Calculation of the energy absorption efficiency function of selected detector arrays using the MCNP code

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Report 103

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August, 2007
# Contents

1 Introduction ........................................... 2

2 Theory .............................................. 2
   2.1 Energy imparted to a single detector element ................. 2
   2.2 Energy absorption efficiency function .......................... 4
   2.3 Cross talk between detector elements .......................... 5

3 Methods ............................................ 6
   3.1 Case A: an infinite slab .................................. 6
   3.2 Case B: a detector array without a collimator .................. 6
   3.3 Case C: a detector array with a collimator .................... 7

4 Results ............................................ 8

5 Discussion ......................................... 11
   5.1 Comments on approximations .................................. 11
      5.1.1 Dependence on the azimuthal angle ....................... 11
      5.1.2 Alternative angle-energy grid ............................ 11
   5.2 Alternative scoring scheme .................................. 12

6 Summary ............................................ 12

7 Appendix: MCNP input files ............................ 18
   7.1 Case A: an infinite slab .................................. 18
      7.1.1 Photon transport only ................................ 18
      7.1.2 Photon and electron transport ........................ 18
   7.2 Case B: a detector array without a collimator .............. 19
      7.2.1 Photon transport only ................................ 19
      7.2.2 Photon and electron transport ........................ 20
   7.3 Case C: a detector array with a collimator .................. 21
      7.3.1 Photon transport only ................................ 21
      7.3.2 Photon and electron transport ........................ 22
      7.3.3 Simplified geometry .................................... 23
      7.3.4 The source subroutine ................................. 24
1 Introduction

This report describes a method for the calculation of the energy absorption efficiency function. It gives a theoretical justification of the method and presents results obtained using the MCNP4C code for (i) an infinite slab, (ii) a detector array without a collimator, and (iii) a detector array with a collimator. Moreover, it discusses an alternative method of scoring of the energy imparted per unit surface area in CTmod. This report is a supplement to the article “CTmod—a toolkit for Monte Carlo simulation of projections including scatter in computed tomography” by A. Malusek, M. Sandborg, and G. Alm Carlsson [1].

2 Theory

A method for the calculation of the energy imparted to the active volume of a single detector element is described in section 2.1. The relation between this quantity and the energy absorption efficiency function is discussed in section 2.2. An alternative concept of scoring of energy imparted per unit surface area based on cross talk matrices is presented in section 2.3.

2.1 Energy imparted to a single detector element

Consider an infinite detector array consisting of one layer of identical hexahedral elements, see figure 1. Suppose the angle-energy distribution of photon fluence is known at the center of the entrance surface of each detector element. The task is to estimate the energy imparted to a given detector element.

A straightforward approach would be to calculate the energy imparted to the given detector element from a photon field obtained by interpolation from its known values at the entrance surface of the detector array. But the corresponding Monte Carlo simulation would be inefficient. In the following, we introduce an alternative method which is more efficient but its application is limited to photon fields that are sufficiently uniform and to detector arrays that have low cross talk between detector elements.
First, we expand the photon field from the single point (the center of the entrance surface of the considered detector element) to the whole space outside the detector array, see figure 2a. For more information about the concept of an expanded field see [2]. Note that any expanded field can be created by a superposition of wide parallel beams with different directions of flight of photons. The detector response to this field can be simulated using a virtual surface-source of photons that covers the entrance surface of the detector array. (c) Energy imparted to a single detector element from a virtual source covering the whole entrance surface is the same as the energy imparted to all detector elements from a virtual surface-source covering a single detector element.

Lemma 1: The energy imparted to a detector element \( D_{(0,0)} \) by photons generated by a surface-source covering the entrance surface of all detector elements equals the energy imparted to all detector elements by photons generated by a surface-source covering the entrance surface of the detector element \( D_{(0,0)} \). The proof is based on the symmetry of the repeated structure. The mean contribution of photons impinging on \( D_{(i,j)} \) to the energy imparted to \( D_{(0,0)} \) equals the contribution of photons impinging on \( D_{(0,0)} \) to the energy imparted to \( D_{(-i,-j)} \). Thus the sum of contributions from photons impinging on all detector elements to the energy imparted to \( D_{(0,0)} \) equals the sum of energies imparted to individual detector elements by photons impinging on \( D_{(0,0)} \).

Lemma 2: Energy imparted to all detector elements by photons generated by a surface-source covering the entrance surface of the detector element \( D_{(0,0)} \) does not change when surface source is shifted horizontally. The proof is based on a one-to-one mapping between regions A, B, C, and D in the shifted and original surface-source, see figure 3a.

