Conflict Detection Metrics for Aircraft Sense and Avoid Systems^{*}

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Abstract: The task of an airborne collision avoidance system is to continuously evaluate the risk of collision and in the case of too high risk initiate an evasive action. The traditional way to assess risk is to focus on a critical point of time. A recently proposed alternative is to evaluate the cumulated risk over time. It is the purpose of this contribution to evaluate the difference between these two concepts and also to validate an approximate method for computing the cumulated risk, suitable for real-time implementations. For this purpose, random scenarios are generated from stochastic models created from observed conflicts. A realistic tracking filter, based on angle-only measurements, is used to produce uncertain state estimates which are used for risk assessment. It is shown that the cumulated risk is much more robust to estimation accuracy than the maximum of the instantaneous risk. The intended application is for unmanned aerial vehicles to be used in civilian airspace, but a real mid-air collision scenario between two traffic aircraft is studied as well.

1. INTRODUCTION

Unmanned aerial vehicles (UAVs) are likely to be a much more common element in aviation in the near future. There are many possible applications which will require the UAV to operate in the same airspace as other aircraft, both manned and unmanned. One critical problem that has to be dealt with is therefore how to avoid collisions between the UAV and other aircraft.

A near mid-air collision (NMAC) occurs if the distance between two aircraft ever becomes less than a predefined safety distance of $R = 150$ m. For the UAV to be able to avoid NMACs it needs to be equipped with a collision avoidance system. One critical component of this is the risk assessment module for conflict detection Kuchar and Yang [2000]. The idea is to use information supplied by sensors, for a UAV limited basically to an electro-optical (EO) camera, to determine the probability of NMAC. Computing the analytical probabilities is in general impossible and numerical approximations are therefore necessary. An arbitrarily accurate approach is to apply Monte Carlo approximation Yang et al. [2004], Jansson and Gustafsson [2008]. This might however not be feasible in a real-time implementation.

A common approach to risk assessment is to focus on a critical point of time (time of closest point of approach, time of maximum risk) as investigated in Chan [2001, 2003], Krozel and Peters [1997], Prandini et al. [2000]. This time instant becomes stochastic when it is based on uncertain state estimates and the mode or mean of the distribution can be quite misleading for decisions. A sounder approach is to use the cumulated risk over a critical time horizon. This concept is studied in Nordlund and Gustafsson [2008a,b] for a UAV application, and a sound numerical approximation is proposed.

It is the purpose of this contribution to evaluate the sensitivity of instantaneous risk and cumulated risk for realistic conflict situations. The two different concepts are presented in Section 2. It is also the purpose to evaluate the accuracy of the numerical approximation for risk computation proposed in Nordlund and Gustafsson [2008a,b]. Section 3 briefly presents this approximation and also shows how the probabilities can be calculated using Monte Carlo approximation. For generating conflict scenarios an encounter model, presented in Section 4, is used. The model is based on physical reasoning about the problem together with the statistical modeling from a large amount of surveyed data done in Kochenderfer et al. [2008]. In Section 5 simulation results are given and Section 6 evaluates a scenario based on a real Brazilian accident NTSB [2006]. Finally, in Section 7 conclusions are drawn.

2. RISK ASSESSMENT CONCEPTS

2.1 Maximum instantaneous and cumulated risks

Once a possible threat is detected by the UAV, it is continuously monitored by a target tracking system. This system delivers an estimate of the intruder's state, $\hat{x}$, and a corresponding covariance matrix, P . Based on these, the true state of the intruder can be seen as stochastic

$$x \sim N(\hat{x}, P) \quad (1)$$

The purpose of the risk calculation step is to use the tracking output to determine the risk for NMAC. The first problem that needs to be dealt with is how to define the probability of NMAC. A proper definition is necessary to get a method for risk computation which is robust to the large uncertainties associated with angle-only (i.e. camera based) tracking. Two different viewpoints will be analyzed. First, when the event of NMAC is considered to occur only momentarily, second when the event of NMAC is considered over a period of time.

