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## Liquid crystal light deflecting devices based on nonuniform anchoring

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Tunable liquid crystal light deflecting devices based on nonuniform anchoring energy are proposed. These devices have uniform thicknesses of the layers they are composed of, and beam deviation is controlled with a uniform electrical field. Potential applicability of such an approach in beam deflectors and active lenses is investigated. It is shown that the approach is a competitive alternative to liquid crystal light deflecting devices, in which the needed spatial distribution of liquid crystal molecules is achieved either due to nonuniform thickness or due to generation of nonuniform electrical field. © 2010 American Institute of Physics. [doi:10.1063/1.3526311]

Various liquid crystal (LC) light deflecting devices for manipulation of light propagation, such as active lenses, spatial light modulators, switchable diffraction gratings, have been developed.<sup>1–6</sup> The needed spatial distribution of LC molecules (director) in the earlier proposed methods<sup>1–6</sup> is achieved mainly by applying a nonuniform electrical field or by introducing thickness variations in the layered structure of a LC device. In this letter, we propose another approach, in which nonuniform anchoring energy plays the main role in achieving the desired LC director distribution. The main point is that the bulk distribution of the LC director under certain circumstances is defined by the anchoring strength.<sup>7–9</sup>

The benefits of such an approach are clear: a LC light steering device can have uniform thicknesses of the composing layers and, what is important, it can be controlled with a uniform external electric field, i.e., the electrodes of the device are continuous. This means that, in addition to simplicity of electrical driving, there is no initial periodical structure, which is a source of the undesired diffraction effects in the majority of the known LC light steering devices. We assume that further method development will lead to that the needed distribution of the anchoring energy will be possible to produce with photoalignment techniques.<sup>10–12</sup> The alignment energy can be controlled by the dose of UV light irradiation<sup>11</sup> or by the relief of the alignment layer formed with a holographic exposure method.<sup>12</sup> Moreover, it is possible to control the anchoring energy by varying the concentration of two materials in a mixture composing the alignment layer<sup>13</sup> or by the thickness of this layer.<sup>10</sup>

This letter has the following structure. First, we consider how a distribution of nonuniform anchoring energy can affect the phase profile of the outgoing light. Then, we examine nondiffractive elements and make an attempt to determine optimal geometrical parameters of a LC beam deflector and a LC lens, in particular, the thickness of the LC layer and the width of the working zone. Finally, we investigate optical phase arrays producing phase gratings with rectangular as well as sawtoothlike profiles.

Let us consider a LC cell having monotonically varying anchoring energy along the covered substrates. We introduce a coordinate system with the  $z$ -axis normal to the substrates and the  $x$ -axis in the direction of variation of the anchoring

energy. Evidently, this LC cell as a light deflecting element has the maximum effect when the threshold voltage of the Fredericksz transition for the area with the strong anchoring equals the saturation voltage of the area with the weak anchoring. The threshold voltage of a nematic LC in the case of splay deformation, zero pretilt angle, and elastic boundary coupling<sup>14</sup> is described by the equation<sup>7–9</sup>

$$\lambda u_F \tan(\pi u_F/2) = 1. \quad (1)$$

Here  $\lambda$  is the reduced surface-coupling parameter defined as  $\lambda = \pi K_{11}/Wd$ , where  $K_{11}$  is the splay elastic constant,  $W$  is the polar anchoring strength, and  $d$  is the thickness of the LC layer. Furthermore,  $u_F = V/V_c$ , where  $V$  is the voltage applied to the cell and  $V_c$  is the threshold voltage for rigid boundary coupling, which is given by  $V_c = \pi \sqrt{K_{11}/\epsilon_0 \Delta\epsilon}$ , where  $\Delta\epsilon$  is the dielectric anisotropy of the LC material and  $\epsilon_0$  the permittivity of free space.