We use the term reference area for the area according to which we sample impinging photons, i.e. for the area where we known the angle-energy distribution of photon fluence \( \Phi_{\Omega,E}(\Omega,E) \). So far, the reference area was the same as the entrance surface of the detector element \( D_{(0,0)} \). In Lemma 2, we showed that the reference area could be shifted horizontally. In Lemma 3, we show that it can be shifted vertically. In this case, the virtual source of photons is still located on the surface of the detector array. Its angle-energy distribution of generated photons is defined so that, in free space, the resulting angle-energy distribution of photons at the reference area is the same as \( \Phi_{\Omega,E}(\Omega,E) \). In practice, the photons can be generated at the reference area and their position can be back-projected to the entrance surface.
Figure 3: (a) Top view of a detector array. Photons impinging on the area $A$ impart the same amount of energy into all detector elements as photons impinging on the area $A'$. Similarly for areas $B$ and $C$. (b) Side view of a detector array. Trajectories of photons with the same direction of flight back-projected from the reference area $A$ to the surface of the detector array form a new reference area $A'$ there.

Lemma 3: The energy imparted to all detector elements does not change when the reference area is shifted vertically, see figure 3b. The proof consists of two steps. First, consider a parallel beam of photons expanded over the reference area. To simulate such field, we back-project positions of photons from the original reference area to a new one on the detector’s surface. We know from lemma 2 that this new, horizontally shifted reference area does not change the energy imparted to all detector elements. Second, a general field expanded over the reference area can be considered as a superposition of parallel beams. For each of the parallel beams, the statement is true. Since the energy imparted to all detector elements is an additive function with respect to the decomposition to the parallel beams, the statement is true for the general case too.

Lemmas 1–3 give us a recipe on how to calculate the energy imparted to a detector element from a field expanded from the center of the entrance surface of a detector element to the whole space: We select a reference area for the detector element. This reference area serves as a virtual source of photons. If it is inside the detector element then we back-project positions of photons on the detector array surface. We score the energy imparted to all detector elements.

2.2 Energy absorption efficiency function

Let $\Phi_{\Omega,E}(r, \Omega, E)$ be the angle-energy distribution of photon fluence at the point $r$, i.e. let $\Phi_{\Omega,E}(r, \Omega, E) \, d\Omega dE$ be the number of photons with energy between $E$ and $E + dE$ passing through a unit surface area at point $r$ perpendicular to the direction $\Omega$ into the solid angle $d\Omega$.

Let a function $f(E, \Omega)$ be defined so that the mean energy imparted to all detector elements, $\bar{\epsilon}$, by photons with $\Phi_{\Omega,E}(\Omega, E)$ that impinge on the reference area $A$ is

$$\bar{\epsilon} = \int_{\Delta A} \int_{4\pi} \int_{0}^{\infty} E\Phi_{\Omega,E}(\Omega, E) |\Omega n| f(E, \Omega) \, d\Omega dE. \quad (1)$$

We call it the energy absorption efficiency function of the detector element. To calculate
\( f(E, \Omega) \), assume a parallel beam of \( N \) photons with energy \( E \) and direction \( \Omega \) impinging on the reference area \( \Delta A \). The corresponding \( \Phi_{\Omega,E}(\Omega, E) \) is

\[
\Phi_{\Omega,E}(\Omega, E) = \frac{N}{\Delta A |\Omega n|} \delta(\Omega - \Omega_0)\delta(E - E_0),
\]

where \( \delta \) is the Dirac’s function and \( n \) is the normal to the entrance surface. (In practical applications, no photons are impinging on the bottom side of the detector array and thus \( n \) and \( \Omega \) are always antiparallel.) By inserting (2) to (1) we get

\[
\bar{\epsilon} = \int_{\Delta A} E_0 \frac{N}{\Delta A} f(E_0, \Omega_0) dA = E_0 N f(E_0, \Omega_0).
\]

Thus the energy absorption efficiency function can be calculated as

\[
f(E_0, \Omega_0) = \frac{\bar{\epsilon}}{E_0 N}.
\]

The energy imparted to all detector elements per one incident photon, \( \bar{\epsilon}/N \), can be calculated using Monte Carlo codes like MCNP.