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Let t be the absolute time, t_p the prediction time and T_p the prediction time horizon. At every point of time t the risk will be calculated over the prediction horizon $0 < t_p < T_p$. Using the maximum instantaneous probability we wish to compute

$$\max_{0 < t_p < T_p} P(NMAC_{t_p}) = \max_{0 < t_p < T_p} P(|s(t_p)| < R) \quad (2)$$

where $s(t_p)$ is the predicted relative distance between the two aircraft (at time t_p) derived from the current state, $x(t)$. This variable will thus be stochastic. $R = 150$ m is the predefined radius of the safety zone. This probability can be interpreted as; *from the current time and during the specified time horizon, how high is the risk of NMAC when the risk is at its peak?*

In contrary, the probability of NMAC over a period of time (cumulated risk) is

$$P(NMAC_{(0,T_p)}) = P\left(\min_{0 < t_p < T_p} |s(t_p)| < R\right) \quad (3)$$

which can be interpreted as; *from the current time and during the specified time horizon, how high is the risk that NMAC will occur at any time?*

An interesting fact is that

$$\int_0^{T_p} P(NMAC_{t_p}) dt \neq P(NMAC_{(0,T_p)}) \quad (4)$$

The cumulated collision risk can thus not be calculated in any simple manner based on the instantaneous risk.

2.2 One-dimensional illustration

It is believed that the probability according to (2) will be diminished due to the large uncertainties associated with angle-only tracking. To exemplify this, assume that the scene is one-dimensional, i.e. that all motion takes place along a single line. The relative distance between the aircraft, $s(t_p)$, is assumed to have the PDF (probability density function) $p_{s(t_p)}(s)$. The probability of NMAC according to (2) can then be calculated from

$$P(NMAC_{t_p}) = P(|s(t_p)| < R) = \int_{-R}^R p_{s(t_p)}(s) ds \quad (5)$$

Now, consider Figure 1 where the PDF is depicted at three different prediction time instants. The figure shows two instances of the PDF, one when the uncertainty is large and one when it is small. It is apparent that the collision risk according to (5) will be different depending on which one of the two PDFs that is used. Possible values of the integral are shown in Figure 2.

The maximum probability will be lower when the uncertainty is large, even though the actual collision risk might be high. It is expected that the definition of NMAC according to (3) does not suffer from this drawback since the entire time horizon is taken into account.

2.3 Application to collision avoidance systems

The purpose of the collision avoidance system is to initiate an evasive maneuver if the computed risk is considered to be too high. However, even if the indicated risk is high for the current state, it is not always desirable to trigger the avoidance. It could very well be that this risk decreases in the near future, for instance if better measurements become available or if the intruder changes its course. A better approach is to reason like this; if the evasive maneuver should be triggered now, then what would the

Fig. 1. The PDFs for aircraft separation at three points of time. Two functions are depicted, one where the uncertainty is large (dashed line) and one where it is small (solid line).

Fig. 2. $\int_{-R}^R p_{s(t_p)}(s) ds$ for two probability density function, one with large and one with small variance.

Fig. 3. Risk calculation at three subsequent points of time. The evasive maneuver is triggered in the last view, where $P(NMAC|evasion) \geq 0.02$.

probability of NMAC be? If this probability is too high, initiate the maneuver, otherwise keep to the mission. This strategy is illustrated in Figure 3.

The choice of threshold level, at which the evasive action is triggered, needs to be a tradeoff between obtaining robustness in the system and keeping the NMAC frequency low. If the threshold is set too low, evasive maneuvers

will be triggered even when it is unnecessary which is an undesired and possibly dangerous behavior. TCAS (traffic collision avoidance system) which is applied in commercial aviation can detect and avoid NMAC, given a collision scenario, with a probability of approximately 0.91 Arino et al. [2002]. By using this level as a point of reference, an appropriate threshold can be set. The value 0.09 should however be seen as a total outcome acceptance. Since there are other aspects affecting the performance of the collision avoidance system, such as inaccuracies in the tracking and missed detections, the threshold must be set lower. A value of 0.02 will thus be used throughout this article.

3. METHODS FOR RISK COMPUTATION

Regardless of which one of the two risk assessment concepts that is used, a method for computing the probabilities is needed. This section presents two such methods, first a sampling based approximation which can be used for both concepts, (2) and (3), second a sound geometrical and numerical approximation which is designed for (3).

3.1 Monte Carlo approximation

In Monte Carlo approximation (not to be confused with Monte Carlo simulations that are used for evaluation, Section 4), a large number of samples (N) are drawn from (1). For each of these the relative distance $s_i(t_p)$, $i = 1, \dots, N$ will be deterministic.

The probability of NMAC according to either (2) or (3) can then be approximated with the outcome of the sampling

$$P(NMAC_{t_p}) \approx \frac{1}{N} \sum_{i=1}^N (|s_i(t_p)| < R) \quad (6)$$

and

$$P(NMAC_{(0,T_p)}) \approx \frac{1}{N} \sum_{i=1}^N \left(\min_{0 < t_p < T_p} |s_i(t_p)| < R \right) \quad (7)$$

This way of computing the probabilities can be made arbitrary accurate by choosing a sufficiently large N . To get an appropriate accuracy for this specific problem it is required to use $N \geq 90000$ Nordlund and Gustafsson [2008a].

