) + d*(PD_crack(i))^2; % A_crack[mm^2] AND PD_crack[v]
area(i) = A_crack(i) + (pi/2)* (pre_crack_length)^2;

end

% NORMALIZE AREA AND CYCLES
run_av = 2;
s=0;
run_A = zeros((length(area)-run_av),2);
for i = 1:(length(area)-1)
    s = s + 1;
    run_A(s,:) = [ mean(antal_cycle((i):(i+1))) mean(area((i):(i+1)))];
    crack = sqrt(2.*run_A(:,2)./(pi));
end

%ROUNDING UP THE CRACK LENGTHES
for i=1:length(crack)
    round_crack(i) = round(100*crack(i))/100;
end

% FINDING THE IDENTICAL VALUES OF CRACK LENGTH AND PUT IN crack_size VECTOR
crack_size = unique(round_crack);
crack_size = sort(crack_size);

%CALCULATING THE NUMBER OF CYCLES FOR EACH IDENTICAL CRACK LENGTH
for i=1:length(crack_size)
    cycle_number=[];
    positions = find(round_crack == crack_size(i));
    for j=1:length(positions)
        cycle_number(j) = antal_cycle(positions(j));
        time_crack(j) = time(positions(j));
    end
    cyc_mv(i) = mean(cycle_number);
time_mv(i) = mean(time_crack);
end

%CALCULATING THE AVERAGE OF MAXIMUM AND MINIMUM APPLIED FORCES IN [N]
med_force_max = mean(max_force).*1000;
med_force_min = mean(min_force).*1000;

%CALCULATION OF da/dN AND DELTA_K
ds = ((med_force_max-med_force_min)./(T.*W))'*100000; % [MPa/sqrt(m)]
ds_max = ((med_force_max)./(T.*W))'*1000000;
da_dN = zeros(length(crack_size)-1,1);
da_medel = zeros(length(crack_size)-1,1);
c_medel = zeros(length(crack_size)-1,1);
Delta_K = zeros(length(crack_size)-1,1);
da_dt = zeros(length(crack_size)-1,1);
time_medel = zeros(length(crack_size)-1,1);
K_MAX = zeros(length(crack_size)-1,1);
for i = 1:length(da_dN)
da_dN(i) = (crack_size(i + 1) - crack_size(i)) / (cyc_mv(i + 1) - cyc_mv(i));
da_dt(i) = (crack_size(i + 1) - crack_size(i)) / (time_mv(i + 1) - time_mv(i));
a_medel(i) = (crack_size(i + 1) + crack_size(i)) / 2;
c_medel(i) = (cyc_mv(i + 1) + cyc_mv(i)) / 2;
M = 1.04+0.202.*(a_medel(i)./T)^2-0.106.*(a_medel(i)./T)^4;
Delta_K(i) = (ds.*sqrt(pi.*a_medel(i)*1E-3).*M./1.57)/1000000;
K_MAX(i) = (ds_max.*sqrt(pi.*a_medel(i)*1E-3).*M./1.57)/1000000;
time_medel(i) = (time_mv(i+1)+time_mv(i))/2;
end