### 2.3 Cross talk between detector elements

Let \( \bar{\epsilon}_{ij} \) be the mean energy imparted to the active volume of a detector element per one impinging photon with energy \( E \) and incidence angle \( \theta \) (the azimuthal angle has a uniform distribution in \([0, 2\pi])\. Let \( \bar{\epsilon}_{ij}^{\text{tot}} = \sum_{ij} \bar{\epsilon}_{ij} \) be the total mean energy imparted to the considered detector elements. In this report, we define a cross talk matrix \( M = m_{ij} \) as

\[
m_{ij} = \frac{\bar{\epsilon}_{ij}}{\bar{\epsilon}_{ij}^{\text{tot}}}. \tag{5}
\]

Thus \( m_{ij} \) gives the fraction of the energy imparted to active volumes of all considered detector elements that is imparted to the detector element \((i, j)\). In cases A, B, and C (see table 1) considered in this report, the \( 5 \times 5 \) matrix is symmetric about the element \( m_{33} \):

\[
\begin{bmatrix}
  m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\
  m_{21} & m_{22} & m_{23} & m_{24} & m_{25} \\
  m_{31} & m_{32} & m_{33} & m_{34} & m_{35} \\
  m_{41} & m_{42} & m_{43} & m_{44} & m_{45} \\
  m_{51} & m_{52} & m_{53} & m_{54} & m_{55}
\end{bmatrix}
\begin{bmatrix}
  m_{11} & m_{21} & m_{31} & m_{21} & m_{11} \\
  m_{21} & m_{22} & m_{32} & m_{22} & m_{21} \\
  m_{31} & m_{32} & m_{33} & m_{32} & m_{31} \\
  m_{41} & m_{42} & m_{43} & m_{44} & m_{45} \\
  m_{51} & m_{52} & m_{53} & m_{54} & m_{55}
\end{bmatrix}
\]

To save space, we report just 6 elements of the triangular matrix

\[
\begin{bmatrix}
  m_{11} & 0 & 0 \\
  m_{21} & m_{22} & 0 \\
  m_{31} & m_{32} & m_{33}
\end{bmatrix}
\]

The normalization condition following from (5) is

\[
4m_{11} + 8m_{21} + 4m_{22} + 4m_{31} + 4m_{32} + m_{33} = 1. \tag{8}
\]
3 Methods

Simulations were performed using the MCNP4C code [3]. Three cases were studied, see table 1. In all cases, the active volume was a 3 mm thick ceramic scintillator $Y_{1.34}Gd_{0.6}Eu_{0.06}O_3$.

<table>
<thead>
<tr>
<th>case</th>
<th>detector array description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>an infinite slab (approximated with a cylinder)</td>
</tr>
<tr>
<td>B</td>
<td>a hexahedral array of detector elements without a collimator</td>
</tr>
<tr>
<td>C</td>
<td>a hexahedral array of detector elements with a collimator</td>
</tr>
</tbody>
</table>

Table 1: List of studied cases.

[4, 5] also known as $(Y,Gd)_2O_3:Eu$ with density of 5.92 g/cm$^3$. In case A, the active volume of the detector element was an infinite slab that was approximated with a large cylinder. In case B, the active volume of the detector element was a 0.9 mm × 0.9 mm × 3 mm box. In case C, a collimator was placed in front of the detector array of case B and the number of detector elements was increased. More information about the configuration is in sections 3.1–3.3.

Two independent sets of simulations were performed; their results are presented in table 2 and figures 9–11. Results presented in table 2 were calculated for the impinging photon energy $E = 100$ keV and incidence angles of 0°, 30°, or 60° both with and without the transport of electrons. Results presented in figures 9–11 were calculated for an equidistant grid of 150 × 64 points. The photon energy, $E$, varied from 1 keV to 150 keV with 1 keV step. The cosine of the incidence angle, $\xi$, varied from 1/64 to 1.0 with the step of 1/64.

3.1 Case A: an infinite slab

The detector element was approximated with a cylinder with a height of 3 mm and a radius of 100 cm. Photons with energy $E$ and incidence angle $\theta$ were impinging on the center of the entrance surface. The azimuthal angle was sampled from the uniform distribution on the interval $[0, 2\pi]$. Results presented in table 2 and figure 9 were calculated with $10^6$ and $10^5$ photons emitted from the source, respectively. Energy imparted to the cylinder, $\epsilon$, was scored. For a given energy of impinging photons, $E$, and cosine of the incidence angle, $\xi$, the energy absorption efficiency function was calculated as

$$ f(E, \xi) = E^{-1}\epsilon. \quad (9) $$

MCNP input files are in Appendix 7.1.