3.2 Geonumerical approximation

Monte Carlo approximation is in general not suitable for real-time implementations due to the large number of samples required for evaluating small risks. In Nordlund and Gustafsson [2008a,b] a method for calculating the probability of NMAC according to (3), i.e. $P(NMAC_{(0,T_p)})$, is proposed. This method will perform the risk calculation roughly 1000 times faster than the corresponding Monte Carlo approximation Nordlund and Gustafsson [2008a]. If it can be deduced that the definition of NMAC according to (3) is to prefer, this method would thus induce a great benefit to the risk calculation step of a collision avoidance system.

4. RISK SCENARIOS

The performance of the collision avoidance system is evaluated through Monte Carlo simulations. Figure 4 shows the geometry of a collision scenario. The trajectories of the involved aircraft are defined by their initial positions and the angles of their velocity vectors relative to the axes

of the system. Let the own vehicle initially be placed at the origin and the intruder on the positive x -axis. The own vehicle holds the course given by the angle ψ and descends with the angle γ (or ascends if γ is negative). The intruder's course is given by α and the angle of descent is β .

Fig. 4. Geometry of a collision scenario.

The scenario parameters are summarized in Table 1. The basic idea is as follows. The own vehicle is engaged in level flight at 50 m/s, i.e. $\gamma = 0$ and $v_{own} = 50$ m/s. The angle of approach between the aircraft is $\psi = 20^\circ$. In the nominal case, the aircraft will collide after approximately 32 s. The intruder's state vector is random according to an encounter model based on the extensive statistical modeling done in Kochenderfer et al. [2008]. The climb angle and speed are randomized, and the critical course angle leading to collision is computed and slightly perturbed. Besides the intruders state, also the detection distance for a representative EO camera is random with a model based on Holst [2006].

Table 1. Scenario parameters

Parameter	Value (SI units)
ψ	Own course angle
γ	Own climb angle
v_{own}	Own speed
r	Detection distance
α	Intruder course angle
β	Intruder climb angle
v_{int}	Intruder speed
	Value (SI units)
ψ	0.349 ($= 20^\circ$)
γ	0
v_{own}	50
r	$\Gamma(22.85, 83.68) + 1200$, Figure 5
α	$\arcsin\left(-\frac{v_{own}}{v_{int}} \frac{\sin \psi \cos \gamma}{\cos \beta}\right) + U(-0.04, 0.04)$
β	Figure 5
v_{int}	$\Gamma(5, 8) + 15$, Figure 5

Fig. 5. Detection distance r probability (top), PDF for intruder's speed v_{int} (bottom left) and PDF for intruder's angle of descent β (bottom right).

Figure 6 shows the geometry of the scenario seen from above. Observe that the angle of descent for the intruder, β , is possibly nonzero.

4.1 Tracking framework

To supply the risk calculation with information of the intruder, a tracking system is implemented in the simulation

Fig. 6. Geometry for a collision scenario.

environment. The tracking is based on angle-only information, possibly supplied by a simple sensor such as a digital video camera. The system is implemented as an extended kalman filter working in modified spherical coordinates, a so called MSC-EKF Blackman, S. and Popoli, R. [1999], Allen, R.R. and Blackman, S.S. [1991].

A well known problem with angle-only tracking is that range is unobservable. To cope with this, the own vehicle can perform a platform maneuver. To get a high performance out of the tracking, a quick and large maneuver is to prefer Holtsberg [1992]. From other aspects this can be far from optimal though. The choice of platform maneuver must be a compromise between gaining observability and interfering with the mission as little as possible.

As mentioned in Section 2 it is expected that the definition of NMAC according to (3) is more robust against large uncertainties in the state estimate than the definition according to (2). Due to this it is of interest to investigate whether this method for risk calculation can perform well even for an insignificant platform maneuver. Two different maneuvers, illustrated in Figure 7, will thus be used in this article. The first is a so called SS-turn in the horizontal plane; a maneuver that is chosen especially to increase observability in range. The second maneuver can be seen as an attempt to use the natural variations in the aircraft's motion as an own platform maneuver. The maneuver is a small sinusoidal movement in the vertical plane. This maneuver induces a small disturbance to the UAV's mission.

Fig. 7. SS-turn seen from above (left) and small sinusoidal maneuver seen from the side (right). Observe the different scales on the axes.

To see how the choice of platform maneuver influences the quality of the range estimate, three sets of Monte Carlo simulations have been generated. In one of the simulations no platform maneuver at all is used. The other two make use of the maneuvers presented above. Figure 8 shows the RMSE (root mean squared error) for the relative error in range.