%% PLOTTING
figure
loglog(Delta_K,da_dN,'*')
grid on
xlabel('Stress intensity factor [MPa/sqrt(m)]')
ylabel('da/dN [mm/cycle]')
title('Crack propagation')
%
%% EXPORTING DATA TO EXCEL FILE
% THE INTERESTED DATA CAN BE EXPORTED. THE NAME OF EXCEL FILE, SHEET NAME
% AND COLUMN NAME SHOULD BE DECLARED
xlswrite('evaluation_FCP_550C_2160s.xlsx','Delta_K','A3');
xlswrite('evaluation_FCP_550C_2160s.xlsx','da_dN','B3');
xlswrite('evaluation_FCP_550C_2160s.xlsx','K_MAX','C3');
xlswrite('evaluation_FCP_550C_2160s.xlsx','da_dt','D3');
xlswrite('evaluation_FCP_550C_2160s.xlsx','c_medel','E3');
```
10.1.4 The codes for identification each group of crack sizes in each loading cycle (6.3.2) and fit the regression curve through them (Crack increment technique)

clc
%
clear
eps=0.038;
time=data(:,1);

for i=cont:length{crack}-1
    if abs((crack(i+1)-crack(i)))< eps
        n=n+1;
        a(n) = crack(i+1);
        t(n)=time(i+1);
    else
        t=t(:);a=a(:);
        createfigure(t,a)
        coeff=polyfit(t,a,2);
        sm_crack(r,1)=coeff(1)*(t(1))^2 + coeff(2)* (t(1)) + coeff(3);
        sm_crack(r+1,1)=coeff(1)*(max(t))^2 + coeff(2)*(max(t)) + coeff(3);
        sm_time(r,1)=t(1);
        sm_time(r+1,1)=max(t);
        r=r+2;
        first_crack=crack(i+1);
        t(1)=first_time;
        a(1)=first_crack;
        cont=i+1;
        a=[];
        t=[];
        cont=i+1;
        %pause
    end
end

figure
plot(sm_time,sm_crack,'-*');grid on
plot(time,crack,'rO')

force_min = 1.262697368;
force_max = 28.64793421;

for i=2:2:(length{sm_crack)2-
    a_1 = sm_crack(i);
    a_2 = sm_crack(i+1);
    a_3 = sm_crack(i+2);
    delta_a_f = a_2 - a_1;
    delta_a_t = a_3 - a_2;

    if delta_a_f > 0
        da_dN(j,1) = a_2 - a_1 ;
        a_m = (a_2 + a_1)/2;
        M = 1.04+0.202*(a_m/4.3)^2-0.106*(a_m/4.3)^4;
        D_K(j,1) = (ds^sqrt(pi*a_m/0.001)*(M/1.57))/1000000;
        j=j+1;
    end

    if delta_a_t > 0
        delta_a_t = sm_time(i+2) - sm_time(i+1);
    end
da_dt(k,1) = (a_3 - a_2)/delta_t;
M_MAX = 1.04+0.202*(a_3/4.3)^2-0.106*(a_3/4.3)^4;
K_MAX(k,1) = (ds_max*sqrt(pi*(a_3)*0.001)*(M_MAX/1.57))/1000000;

%%%%%%%%%%%

k=k+1;
end
end

figure
loglog(D_K,da_dN,'

xlswrite('evaluated_crack_growth_rate_smoothed_group_550_2160.xlsx',D_K, '550C-2160s', 'A2');
xlswrite('evaluated_crack_growth_rate_smoothed_group_550_2160.xlsx',da_dN, '550C-2160s', 'B2');
xlswrite('evaluated_crack_growth_rate_smoothed_group_550_2160.xlsx',K_MAX, '550C-2160s', 'C2');
xlswrite('evaluated_crack_growth_rate_smoothed_group_550_2160.xlsx',da_dt, '550C-2160s', 'D2');
xlswrite('evaluated_crack_growth_rate_smoothed_group_550_2160.xlsx',time, 'crack_length', 'A2');
xlswrite('evaluated_crack_growth_rate_smoothed_group_550_2160.xlsx',crack, 'crack_length', 'B2');
xlswrite('evaluated_crack_growth_rate_smoothed_group_550_2160.xlsx',sm_time, 'crack_length', 'D2');
xlswrite('evaluated_crack_growth_rate_smoothed_group_550_2160.xlsx',sm_crack,'crack_length', 'E2');
10.2 Integration in Gayda’s model

Integration procedure [10]:

To evaluate the creep component of crack growth, the integral is broken into three parts, the ramp up, the hold time, and the ramp down, as shown below:

\[
\int \left( \frac{da}{dt} \right) dt = \int_0^{2\varepsilon} AK^n dt + AK_{\text{Max}t_{\text{Hold}}} + \int_{2\varepsilon}^{3\varepsilon} AK^n dt
\]

Since the first and the third term are numerically equivalent they may be combined as follows:

\[
\int \left( \frac{da}{dt} \right) dt = 2 \int_0^{2\varepsilon} AK^n dt + AK_{\text{Max}t_{\text{Hold}}}
\]

\(K\) is a linear function of time, \(t\), as shown below:

\[K = 2\nu\Delta Kt + K_{\text{Min}}\]

Substituting this expression for \(K\) in the integral, one obtains:

\[
\int \left( \frac{da}{dt} \right) dt = 2 \int_0^{2\varepsilon} A(2\nu\Delta Kt + K_{\text{Min}})^n dt + AK_{\text{Max}t_{\text{Hold}}}
\]

Expressing \(K_{\text{Min}}\) and \(K_{\text{Max}}\) in terms of \(\Delta K\) and \(R\), the load ratio, the following expression is obtained:

\[
\int \left( \frac{da}{dt} \right) dt = 2 \int_0^{2\varepsilon} A(2\nu\Delta Kt + (R\Delta K)/(1 - R))^n dt + (A\Delta K^n t_{\text{Hold}})/(1 - R)^n
\]

The results of the integration are shown below:

\[
= \left( \frac{A\Delta K^n}{\nu n + n} \right) \left[ \left( 2\nu t + \frac{R}{1 - R} \right)^{n+1} \right]^{t = 2\varepsilon} - \left[ \frac{A\Delta K^n t_{\text{Hold}}}{(1 - R)^n} \right]^{t = 0}
\]

Note that \(\Delta K\) is constant for a given cycle and has been removed from the integrand as \(\Delta K^n\). Evaluating the limits yields the following expression:

\[
= \left( \frac{A\Delta K^n}{\nu n + n} \right) [(1 + Q)^{n+1} - Q^{n+1}] + \left[ \frac{A\Delta K^n t_{\text{Hold}}}{(1 - R)^n} \right]
\]

where

\[Q = \frac{R}{1 - R}\]
This can be simplified by combining like terms to yield the time dependent term of Equation 6.2:

\[ A\Delta \kappa \left[ \frac{Z}{n(n + 1)} + \frac{t_{Hold}}{(1 - R)^n} \right] \]

where  

\[ Z = \frac{(1 - R^{n+1})}{(1 - R)^{n+1}} \]
10.3 TMF specimen scheme