3.2 Case B: a detector array without a collimator

The detector array was a 5 mm × 5 mm × 3 mm box containing $5 \times 5 = 25$ detector elements with x- and y-indices ranging from -2 to 2. Each detector element was approximated with a
1 mm × 1 mm × 3 mm box, see figure 4. (The figure was produced using the geometry plotter of MCNP4C.) The active volume size was 0.9 mm × 0.9 mm × 3 mm and the thickness of the tantalum septa was 0.1 mm. Photons with energy $E$ and incidence angle $\theta$ were impinging on the 1 mm × 1 mm entrance surface of the detector element with index (0,0). The azimuthal angle was sampled from a uniform distribution on the interval $[0, 2\pi]$. Results presented in table 2 and figure 10 were calculated with $10^6$ and $10^5$ photons emitted from the source, respectively. Energy imparted to the active volume of each detector element, $\epsilon_{ij}$, was scored. For a given energy of impinging photons, $E$, and cosine of the incidence angle, $\xi$, the energy absorption efficiency function was calculated as

$$ f(E, \xi) = E^{-1} \sum_{i=-2}^{2} \sum_{j=-2}^{2} \epsilon_{ij}. $$

(10)

MCNP input files are in Appendix 7.2.

### 3.3 Case C: a detector array with a collimator

The detector array was a 101 mm × 101 mm × 8 mm box containing 101 × 101 detector elements with x- and y-indices ranging from -50 to 50. The size of the detector element was the same as in case B, i.e. 1 mm × 1 mm × 3 mm. The septa protruded 5 mm in front of each detector element, see figure 5, the void space was filled by air. The source subroutine (see section 7.3.4) generated photons with energy $E$ and incidence angle $\theta$ in
two steps. First, a position \( P \) was randomly selected at the 1 mm × 1 mm entrance area of the sensitive volume of the detector element with indices \((0,0)\). The azimuthal angle was sampled from a uniform distribution on the interval \([0, 2\pi]\). Second, the position of the photon was back-projected to a spherical surface with radius \( R = 9 \) cm. Let \( u \) be the direction of flight of the photon. The intersection \( Q \) of the line \( P + tu \), \( t \in \mathbb{R} \), and the spherical surface \( |x|^2 = R^2 \) is

\[
Q = P + tu, \tag{11}
\]

where

\[
t = -B - \sqrt{B^2 - C}, \quad B = Pu, \quad C = |P|^2 - R^2, \tag{12}
\]

is a solution of the resulting quadratic equation. Results presented in table 2 and figure 11 were calculated with \( 10^7 \) and \( 10^5 \) photons emitted from the source, respectively. Energy imparted, \( \epsilon_{ij} \), to active volumes of 25 detector elements with indices \(-2 \leq i \leq 2, -2 \leq j \leq 2\) was scored. The energy absorption efficiency function was calculated from (10).

4 Results

The energy absorption efficiency function, \( f(E, \xi) \), for photons with energy \( E = 100 \) keV impinging with incidence angles of 0°, 30°, and 60° for cases A, B, and C is in table 2. Calculations were performed with the photon transport only (mode \( p \)) and with the transport of both photons and electrons (mode \( p e \)). The additional transport of electrons slowed
Table 2: The energy absorption efficiency function, \( f(E, \xi) \), for the energy of the impinging photon \( E = 100 \, \text{keV} \) and incidence angles of 0°, 30°, or 60°. Modes “p” and “e” correspond to photon and electron transport, respectively. All values are multiplied by 100, the coverage factor is \( k = 1 \).

<table>
<thead>
<tr>
<th>case</th>
<th>mode</th>
<th>incidence angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0°</td>
</tr>
<tr>
<td>A</td>
<td>p</td>
<td>91.17 ± 0.03</td>
</tr>
<tr>
<td>A</td>
<td>p e</td>
<td>91.16 ± 0.03</td>
</tr>
<tr>
<td>B</td>
<td>p</td>
<td>70.22 ± 0.04</td>
</tr>
<tr>
<td>B</td>
<td>p e</td>
<td>70.29 ± 0.04</td>
</tr>
<tr>
<td>C</td>
<td>p</td>
<td>67.63 ± 0.01</td>
</tr>
<tr>
<td>C</td>
<td>p e</td>
<td>67.64 ± 0.01</td>
</tr>
</tbody>
</table>

Table 3: Simulation speed in number of source particles per second for photons with energy \( E = 100 \, \text{keV} \) and incidence angles of 0°, 30°, or 60°. Modes “p” and “e” correspond to photon and electron transport, respectively.

