

Linköping University Post Print

**Stimulated Brillouin scattering of
electromagnetic waves in magnetized plasmas**

P K Shukla and Lennart Stenflo

N.B.: When citing this work, cite the original article.

Original Publication:

P K Shukla and Lennart Stenflo, Stimulated Brillouin scattering of electromagnetic waves in magnetized plasmas, 2010, JOURNAL OF PLASMA PHYSICS, (76), Part 6 Sp. Iss. SI, 853-855.

<http://dx.doi.org/10.1017/S0022377810000504>

Copyright: Cambridge University Press

<http://www.cambridge.org/uk/>

Postprint available at: Linköping University Electronic Press

<http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-62731>

Stimulated Brillouin scattering of electromagnetic waves in magnetized plasmas

P. K. SHUKLA¹ and L. STENFLO²

¹International Centre for Advanced Studies in Physical Sciences, Fakultät für Physik und
Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany
(profshukla@yahoo.de, ps@tp4.rub.de)

²Department of Physics, Linköping University, SE-58183 Linköping, Sweden

(Received 13 July 2010; revised 14 July 2010; accepted 14 July 2010,
first published online 17 August 2010)

Abstract. A previous theoretical result on stimulated Brillouin scattering is corrected.

Using the Jicamarca facility in Peru, it has been verified [1] that the threshold values for stimulated Brillouin scattering [2, 3] can be exceeded in ionospheric experiments. Recently, Bernhardt et al. [4] presented additional experimental observations (from the High Frequency Active Auroral Research Program (HARP) facility in Alaska) and a theoretical model for stimulated Brillouin scattering instabilities (SBSIs) for a magnetized ionospheric plasma. The SBSIs reveal that a large-amplitude electromagnetic (EM) pump wave (with frequency $f_0 = 4.5$ MHz) decays into a daughter EM wave and low-frequency electrostatic ion-acoustic (EIA) and electrostatic ion-cyclotron (EIC) waves. In this comment paper, we argue that (1)–(4) of [4] do not correctly describe the nonlinear couplings between the EM pump and daughter EM waves and the EIA/EIC wave in the magnetized ionospheric plasma. Specifically, we notice that the nonlinear coupling constant appearing in the right-hand side of (3) is erroneous, since it contains the factor $(\Omega_e^2 - \omega_p^2)$ in the denominator, and that the coupling constant contains asymmetric terms involving the electron gyro and EM pump wave frequencies in the numerator of (3). Hence, the underlying physics of the low-frequency ponderomotive force, which comes from the averaging (over the high-frequency EM wave periods) of the advection and nonlinear Lorentz forces in the inertialess electron momentum equation, has to be corrected in [4].

In what follows, we shall remedy the theoretical model of [4] and present a correct description of the SBSIs that have been observed by the HARP facility and that have been reported in [4, 5]. At this point, it is worth emphasizing that in the past many authors [6–8] have presented rigorous investigations of scattering instabilities of high-frequency EM pumps of arbitrary polarizations, and that Mendonça et al. [9] in addition have examined stimulated Raman and Brillouin backscattering instabilities of EM beams carrying orbital angular momentum in an unmagnetized plasma. Unfortunately, it appears that the authors of [4] have overlooked the work in [8], which dealt with a comprehensive treatment of SBSIs in magnetized plasmas that is relevant for the observations reported in [4].

The plasma parameters at 178 Km altitude in the Earth's ionosphere are such that the square of the pump wave frequency is much larger than the square of the electron gyrofrequency Ω_e . In such a situation, the nonlinear dispersion relation for

SBSIs is [6–8]

$$\frac{1}{\chi_e(\omega, \mathbf{k})} + \frac{1}{1 + \chi_i(\omega, \mathbf{k})} \approx \frac{k^2 |\mathbf{k}_s \times \mathbf{u}_0|^2}{k_s^2 D_s}, \quad (1)$$

where $\chi_e(\omega, \mathbf{k})$ and $\chi_i(\omega, \mathbf{k})$ are the low-frequency electron and ion susceptibilities for EIA and EIC waves, ω and \mathbf{k} are the frequency and the wave vector, $\mathbf{k}_s = \mathbf{k} - \mathbf{k}_0$ is the wave vector of the daughter EM sideband, \mathbf{k}_0 is the wave vector of the EM pump wave, $\mathbf{u}_0 = e\mathbf{E}_0/m_e\omega_0$ is the electron quiver velocity in the EM pump with the electric field \mathbf{E}_0 and the frequency $\omega_0 = (k_0^2 c^2 + \omega_{pe}^2)^{1/2}$, e is the magnitude of the electron charge, m_e is the rest mass of the electron, c is the speed of light in vacuum, and ω_{pe} is the electron plasma frequency. Furthermore, $D_s = k_s^2 c^2 - \omega_s^2 + \omega_{pe}^2 + i\nu_e \omega_{pe}^2/\omega_0 \approx 2\omega_0(\omega + i\Gamma - \delta)$, where $\omega_s = \omega - \omega_0$, $\Gamma = \nu_e \omega_{pe}^2/2\omega_0^2$, ν_e is the electron collision frequency, $\delta = \mathbf{k} \cdot \mathbf{v}_g - \Delta$, $\mathbf{v}_g = \mathbf{k}_0 c^2/2\omega_0$ is the group velocity of the EM pump wave, and $\Delta = k^2 c^2/2\omega_0$. The external magnetic field is $B_0 \hat{\mathbf{z}}$, where B_0 is the strength of the geomagnetic field and $\hat{\mathbf{z}}$ is the unit vector along the z -axis in a Cartesian coordinate system.