The equation for the saturation voltage can be obtained from the results derived by Zhao *et al.*<sup>9</sup> Adaptation of Eqs. (39)–(41) in Ref. 9 to the splay deformation yields

$$\coth\left(\frac{\pi \gamma u_S}{2}\right) = \frac{\lambda u_S}{\gamma}. \quad (2)$$

Here  $\gamma = \sqrt{K_{11}/K_{33}}$ , where  $K_{33}$  is the bend elastic constant and  $u_S = V/V_c$ . In the equal-elastic-constant approximation, Eq. (2) transforms to the result obtained by Nehring *et al.*<sup>7</sup>

Combining Eqs. (1) and (2) when  $u_S = u_F$  enables one to express  $\lambda$  corresponding to the area with weak anchoring ( $\lambda^w$ ) through the  $\lambda$  corresponding to the area with strong anchoring ( $\lambda^s$ ), as well as through  $V$  and  $\gamma$ . If  $\lambda^s$  tends to zero, the requirement for  $W$  for the weak anchoring area is

$$W = \frac{\pi K_{11}}{d\gamma} \tanh\left(\frac{\pi \gamma}{2}\right). \quad (3)$$

In order to find the phase of the output light for an arbitrary director distribution, it is necessary to calculate the effective refractive index, which at normal incidence is expressed as

$$n_{\text{eff}} = \frac{1}{d} \int_0^d \frac{n_o n_e dz}{\sqrt{(n_o \sin \theta(z))^2 + (n_e \cos \theta(z))^2}}, \quad (4)$$

where  $n_o$ ,  $n_e$  are the principal refractive indices and  $\theta(z)$  is the zenithal angle of the director. The distribution  $\theta(z)$  is

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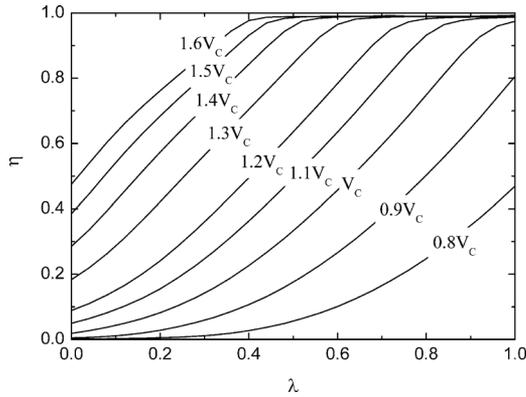


FIG. 1.  $\eta(\lambda)$  at different applied voltages relative to the threshold voltage for rigid boundary coupling  $V_c$ .

found by minimizing the free energy per unit area of the bounding plates<sup>14</sup> by taking into account elastic boundary coupling.<sup>15</sup>

To evaluate the influence of the reduced surface-coupling parameter  $\lambda$  on the phase of the outgoing light, we have computed the parameter  $\eta(\lambda) = (n_e - n_{\text{eff}}(\lambda)) / (n_e - n_o)$  at different values of  $V$  as presented in Fig. 1. The calculations were carried out for the pretilt angle of  $2^\circ$ , the elastic constants  $K_{11} = 1.55 \times 10^{-11}$  N,  $K_{33} = 2.8 \times 10^{-11}$  N, and  $\Delta\epsilon = 16.8$ , corresponding to the LC material E44 (Merck Co.).

Having the function  $\eta(\lambda)$  [or  $n_{\text{eff}}(\lambda)$ ], one can derive  $\lambda(x)$ , such that  $\eta(\lambda(x))$  [or  $n_{\text{eff}}(\lambda(x))$ ] gives the required phase profile of the output light: linear for a beam deflector or parabolic for a lens. If the distance  $D$  between the areas with weak and strong anchoring strengths is much larger than the thickness of the LC layer  $d$ , i.e.,  $D \gg d$ , a linear phase profile is achieved when

$$n_{\text{eff}}(x) = \frac{x \sin \alpha}{d} + n_{\text{eff}}(0), \quad (5)$$

where  $\alpha$  is the angle of light deflection. Usually  $\alpha$  is sufficiently small so that  $\sin \alpha$  in Eq. (5) can be replaced by  $\alpha$ . For birefringence  $\Delta n = 0.25$  and  $D \sim 10d$ , the maximum angle of the deflection is  $1.5^\circ$  at normal incidence and transmission mode.