<table>
<thead>
<tr>
<th>case</th>
<th>mode</th>
<th>incidence angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0°</td>
</tr>
<tr>
<td>A</td>
<td>p</td>
<td>(2.24 \times 10^4)</td>
</tr>
<tr>
<td>A</td>
<td>p e</td>
<td>(2.00 \times 10^6)</td>
</tr>
<tr>
<td>B</td>
<td>p</td>
<td>(1.31 \times 10^4)</td>
</tr>
<tr>
<td>B</td>
<td>p e</td>
<td>(1.56 \times 10^6)</td>
</tr>
<tr>
<td>C</td>
<td>p</td>
<td>(1.11 \times 10^4)</td>
</tr>
<tr>
<td>C</td>
<td>p e</td>
<td>(1.11 \times 10^4)</td>
</tr>
</tbody>
</table>

Values of the energy absorption efficiency function strongly depend on the geometry of the detector array. For perpendicularly impinging photons, the geometric efficiency of the detector element equals the ratio of the entrance area of the active volume to the entrance area of the detector element, \(0.9^2/1.0^2 = 0.81\). The predicted energy absorption efficiency function of case B, \(0.81 \times 91.2 = 73.8\), is slightly higher than the simulated one, 70.3. This
can be explained by the higher probability of scatter (per one photon interaction) in the active volume than in the septa—the septa acts more like an absorber than a scatterer of photons. For obliquely incident photons, the function $f(E, \xi)$ has a complicated shape, see figures 6 and 9–11. It is affected by several competing effects. The K- and L-edges of Y, Gd and Eu, see table 4, cause sharp drops of $f(E, \xi)$ as a function of energy since the emitted characteristic radiation may escape from the active volume. The K-edge of Ta at 67.4 keV causes a sharp drop of $f(E, \xi)$ for case C and $\theta$ equals 30° or 60°, see figure 6, because the probability of a photoelectric effect in the Ta septa strongly increases for energies above the K-edge. For cases A and B, photons with high energy $E$ have lower probability of interaction inside the active volume and thus $f(E, \xi)$ is a decreasing function of $E$. The exception from this rule is the vicinity of absorption edges and large incidence angles where secondary electrons may escape from the active volume. For case C, the low energy photons are absorbed by the collimator and thus $f(E, \xi)$ is an increasing function of energy for a given $\xi$. For large incidence angles ($\xi \approx 0$), the collimator (case C) absorbed all impinging photons. Therefore the relative errors in figure 11b increase sharply with increasing $\xi$ and then become undefined as almost no energy was imparted to the active volume.

Figures 9–11 clearly demonstrate the importance of a proper model of the detector array for scatter projection calculations. For photons impinging with the incidence angle

<table>
<thead>
<tr>
<th></th>
<th>O</th>
<th>Y</th>
<th>Eu</th>
<th>Gd</th>
<th>Ta</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z$</td>
<td>8</td>
<td>39</td>
<td>63</td>
<td>64</td>
<td>73</td>
</tr>
<tr>
<td>L1-edge (keV)</td>
<td>0.024</td>
<td>2.37</td>
<td>8.05</td>
<td>8.38</td>
<td>11.7</td>
</tr>
<tr>
<td>K-edge (keV)</td>
<td>0.532</td>
<td>17.04</td>
<td>48.5</td>
<td>50.2</td>
<td>67.4</td>
</tr>
</tbody>
</table>

Table 4: K- and L1-edges of elements contained in materials of the detector array. Data were taken from [6].
of $0^\circ$, the septa reduces $f(E, \xi)$ approximately according to the geometric efficiency. But for obliquely impinging photons (these produce the scatter projection), $f(E, \xi)$ increases in case A by the factor $93.2/91.2 = 1.02$ and $95.6/91.2 = 1.05$ for incidence angles of $30^\circ$ and $60^\circ$, respectively, and decreases by $52.4/70.3 = 0.75$ and $47.1/70.3 = 0.67$, respectively, for case B. For case C, the corresponding values are $0.42/67.6 = 6.2 \times 10^{-3}$ and $0.0074/67.4 = 1.1 \times 10^{-4}$, respectively.

5 Discussion

In CTmod, so far, we preferred a simple computational model with fast algorithms and small computer memory requirements. As the processing power of new computers has increased over the time, there is a room for improvement of the model. Several suggestions are discussed in following sections.

5.1 Comments on approximations

5.1.1 Dependance on the azimuthal angle

The dependence of the energy absorption efficiency function, $f(E, \xi)$, on the azimuthal angle, $\phi$, was neglected. So far, we published results for detector elements approximated by an infinite slab only. In this case, the simplification is justified because of the rotational symmetry of the geometry. For detectors arrays with a significant asymmetry with respect to the azimuthal angle, the energy absorption efficiency function can be approximated by a three-dimensional array $f(E, \xi, \phi)$. It will take more time to pre-calculate the function and it will occupy more space in the computer’s memory but there are no principal problems with this approach.