Since the parallel (to $\hat{\mathbf{z}}$) phase speed of the electrons in the low-frequency electrostatic EIA and EIC waves is much smaller than the electron thermal speed, the electron susceptibility for $\omega \ll \Omega_e$ is

$$\chi_e \approx 1/k^2 \lambda_{De}^2, \quad (2)$$

where $\lambda_{De} = C_s/\omega_{pi}$ is the electron Debye radius, C_s is the ion sound speed, and ω_{pi} is the ion plasma frequency. For $\omega \gg kV_{Ti}$ and $k_\perp \rho_i \ll 1$, we have

$$\chi_i(\omega, \mathbf{k}) \approx -[\omega_{pi}^2 k_\perp^2 / (\omega^2 - \Omega_i^2) k^2] - \omega_{pi}^2 k_z^2 / \omega^2 k^2, \quad (3)$$

where V_{Ti} is the ion thermal speed, $\rho_i = V_{Ti}/\Omega_i$ is the ion thermal gyroradius, Ω_i is the ion gyrofrequency, $k = (k_\perp^2 + k_z^2)^{1/2}$, and the subscripts \perp and z stand for the components of the wave vector across and along $\hat{\mathbf{z}}$, respectively. Accordingly, (1) gives

$$(\omega + i\Gamma - \delta)(\omega^4 - \omega^2 \Omega_{IC}^2 + k_z^2 C_s^2 \Omega_i^2) = (\Omega_i^2 k_z^2 - \omega^2 k^2) \omega_{pi}^2 u_0^2 \sin^2 \varphi / 2\omega_0, \quad (4)$$

where φ is the angle between \mathbf{k}_s and \mathbf{u}_0 and $\Omega_{IC} = (\Omega_i^2 + k^2 C_s^2)^{1/2}$. For SBSI involving an obliquely propagating EIA wave (with $\omega \ll \Omega_{IC}$ and $k_z \ll k_\perp$), we let $\omega \sim \Omega_{IA} \approx k_z C_s / (1+b)^{1/2} \sim \delta$, where $b = k^2 C_s^2 / \Omega_i^2$, to obtain from (4) the maximum growth rate ($\gg \Gamma$):

$$\gamma_{IA} \approx k_z u_0 \omega_{pi} / 2(1+b) \sqrt{\omega_0 \Omega_{IA}}, \quad (5)$$

while for SBSI involving the EIC wave, with the frequency $\omega (\gg \sqrt{k_z C_s \Omega_i}) = \Omega_{IC} \sim \delta$, the maximum growth is

$$\gamma_{IC} \approx k_\perp u_0 \omega_{pi} [1 + bk_z^2/k_\perp^2 (1+b)]^{1/2} / 2\sqrt{\omega_0 \Omega_{IC}}. \quad (6)$$

Thus, the ratio γ_{IC}/γ_{IA} differs significantly from that given by (4) in [4].

In conclusion, we mention that the present investigation also holds for studies of SBSI involving an EM pump wave-carrying orbital angular momentum, when the EM pump wave has a Laguerre–Gaussian profile [10].

Acknowledgements

The authors dedicate this paper to Jose Tito Mendonça to honor him on the occasion of his 65th birthday on 28 December, 2010. Tito is a great scholar of our modern times and has significantly contributed to the advancement of the frontiers of nonlinear physics.

This research was partially supported by the Deutsche Forschungsgemeinschaft through the project SH21/3-1 of the Research Unit 1048.

References

- [1] Fejer, J. A., Rinnert, K. and Woodman, R. 1978 *J. Geophys. Res.* **83**, 2133.
- [2] Sjölund, A. and Stenflo, L. 1967 *Appl. Phys. Lett.* **10**, 201.
- [3] Dysthe, K. B., Leer, E., Trulsen, J. and Stenflo, L. 1977 *J. Geophys. Res.* **82**, 717.
- [4] Bernhardt, P. A., Selcher, C. A., Lehmberg, R. H., Rodriguez, S. P., Thomason, J. F., Groves, K. M., McCarrick, M. J. and Frazer, G. J. 2010 *Phys. Rev. Lett.* **104**, 165004.
- [5] Bernhardt, P. A., Selcher, C. A., Lehmberg, R. H., Rodriguez, S., Thomason, J., McCarrick, M. and Frazer, G. 2009 *Ann. Geophys.* **27**, 4409.
- [6] Yu, M. Y., Spatschek, K. H. and Shukla, P. K. 1974 *Z. Naturforsch.* **29A**, 1736; Stenflo, L. 1981 *Phys. Rev. A* **23**, 2730.
- [7] Shukla, P. K. and Tagare, S. G. 1979 *J. Geophys. Res.* **84**, 1317.
- [8] Stenflo, L. 1995 *J. Plasma Phys.* **53**, 213; 2004 *Phys. Scr.* **T107**, 262.
- [9] Mendonça, J. T., Thidé, B. and Then, H. 2009 *Phys. Rev. Lett.* **102**, 185005.
- [10] Allen, L., Beijersbergen, M. W., Spreeuw, R. J. C. and Woerdman, J. P. 1992 *Phys. Rev. A* **45**, 8185.