

# Histogram Filters for Noise Reduction

Andreas Wrangsjö and Hans Knutsson

Medical Informatics, Department of Biomedical Engineering, Linköping University

[www.imt.liu.se/mi](http://www.imt.liu.se/mi)

## Abstract

A class of filters based on histograms are presented. The signal probability density function is estimated and filtering is performed in the pdf domain. Such filters can be designed to preserve signal features such as sharp edges while suppressing stochastic variations. One particular histogram filter scheme is evaluated and compared to a median filter and a normal gaussian blurring filter.

## 1 Introduction

Noise reduction is hardly a new concept. Neither would anyone be surprised by the fact that histograms are used to perform noise reduction. Rank filters, particularly the special case *median filters*[5] are widely in use by the image processing community. Histogram filters as such can, however, be defined as a much more general class of filters than just rank filters.

In this paper, one particular type of histogram filter is proposed for noise reduction. It is based on deconvolution of the histogram. The motivation to do so is that additive noise has performed a type of low-pass filtering on the original signals histogram.

## 2 Theory

### Histogram Based Filters

The main idea behind histogram filters is to use the statistics of the local surrounding to determine the, in some sense, most probable signal value. It is, of course, difficult to determine what is signal and what is noise with no prior knowledge about the noise model. The histograms are, of course, a means to find an estimate of the noisy signal probability density function (pdf). Once the pdf is known (or at least estimated) it can be used to find the most probable intensity value at the centre of the region.

If we assume an additive noise model, the pdf of the noisy signal can be found as the convolution between the original signal and the noise, all according to basic statistics theory. In most situations, noise has a pdf in the shape of some bump. The resulting pdf then turns into a low-pass filtered version of the input signal pdf. With this in mind, we would like to estimate the histogram of the original signal by some clever deconvolution of the noisy signal

histogram. Once the original noise-free histogram has been estimated, we need to decide how to interpret it into an output value. The following three sections describe the three processes of estimating the local signal pdf, de-noising the estimated pdf and determining the output value.

### Weighted Histograms

If the input signal is a stationary stochastic process, the histogram is best found by simply dividing the value space into a number of bins and adding 1 to a bin for each occurrence of a value within its bounds. If, however, the input signal is an image, this scheme will not necessarily yield the best possible estimate. We need to accommodate for the spatial variations in the image statistics. We need spatially weighted histograms.

Instead of simply adding 1 to the corresponding bin, we introduce a certainty measure to the estimate. This measure should decrease with the distance to the centre of the local region and thus make the nearest values more important than the values farther away.

### Histogram Deconvolution

The histogram corresponds to the estimated probability density function of the noisy data. As mentioned above, adding noise to the signal yields a pdf equal to a convolution between the noise and the signal pdfs. With gaussian noise, this convolution means gaussian low-pass filtering. In the histogram domain, adding gaussian noise is thus equivalent to normal gaussian blurring of the histogram.

With a suitable inverse filter, the original signal histogram could be found. This histogram could then be used to find a suitable estimate of the most probable centre value. If there is no prior knowledge about the noise statistics, the inverse filter needs to be replaced by a filter that at least counters the effects of the noise. These effects are that impulses turn into bumps, bumps go wider and adjacent bumps merge into one. To reduce these effects, we would want to make peaks more narrow and pointy.

Before performing any filtering of the histogram, it is recommended that certain smoothing is performed. The histogram is a quantized version of the true pdf, which may very well contain pointy peaks and other undesired high frequency components, even for signals with smooth and well-behaved pdfs. Smoothing of the histogram has the effect that the bins are no longer rectangular channels in the

pdf domain, but overlapping soft bumps. Similar strategies have been proposed by e.g. Parzen[4] and have been used for image analysis by e.g. Forssén[1]. It is difficult to say how much the histogram should be smoothed. It will depend on the number of bins, the statistics of the signal and the size of the spatial kernel. Notice, however, that another method to obtain well-behaved histograms is dithering. The most basic form of dithering is adding Gaussian noise to the input signal. See e.g. Gray[2] for more on dithering.

The smoothed histogram can be seen as a channel representation of the data, with the channels being the overlapping bumps resulting from LP-filtering of the histogram bins. The channel representation has been used in image analysis and vision by e.g. and [3].

To increase the importance of peaks, a simple difference filter is used.

$$h_1(x) = h(x) - \alpha \cdot \frac{d^2 h(x)}{dx^2}$$

Where  $h(x)$  is the smoothed histogram. This filter exaggerates peaks (local maxima). To increase the importance of frequent intensity levels, the dynamics of the histogram can be modified through gamma correction:

$$h_2(x) = h_1(x)^\gamma$$

This will make large values larger and small values smaller (in relation to each other).