5.1.2 Alternative angle-energy grid

The energy absorption efficiency function, $f(E, \xi)$, was pre-calculated in an equidistant grid of 64 values of the cosine of the incidence angle, $\xi$, and 150 values of the impinging photon energy, $E$. This approach was designed for detector elements approximated by an infinite slab. For a detector array without a frontal collimator (case B), the steep increase in $f(E, \xi)$ for $\xi$ approaching 1 indicates that a denser grid should be used for large values of $\xi$. For a detector array with a collimator (case C), a different approach would be better. The energy absorption efficiency function $f(E, \Omega)$ could be written as $f(E, \Omega) = T(E, \Omega) f^*(E, \Omega)$, where $T(E, \Omega)$ is the analytical transmission formula of the collimator. It is expected, that the function $f^*(E, \Omega)$ will change much less than $T(E, \Omega)$. An optimal approximation of $f(E, \Omega)$ is still an open question.
5.2 Alternative scoring scheme

Currently, each scattering interaction contributes $\Delta \epsilon_A (r_d)$

$$\Delta \epsilon_A (r_d) = \frac{p(\Omega_{i-1}, \Omega_d)}{4\pi d^2} \exp\left[-\lambda(E_i, r_d, r_i)\right] w_i E \Omega n f(r_d, E, \Omega),$$

(13)

for a point detector at the position $r_d$, see [1] for more information. The energy absorption efficiency function, $f(r_d, E, \Omega)$, estimates the energy imparted to a single detector element via the photon field which is expanded from the point $r_d$ to the whole space outside the detector array.

An alternative approach is to use the cross talk matrices to score $\Delta \epsilon_A (r_d)$. In this case, a contribution, $\Delta \epsilon_A (r_d)$, to the point detector at $r_d$ would be accompanied with contributions $\Delta \epsilon_A (r_d, k, l)$ to surrounding detector elements with indices $(k, l)$. The formula is

$$\Delta \epsilon_A (r_d, k, l) = \frac{p(\Omega_{i-1}, \Omega_d)}{4\pi d^2} \exp\left[-\lambda(E_i, r_d, r_i)\right] w_i E \Omega n f_{kl}(r_d, E, \Omega),$$

(14)

Instead of a single energy absorption efficiency function $f(r_d, E, \Omega)$ calculated from (4), where $\bar{\epsilon}$ is the energy imparted to all detector elements from a wide parallel beam of photons impinging on the entrance surface of the detector element with index $(0, 0)$, we have a matrix of $f_{kl}(r_d, E, \Omega)$ values defined as

$$f_{kl}(r_d, E, \Omega) = \frac{\bar{\epsilon}_{kl}}{E_0 N},$$

(15)

where $\bar{\epsilon}_{kl}$ is the energy imparted to detector element $(k, l)$ by a parallel beam of photons impinging on the detector element $(0, 0)$. Clearly,

$$f_{kl}(r_d, E, \Omega) = m_{ij} f(r_d, E, \Omega),$$

(16)

where $m_{ij}$ is the cross talk matrix defined in (5).

6 Summary

In CTmod, the implementation of scoring of energy imparted per unit surface area of a detector element, $\epsilon_A$, based on the energy absorption efficiency function, $f(E, \xi)$, pre-calculated in an equidistant grid of 64 values of the cosine of the incidence angle, $\xi$, and 150 values of the impinging photon energy, $E$, was designed for a detector element approximated by an infinite slab (case A). Presented results indicate that a similar approach could be used for a detector array without a collimator (case B). The steep increase in $f(E, \xi)$ for $\xi$ approaching 1 could be addressed with a denser grid for large values of $\xi$. Results obtained for the detector array with a collimator (case C) indicate that the energy absorption efficiency function $f(E, \Omega)$ could be evaluated as $f(E, \Omega) = T(E, \Omega) f^*(E, \Omega)$, where $T(E, \Omega)$ is the analytical transmission formula of the collimator. We believe that this factorization could significantly improve the accuracy of the approximation since the function $f^*(E, \Omega)$ is not expected to change as much as $T(E, \Omega)$.
Figure 7: Case B: Elements of cross talk matrices, $m_{ij}$, (multiplied by 100) for incidence angles $\theta$ of 0°, 30°, and 60° and 100 keV photons calculated using photon and electron transport.

<table>
<thead>
<tr>
<th></th>
<th>photon transport only</th>
<th>photon and electron transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.84 × 10^{-4}</td>
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<td>6.16 × 10^{1}</td>
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</tbody>
</table>

Table 5: Case B: Elements of cross talk matrices, $m_{ij}$, (multiplied by 100) for 100 keV photons and incidence angles 0°, 30°, and 60° calculated with and without electron transport. The data are visualized in figure 7.
The data are visualized in figure 8.