The outgoing light possesses a parabolic phase profile under the condition

$$n_{\text{eff}}(x) = \frac{x^2}{2f} + n_{\text{eff}}(0), \quad (6)$$

where  $f$  is the focal distance of a LC lens.

From Eqs. (5) and (6) follows that in order to increase the deflecting angle of the beam deflector or to decrease the focal distance of the lens, it is necessary to reduce the distance  $D$  or to increase the thickness  $d$ . However, the benefits of reducing  $D$  are outweighed by diffraction effects, whereas increasing  $d$  requires very small anchoring energy (less than  $10^{-6} - 10^{-5}$  J/m<sup>2</sup>) which is not enough to affect molecular orientations. Moreover, increasing  $d$  leads to increasing the switching time. To overcome these difficulties, one has to design a light deflecting element based on diffractive optics.

When  $D$  is of the same order of magnitude as  $d$ , the flyback zone,<sup>6</sup> i.e., the transit zone between contiguous areas with different LC configurations, has to be taken into account. In order to evaluate the flyback zone, as well as influ-

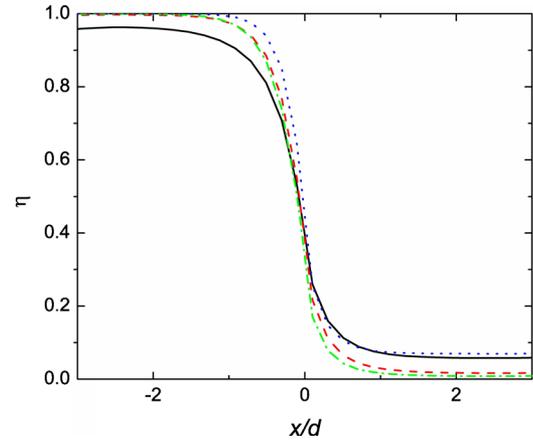


FIG. 2. (Color online)  $\eta$  vs the normalized coordinate  $x/d$  near the border of two areas with different anchoring energies under different conditions as discussed in the text.

ence of the boundary conditions on it, we have computed a two-dimensional director distribution near the border of two areas with different anchoring energies. The function  $\eta(x/d)$  was evaluated in the following four cases: (1) the first and the second areas have respective anchoring strengths of  $W_o$  and  $10W_o$ , as determined by Eq. (3), and  $V = V_c$ ; (2) the first area has anchoring energy  $0.8W_o$ , the second area has  $50W_o$ ,  $V = V_c$ ; (3) the first area has  $0.1W_o$ , the second area has  $100W_o$ , and the length of the electrical coherence<sup>14</sup> is more than half of the LC thickness,  $V = 1.2V_c$ ; (4) the same as case (3), but with  $V = 1.5V_c$ . The obtained results using the same LC-parameters as above are presented in Fig. 2 with the solid, dashed, dash-dotted, and dotted curves corresponding to case 1, 2, 3, and 4, respectively. The border between the areas with different anchoring is placed at  $x = 0$ .

From the results in Fig. 2, one can see that the flyback zones of all considered cases are more or less similar: the most significant changes of  $\eta$  occur in the range  $x/d \in ]-1, 1[$ , i.e.,  $x \in ]-d, d[$ . It is interesting to note that approximately the same value of the flyback zones (around  $2d$ ) is

