Quantum contextuality for rational vectors

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The Kochen-Specker theorem states that noncontextual hidden variable models are inconsistent with the quantum predictions for every yes-no question on a qutrit, corresponding to every projector in three dimensions. It has been suggested [D. A. Meyer, Phys. Rev. Lett. **83**, 3751 (1999)] that the inconsistency would disappear when we are restricted to projectors on unit vectors with rational components; that noncontextual hidden variables could reproduce the quantum predictions for rational vectors. Here we show that a qutrit state with rational components violates an inequality valid for noncontextual hidden-variable models [A. A. Klyachko *et al.*, Phys. Rev. Lett. **101**, 020403 (2008)] using rational projectors. This shows that the inconsistency remains even when using only rational vectors.

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The Kochen-Specker theorem from 1967 [1] states that the quantum predictions from a three-dimensional quantum system (a qutrit) are inconsistent with noncontextual hidden variables. The proof uses 117 directions in three dimensions, arranged in a pattern such that they cannot be colored in a particular manner, see [1] for details. Later proofs use less directions, but one common feature (in the three-dimensional versions) is that the set of unit vectors includes irrational components. It was noted in [2] that the Kochen-Specker proof needs these irrational vectors to be completed. Indeed, when using only the rational subset of vectors, the set is colorable in the manner required by quantum mechanics. It was also suggested [2] that, for this reason, the inconsistency between quantum mechanics and noncontextual hidden variables disappears, and that quantum mechanics can be imitated by noncontextual hidden variable models restricted to rational vectors.

It has been recently shown [3] that the following inequality is a necessary and sufficient condition for qutrit noncontextual hidden variables, for measurements A_i with possible outcomes -1 and $+1$, such that A_i and A_{i+1} (modulo 5) are compatible:

$$\sum_{i=0}^{4} \langle A_i A_{i+1} \rangle \geq -3. \tag{1}$$

Using the rational qutrit state

$$\langle \psi | = \left(\frac{354}{527}, \frac{357}{527}, -\frac{158}{527} \right), \tag{2}$$

and the observables

$$A_i = 2|v_i\rangle\langle v_i| - \mathbb{1}, \tag{3}$$

associated to the rational vectors

$$\langle v_0| = (1, 0, 0)\,, \tag{4a}$$

$$\langle v_1| = (0, 1, 0)\,, \tag{4b}$$

$$\langle v_2| = \left(\frac{48}{73}, 0, -\frac{55}{73} \right), \tag{4c}$$

$$\langle v_3| = \left(\frac{1925}{3277}, \frac{2052}{3277}, \frac{1680}{3277} \right), \tag{4d}$$

$$\langle v_4| = \left(0, \frac{140}{221}, -\frac{171}{221} \right), \tag{4e}$$

we obtain a value of -3.941 for the left-hand side of (1), which deviates very little from the maximum violation at -3.944. Thus, even when using only rational vectors, the inconsistency is not nullified. The violation shows that the (physical content of) the Kochen-Specker theorem remains, namely, that the quantum-mechanical predictions cannot be reproduced by noncontextual hidden variables.

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