Table 6: Case C: Elements of cross talk matrices, $m_{ij}$, (multiplied by 100) for 100 keV photons and incidence angles $\theta$ of 0°, 30° and 60° calculated using photon and electron transport. The data are visualized in figure 8.
Figure 9: Case A: (a) The energy absorption efficiency function, $f(E, \xi)$, of an infinite slab as a function of the incident photon energy, $E$, and the cosine of the incident angle, $\xi$. (b) The corresponding relative error for the coverage factor $k = 3$. Note the different orientation of axes.
Figure 10: Case B: (a) The energy absorption efficiency function, $f(E, \xi)$, of the detector array without a collimator as a function of the incident photon energy, $E$, and the cosine of the incident angle, $\xi$. (b) The corresponding relative error for the coverage factor $k = 3$. Note the different orientation of axes.
Figure 11: Case C: (a) The energy absorption efficiency function, $f(E, \xi)$, of the detector array without a collimator as a function of the incident photon energy, $E$, and the cosine of the incident angle, $\xi$. Note the different orientation of axes and the log scale. (b) The corresponding relative error for the coverage factor $k = 3$. 
7 Appendix: MCNP input files

7.1 Case A: an infinite slab

7.1.1 Photon transport only

Cell 1 is the cylinder approximating the infinite slab. A point source inside the cell 1 generates photons at the depth of $10^{-6}$ cm with the energy $<E>$ MeV and the cosine of the incident angle $<\xi>$. The small shift of $10^{-6}$ cm guarantees that photons impinging parallel to the entrance surface are inside the cell 1. The azimuthal angle is sampled randomly. Photons leaving the cylinder are killed. Energy imparted into the cylinder is scored. Only photons are transported, electrons are deposited locally.

```
energy deposition in a slab by a photon pencil beam
1 1 -5.92 -1 2 -3
2 0 #1

1 pz 0
2 pz -0.3
3 cz 100

m1 39089 1.34 64157 0.6 63152 0.06 8016 3
mode p
imp:p 1 0
sdef erg <E> pos 0 0 -1e-6 vec 0 0 -1 dir <\xi> cel 1
nps 1000000
print
prdmp 2j -1
*f8:p 1
```

7.1.2 Photon and electron transport

The transport of electrons was added to the input file in section 7.1.1.

```
energy deposition in a slab by a photon pencil beam
1 1 -5.92 -1 2 -3
2 0 #1

1 pz 0
2 pz -0.3
3 cz 100

m1 39089 1.34 64157 0.6 63152 0.06 8016 3
mode p e
imp:p 1 0
imp:e 1 0
```
7.2 Case B: a detector array without a collimator

7.2.1 Photon transport only

Cell 1 is a box containing the detector array. It is filled by universe 1 which is a $5 \times 5$ hexahedral lattice. Each cell of the lattice is filled by universe 2 which is a box containing the active volume (cell 3) and the septa (cell 4). The source generates photons with energy $<E>$ and incidence angle $<\xi>$ at the entrance surface of the detector element (0,0,0). They are generated at the depth of $10^{-6}$ cm to avoid rounding errors when $<\xi>$ equals 90°. Their x and y coordinates are sampled from uniform distributions (lines 32–35).
mode p
sdef erg=<E> x=d1 y=d2 z=-1e-6 vec 0 0 -1 dir=<Xi>
si1 -0.05 0.05
sp1 0.0 1.0
si2 -0.05 0.05
sp2 0.0 1.0
nps 1000000
print
prdmp 2j -1
*f8:p (3<2[-2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0])
   (3<2[-2 -1 0 -1 -1 0 0 -1 0 1 -1 0 2 -1 0])
   (3<2[-2 0 0 -1 0 0 0 1 0 0 2 0 0])
   (3<2[-2 1 0 -1 1 0 0 1 1 0 2 1 0])
   (3<2[-2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0])

7.2.2 Photon and electron transport

The transport of electrons was added to the input file in section 7.2.1.

energy deposition into an array of detector elements
1 0 -21 22 -23 24 -25 26 fill=1 imp:p,e=1
2 0 -301 302 -303 304 -305 306 lat=1 u=1 imp:p,e=1 fill=-2:2 -2:2 0:0
2 24r
3 1 -5.92 -11 12 -13 14 -15 16 u=2 imp:p,e=1
4 2 -16.59 #3 imp:p,e=1 u=2