When the SS-turn is used the estimate converges fast during the maneuver. After the maneuver the error starts to increase though. When the maneuver no longer is performed, range once again becomes unobservable. For the small sinusoidal platform maneuver the estimate converges, but not as fast as when the SS-turn is used. When no platform maneuver at all is used, range is unobservable and the relative error increases from the start. The platform maneuver seems thus to have the desired effect on the state estimation.

Fig. 8. RMSEs in $\hat{r}_{rel}$ when an SS-turn (solid line), a small sinusoidal movement (dashed line) and no platform maneuver (dash-dotted line) are used respectively.

5. EVALUATION STUDY

To compare the definitions and algorithms for maximum instantaneous and cumulated risk respectively, a large number of Monte Carlo simulations have been performed. The risk calculation is based on information supplied by the tracking filter output. These state estimates are supposed to resemble those in a real UAV application. In each simulation the own vehicle uses one of the two considered platform maneuvers, either an SS-turn or a small sinusoidal maneuver. The risk is computed with Monte Carlo approximation. When the indicated risk, given an evasive action, becomes larger than the threshold of 0.02, the maneuver is initiated.

5.1 Comparing instantaneous and cumulated risk

The resulting outcome frequencies for 10000 Monte Carlo simulations are given in Table 2.

Table 2. Outcome frequencies

Conditions	Nr of NMACs	Frequency
Maximum instantaneous risk		
SS-turn	579	0.058
Small sinusoidal maneuver	710	0.071
Cumulated risk over a horizon of time		
SS-turn	436	0.044
Small sinusoidal maneuver	394	0.039

An interesting observation is that the NMAC frequencies obtained with the cumulated risk concept appear to be approximately the same for the two platform maneuvers. This is not the case for the definition of NMAC using instantaneous risk. In this case the NMAC frequency is higher for the small sinusoidal maneuver than it is for the SS-turn.

To verify the significance in this observation, statistical hypothesis tests are conducted. Let the observed number of NMACs for one of the risk assessment concepts be f_1 and f_2 , for the two platform maneuvers respectively. If the corresponding NMAC probabilities are p_1 and p_2 and the number of simulations is M we have that

$$f_1 \sim \text{Bin}(M, p_1) \approx N(Mp_1, \sqrt{Mp_1(1-p_1)}) \quad (8)$$

$$f_2 \sim \text{Bin}(M, p_2) \approx N(Mp_2, \sqrt{Mp_2(1-p_2)}) \quad (9)$$

A hypothesis test for finding out significant differences in performance, $H_0 : p_1 = p_2 = p$, is then straightforward to perform. These tests confirm the conclusion that the maximum instantaneous risk is affected negatively from the increased uncertainty in the estimate, whereas the cumulated risk is more robust. It should be mentioned that

many other scenarios with different initial states have been performed with the same conclusion.

5.2 Comparing MC and geonumerical approximations

Simulations are here performed to determine the correctness in the geonumerical approximation. Since the geonumerical method is designed for the definition of NMAC according to (3) only this risk assessment concept will be considered throughout this section. The probability of NMAC over a given time horizon is computed with both Monte Carlo approximation and the geonumerical method. Figure 9 shows the calculated probabilities given an evasive action, i.e. $P(NMAC_{(0,T_p)}|evasion)$, for one simulation.

Fig. 9. Calculated probabilities according to Monte Carlo approximation (solid) and geonumerical approximation (dashed) for one simulation.

When the threshold level of 0.02 is reached, the evasive maneuver is triggered. The aircraft will naturally continue to approach each other and the collision risk is continuously evaluated. It is thus possible that the risk will rise above the threshold, as in Figure 9, dependent on the tracking output. However, in this case the avoidance was successful since the probabilities of NMAC decrease towards zero again. As can be seen in the figure, there is a good correspondence between the two methods. The difference between the calculated probabilities according to the two methods is depicted in Figure 10. The conclusion is that there is a small difference when the risk becomes high, but mostly the difference is within the confidence bound of the Monte Carlo method.

Fig. 10. Difference between the calculated probabilities according to Monte Carlo approximation and geonumerical approximation for one simulation. The dotted lines show the 3σ -levels for the Monte Carlo approximation.