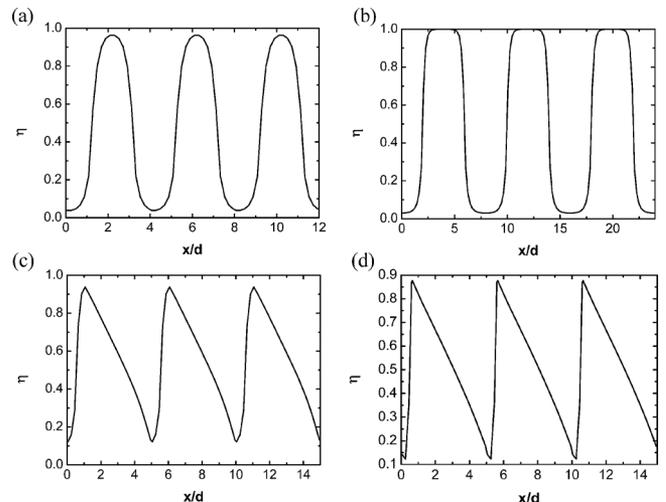


FIG. 3. Profiles of  $\eta$  for phase gratings: (a) square grating with period  $4d$ , (b) square grating with period  $8d$ , (c) sawtooth grating with period  $5d$ , and (d) sawtooth grating with period  $10d$ .

TABLE I. Parameters of the switched gratings based on nonuniform anchoring.

Type of the grating	Max. phase difference	Period	Diffraction efficiency, order				
			-2 (%)	-1 (%)	0 (%)	1 (%)	2 (%)
Rectangular	$\pi$	$4d$	3.9	37.9	7.7	37.9	3.9
Rectangular	$\pi$	$8d$	1.4	40.0	1.9	40.0	1.4
Sawtooth	$2\pi$	$5d$	0.1	0.2	2.3	74.7	9.3
Sawtooth	$2\pi$	$10d$	0.2	0.1	0.1	87.3	2.1

observed at the fringing field.<sup>5,6</sup> This may point at the fact that the flyback zone is determined by the elastic properties of the liquid crystal in first place rather than by the fringing field.

Knowing the value of the flyback zone, it becomes possible to evaluate the minimum period of a phase grating, which can be around  $4d \dots 8d$  for a rectangular grating and around  $5d \dots 10d$  for a sawtooth grating. The minimum value of  $d$  is determined by the maximum required phase delay. For example, gratings with a rectangular profile usually have a maximum phase delay  $\pi$  that for the 500 nm wavelength and  $\Delta n = 0.25$  can be achieved at  $d \geq 1 \mu\text{m}$  in the transmission mode and at normal incidence.

We have also determined  $\lambda(x)$  producing rectangular and sawtooth phase diffraction gratings at  $V_c$ . The calculated profiles of  $\eta(x)$  for rectangular phase gratings with periods  $4d$  and  $8d$ , as well as for sawtooth phase gratings with periods  $5d$  and  $10d$ , are shown in Fig. 3. The calculated diffraction efficiencies for these gratings are summarized in Table I.

Comparing these results with those reported for the phase gratings formed by an electric field gradient,<sup>5,6</sup> one can see that the gratings based on nonuniform anchoring have higher diffraction efficiencies.

By analogy to the calculation of diffraction gratings, it is possible to obtain a Fresnel lens. The parameters of such a lens can also be evaluated from the flyback zone and characteristics of the considered diffraction gratings. In particular, the minimum fragment of the Fresnel lens can have a profile as small as one period of a sawtooth grating (Fig. 3).

Let us evaluate how critical the values of the maximum and minimum anchoring. The maximum change of the retardation of a LC element based on nonuniform anchoring energy is  $\delta_{\text{max}} = d\Delta n(\eta_2 - \eta_1)$ , where 1 and 2 indicate areas with maximum and minimum anchoring energies, respectively. Expressing  $d$  and  $\eta$  in terms of  $\lambda$  and after that in terms of  $W$ , we have

$$\delta_{\text{max}} = \pi K_{11} \Delta n \kappa (1/W_2 - 1/W_1), \quad (7)$$

where  $\kappa = (\eta_2 - \eta_1) / (\lambda_2 - \lambda_1)$ . From Fig. 1 it is seen that the maximum value of  $\kappa$  at the proper voltage is around 1.5.

The foregoing results show that a LC light-deflecting device based on variation of boundary coupling is highly competitive with traditional solutions where the needed distribution of the LC director is achieved by a gradient of an electrical field. Moreover, the device suggested here is controlled with a uniform voltage, the value of which can be several times lower than the driving voltage for conventional LC devices.

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