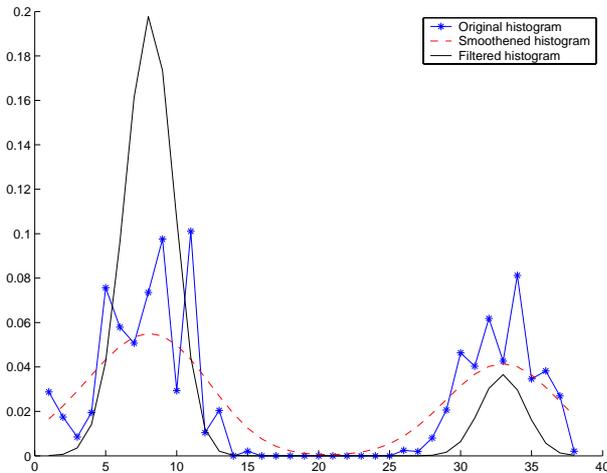


Figure 1: Example histogram before and after filtering

## Output Value Selection

Once the histogram has been estimated, the spatial structure of the local data is lost. Instead, we seek to find the most probable intensity value in the region and use as the new signal value. If we would e.g. choose the mass centre of the histogram as our estimate, we would obtain a result comparable to that of convolution with the spatial weights we used to obtain the histogram. The term *comparable* is used since the result would only be identical if there was

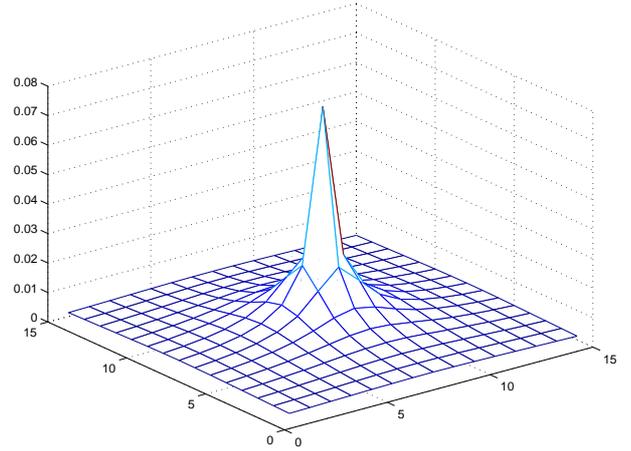


Figure 2: Spatial weights used to create the histogram.

no quantization in the histogram. A standard median filter is obtained by choosing the weights uniformly across the region and by setting the new value as the median in the histogram. If, however, the histogram is filtered as described above, the mass centre will reflect the frequent intensity levels more and the rare ones less. The result is a smoothing filter with edge preserving properties similar to those of a median filter, but softer in the respect that it does not simply choose one most probable value. Instead, it considers all values but weighted according to their frequency in a non-linear but continuous manner.

## 3 Experimental Results

The method was developed with noise reduction in ultrasound images in mind, but is most certainly interesting also in other fields. Here, the filter is applied to two different images. As comparison, the same images have been filtered using a median filter and a simple Gaussian filter. The first image (the ploop test pattern) is an image where Gaussian noise has been added artificially. SNR is approximately 12. The second image is an ultrasound image of a heart. No artificial noise has been added here.

Figure 2 shows the spatial weights used to create the histogram. The histogram was smoothed with a Gaussian filter with standard deviation 4 pixels.

## 4 Conclusions

It has been shown here that the notion of histogram filters can be used to design filters with quite desirable properties. Edges and other high frequency structures of an image are preserved, while more random structures are suppressed.