21 px 0.25
22 px -0.25
23 py 0.25
24 py -0.25
25 pz 0.0
26 pz -0.3
11 px 0.045
12 px -0.045
13 py 0.045
14 py -0.045
15 pz 0.0
16 pz -0.3
301 px 0.050
302 px -0.050
303 py 0.050
304 py -0.050
305 pz 0.0
306 pz -0.3
7.3 Case C: a detector array with a collimator

7.3.1 Photon transport only

energy deposition in a grid of detector elements
1 0 -21 22 -23 24 -25 26 fill=1 imp:p=1
2 0 -301 302 -303 304 -305 306 lat=1 u=1 imp:p=1 fill=-50:50 -50:50 0:0
3 2 10200
4 3 1 -5.92 -11 12 -13 14 -15 16 u=2 imp:p=1
6 5 0 (21 : -22 : 23 : -24 : 25 : -26) -1 imp:p=1
7 6 3 -1.20479E-03 -11 12 -13 14 15 -25 u=2 imp:p=1
8 7 0 1 imp:p=0

1 1 so 10.0
21 px 5.05
22 px -5.05
23 py 5.05
24 py -5.05
25 pz 0.5
26 pz -0.3
11 px 0.045
12 px -0.045
13 py 0.045
14 py -0.045
15 pz 0.0
16 pz -0.3
301 px 0.050
302 px -0.050
7.3.2 Photon and electron transport

deposition in a grid of detector elements
1 0 -21 22 -23 24 -25 26 fill=1 imp:p,e=1
2 0 -301 302 -303 304 -305 306 lat=1 u=1 imp:p,e=1 fill=-50:50 -50:50 0:0
2 10200r
3 1 -5.92 -11 12 -13 14 -15 16 u=2 imp:p,e=1
6 3 -1.20479E-03 -11 12 -13 14 15 -25 u=2 imp:p,e=1
7 0 1 imp:p,e=0
8 1 so 10.0
9 21 px 5.05
10 22 px -5.05
11 23 py 5.05
12 24 py -5.05
13 25 pz 0.5
14 26 pz -0.3
15 11 px 0.045
16 12 px -0.045
17 13 py 0.045
18 14 py -0.045
19 15 pz 0.0
20 16 pz -0.3
7.3.3 Simplified geometry

The following listing describes a $5 \times 5$ detector array where the simulations were performed for perpendicularly impinging photons only. Results are presented in [1].

energy deposition in a grid of detector elements
1 0 -21 22 -23 24 -25 26 fill=1 imp:p=1
2 0 -301 302 -303 304 -305 306 lat=1 u=1 imp:p=1 fill=-2:2 -2:2 0:0
3 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
4 3 1 -5.92 -11 12 -13 14 -15 16 u=2 imp:p=1
5 2 -16.59 11 -12 -13 : -14 : -15 : -26 : 25 imp:p=1 u=2
7 3 -1.20479E-03 -11 12 -13 14 15 -25 u=2 imp:p=1
8 21 px 0.25
9 22 px -0.25
10 23 py 0.25
11 24 py -0.25
12 25 pz 0.5
13 26 pz -0.3
14 11 px 0.045
15 12 px -0.045
16 13 py 0.045
17 14 py -0.045
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subroutine source

... 

Parameters from the RDUM card:
- rdum(1) - x-edge of the detector element
- rdum(2) - y-edge of the detector element
- rdum(3) - z-position of the reference area
- rdum(4) - cosine of the incidence angle measured from (0,0,-1)
- rdum(5) - radius of the sphere where photons are back projected
- rdum(6) - energy of the photon

Parameters from the IDUM card:
- idum(1) - initial cell, icl

Local variables:
- xra, yra, zra - position on the reference area

7.3.4 The source subroutine

The declaration of common blocks (line 2) is omitted to make the listing shorter.
cil=idum(1)
jsu=0
erg=rdum(6)
wgt=1.0
tme=0.0
ipt=2

Sample position on the reference area
xra=rdum(1)*(2.0*rang()-1.0)
yra=rdum(2)*(2.0*rang()-1.0)
zra=rdum(3)

Sample direction of flight
www=-rdum(4)
phi=2.0*pie*rang()
tam=sqrt(1.0-www**2)
uuu=tam*cos(phi)
vvv=tam*sin(phi)

Calculate intersection with spherical surface
bam=xra*uuu+yra*vvv+zra*www
cam=xra**2+yra**2+zra**2-rdum(5)**2
tam=-bam-sqrt(bam**2-cam)
xxx=xra+tam*uuu
yyy=yra+tam*vvv
zzz=zra+tam*www

return
end

References