6. CASE STUDY ON A REAL ACCIDENT

Even though the collision avoidance system evaluated in this article is focused on a UAV application, it is possible to find other scopes of use, such as commercial aviation. Simulations have therefore been performed on a scenario which is a replication of a real mid-air collision which occurred over the Brazilian Amazon jungle on September 29, 2006. The two involved aircraft, one Boeing 737-800 and one Embraer Legacy 600 business jet, collided in mid air at 4:57 pm Brasilia standard time NTSB [2006]. The Boeing 737 was destroyed by in-flight breakup and impact forces. All 154 passengers and crew were killed in the accident. The pilots of the Legacy managed to safely land their vehicle and the two crew members and five passengers were all uninjured NTSB [2006]. Figure 11 shows an illustration of the two aircraft just before the collision.

Fig. 11. The two aircraft just before the collision. This picture is created by the Wikimedia Commons user *Anynobody* and it is licensed under the Creative Commons Attribution ShareAlike license versions 3.0, 2.5, 2.0, and 1.0.

Due to several misfortunate events the aircraft were directed to fly on the same altitude in opposite directions, and were thus on a direct head-on course. Both aircraft were equipped with TCAS which uses information shared directly between the aircraft via transponders. In this particular case, the transponder on the Legacy did however not communicate with the Boeing 737, either because it was malfunctioning or turned off by mistake NTSB [2006]. Due to this, *neither* of the two aircraft's collision avoidance systems was working. This is a drawback with TCAS which could be handled by equipping the aircraft with self-contained collision avoidance systems, i.e. systems not dependent on communicating with each other. A tracking based system could for instance serve as a complement to TCAS and hence increase the safety when the primary system fails.

No platform maneuver is used in the simulations on this scenario, since this would be impractical for a manned aircraft. One possible solution to the unobservability problem is to use a range measuring sensor, such as the radar. This option is however not considered in this article. Instead, the same kind of passive angle-only sensor as used for the UAV evaluation is employed to see if this is applicable in this case as well.

During the simulation, the collision risk is computed with the geonumerical approximate method (see Section 3.2). When the risk exceeds the threshold an evasive maneuver is initiated. The maneuver consists of a 0.8g dive with a 1 s delay, i.e. it takes 1 s after the maneuver is triggered until the acceleration is applied. When the threshold was set to 0.02 the simulation resulted in a minimum separation of 205 m, i.e. NMAC was avoided. This is a very

positive result which suggests that the use of a simple self-contained collision avoidance system could have averted the accident.

One problem with this result though is that the evasive maneuver was triggered very soon, merely 1.4 s, after the intruder was detected. This is due to the very rapid course of events in this scenario; if no avoidance is performed the aircraft will collide in just below 10 s. To obtain a robust collision avoidance system it is necessary to give the system more time before the evasive action is triggered. To accomplish this, several additional simulations have been performed on the same scenario with an increasing threshold level. If the threshold is set to a higher value, the system will wait longer before any action is taken which allows it to become more certain of the coming collision. Figure 12 shows the minimum separation between the aircraft as function of the threshold level. The figure also shows the time between detection and initiation of the evasive action.

Fig. 12. Minimum separations (solid-dotted line) and time between detection and evasive action initiation (solid line) as functions of the threshold level.

When the threshold is raised above approximately 0.07 the minimum separation becomes less than 150 m and there will be an NMAC. However, even if the threshold is set as high as 0.9 the minimum separation is 54 m, i.e. an NMAC occurs but the collision is avoided. In this case the evasive action is triggered 5 s after the detection, which is roughly half the time to collision. This result suggests that the use of a self-contained collision avoidance system could serve as a backup for a primary system like TCAS. By using a high threshold level the system will be more robust and it can still be enough to avoid a collision that would have occurred otherwise.

7. CONCLUSION

Risk computations in general and for UAV applications in particular have been studied. The traditional maximum risk, defined as the maximum over time of the instantaneous risk, was compared to the cumulated risk, defined as the total risk over a time horizon. An encounter model was used to generate a large number of random conflict scenarios. The model is based both on reported conflicts as presented in Kochenderfer et al. [2008] and on an additional investigation of typical conflict situations. An angle-only sensor (EO camera) suitable for UAVs was used in combination with an EKF based on modified spherical coordinates to generate realistic state estimates. One important conclusion from a large number of Monte Carlo simulations from different initial conditions is that the cumulated risk

is much more robust to state uncertainty than the maximum instantaneous risk. For that reason, cumulated risk is recommended to be used for risk assessment. However, without an efficient numerical algorithm, its practical use is limited. For that reason, a recent geometrical-numerical (geonumerical) approximation was evaluated, and shown to perform well (within the statistical confidence margins) when compared with a Monte Carlo method for the same class of scenarios. It was also shown that a self-contained collision avoidance system, using a simple EO camera, is applicable in commercial aviation as well and could have averted severe accidents involving manned traffic aircraft.

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