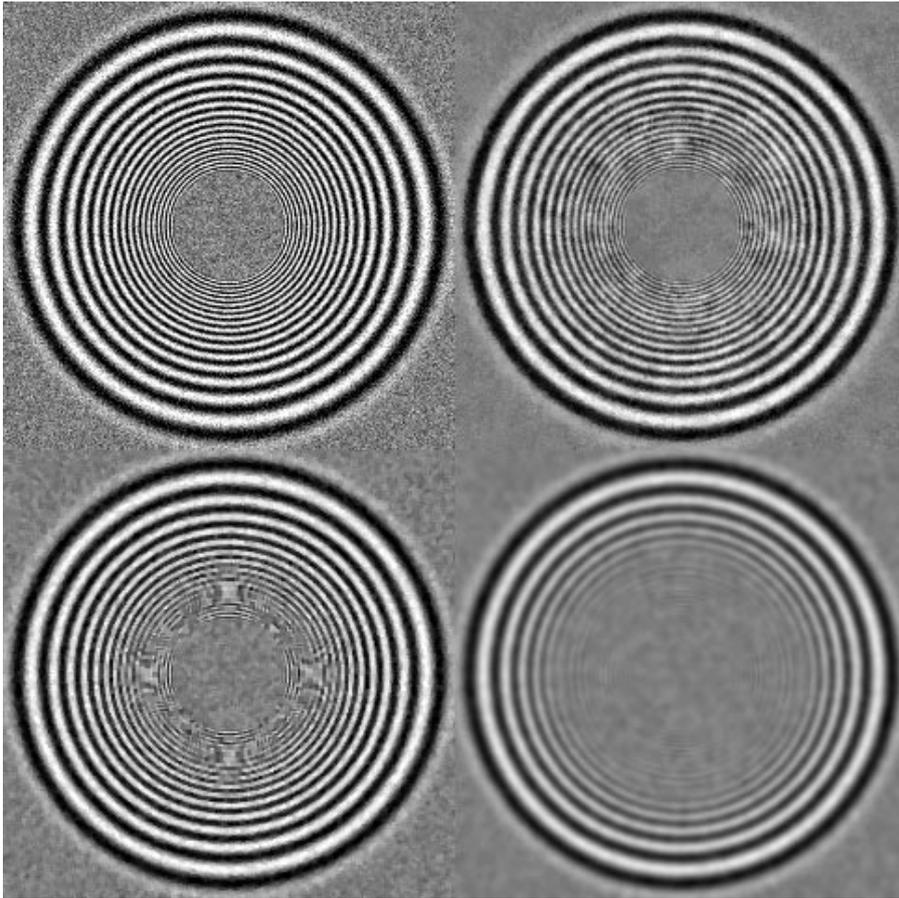


Figure 3: The ploop test image. Top left: Noisy signal, top right: Histogram filter. Bottom left: Median filter. Bottom right: Gaussian filter.

## 5 Future Work

Further work related to histogram filters should probably be focused on statistics and how the method described here relates to theory of mathematical statistics. Another possible focus is on anisotropic spatial weights. With isotropic weights, structures such as thin lines are suppressed due to their modest area.

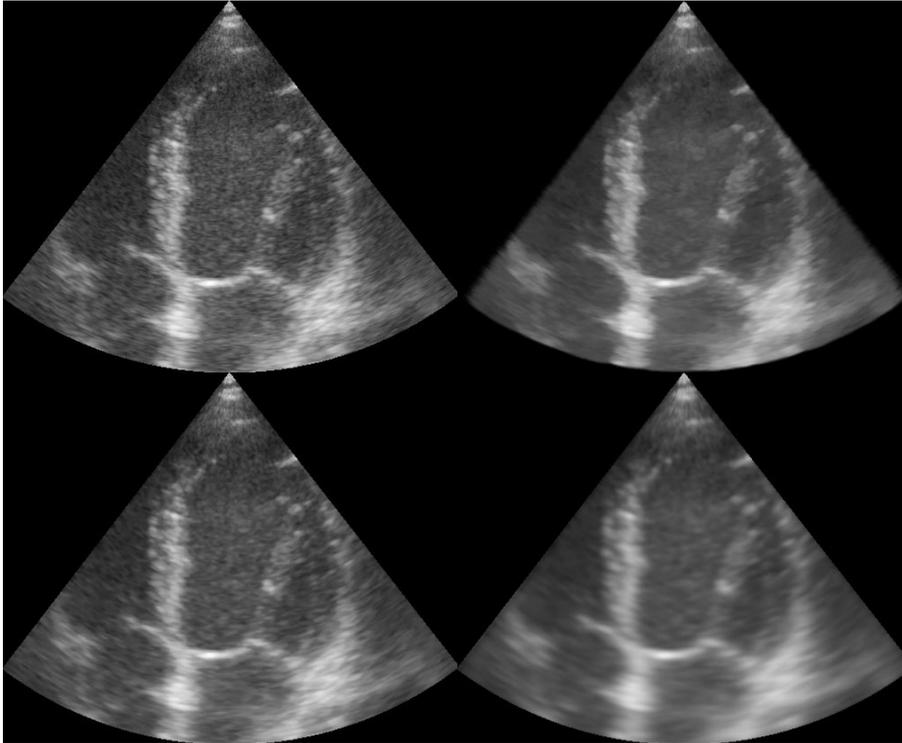


Figure 4: Echocardiography image. Top left: Noisy signal, top right: Histogram filter. Bottom left: Median filter. Bottom right: Gaussian filter.

## References

- [1] P-E. Forssén. Image analysis using soft histograms. In *Proceedings of the SSAB Symposium on Image Analysis*, March 2002.
- [2] R. M. Gray. Dithered quantizers. *IEEE Transactions on Information Theory*, 39:805–812, 1993.
- [3] K. Nordberg, G. Granlund, and H. Knutsson. Representation and learning of invariance. Report LiTH-ISY-I-1552, Computer Vision Laboratory, SE-581 83 Linköping, Sweden, 1994.
- [4] E. Parzen. On estimation of probability density functions and mode. *Annals of Mathematical Statistics*, 33:1065–1076, 1962.
- [5] A. Rosenfeld and A. C. Kak. *Digital Picture Processing*. Academic Press, Inc., second edition, 1982. ISBN 0-12-597301-2.