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Electric-field control of magnetization in biased semiconductor quantum wires and point contacts

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The true origin of the 0.25 and 0.85 conductance features which have been observed in biased split-gate quantum wires and quantum point contacts in semiconductor heterostructures is debated in the literature; one suggestion is that they are caused by spontaneous spin polarization due to the electron-electron interactions. The present work confirms that spontaneous spin splitting may occur within the system and is responsible for both the 0.25 and 0.85 plateaux. We have also shown that the 0.25 plateau consists of two regions, one that is spin polarized, and one that is degenerate with a conductance that remains essentially the same at both sides of the transition. This finding could be of interest for semiconductor spintronics because it opens the possibility for spin manipulation by electric means only.

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I. INTRODUCTION

Conductance in quantum wires and quantum point contacts (QPCs) made from split-gate semiconductor heterostructures is quantized in steps of $2e^2/h$.^{1,2} This phenomenon may be explained in terms of noninteracting electrons, as in Ref. 3, and the stepwise occupation of higher subbands as the electron density is increased. Each occupied subband contributes a fixed amount $2e^2/h$ to the conductance G where the factor of 2 derives from spin degeneracy. Experiments have also revealed other conductance features which may not be explained within a single-particle model. Possible underlying mechanisms are, however, still debated. The most well-known example is the "0.7" conduction anomaly⁴ for which different explanations have been proposed. One advocates spontaneous spin splitting due to direct and exchange interactions,^{5–7} another the formation of quasibound states giving rise to Kondo-like behavior accompanied by the zero-bias anomaly (ZBA).^{8–10} At present these two appear to be the most favored and possibly interrelated models. There is, however, a rich literature on this subject and we refer to the special "0.7 issue" in Ref. 11, the review in Ref. 12, as well as some recent developments including also effects of spin-orbit interactions and localization (Refs. 13-19, and references within).

In addition to the 0.7 anomaly there are the 0.25 and 0.85 nonlinear conductance features.^{12,20} Here we focus on the nonlinear case which has been less explored in theoretical modeling. In doing so we will assume that spontaneous spin polarization is the leading mechanism, a choice which appears natural for the nonlinear biased regime. As already the name ZBA indicates, it is operative only in the very vicinity of zero source-drain bias. In this sense the present case appears more clear cut as ZBA appears to be less of an issue in the nonlinear, biased regime and/or at elevated temperatures (see, e.g., Ref. 13 for some recent experimental data).

In the following we will explore how a finite sourcedrain bias, temperature, and parallel magnetic field B may influence the nonlinear conductance of an extended biased wire. Experiments appear to support such an approach. For example, the 0.25 plateau at high source-drain bias is found to be unaffected by an external magnetic field, which indicates that the plateau is already fully polarized at zero field.²⁰ From this point of view it is interesting that a recent theory assumes the opposite assumption and ties the 0.25 feature to pinning in spin degenerate bands.²¹ This theory indeed predicts the 0.25 conductance at zero magnetic field. Unfortunately the theory in Ref. 21 was only applied to this case. Generally, however, one would expect that a finite *B* field would lift the degeneracy and thereby cause changes within the 0.25 plateau in contradiction to experiments.

II. MODEL SYSTEM

As in previous successful analyses and modeling of the socalled 0.7 conduction analogs^{22,23} in QPCs, we assume that we may replace the real system with an extended (infinite) quasione-dimensional (Q1D) wire at a GaAs/Al_xGa_{1-x} As interface in which electrons propagate freely along the wire, in the x direction, with transmission coefficients 0 or 1 consistent with long wires; the transverse motions (in the y and z directions) are quantized into discrete sublevels because of a confining potential. For simplicity, let the cross section be elliptic as in Ref. 22. For parabolic confinements we then have total external confinement potential U_{ext} ,

$$U_{\rm ext} = \frac{1}{2}m^*\omega_{\rm v}^2 y^2 + \frac{1}{2}m^*\omega_{\rm z}^2 z^2, \tag{1}$$

where ω_y and ω_z are parameters of the confinement potential and m^* is the effective electron mass for GaAs, $m^* = 0.067m_e$, where m_e is the free-electron mass. For a magnetic field $\bar{B} = B\hat{x}$ parallel to the wire and a gauge $\bar{A} = -Bz\hat{y}$ the one-electron Hamiltonian is

$$\hat{H}_{0} = \frac{1}{2m^{*}} \left(-\hbar^{2} \bar{\nabla}^{2} + e^{2} B^{2} z^{2} - 2i\hbar e B z \frac{\partial}{\partial y} \right) + U_{\text{ext}} + \frac{1}{2} g \mu_{B} B \hat{\sigma}, \qquad (2)$$

where the last term is the Zeeman term (g is the effective g factor, μ_B is the Bohr magneton, and $\hat{\sigma}$ is the spin operator). The complex term represents a magnetic coupling between the y and z directions. Assume now that the wire is quite flat, i.e., the cross section is elongated with $\omega_z \gg \omega_y$. The coupling is then weak due to the large difference between sublevels related

to the y and z directions²²) and we therefore simplify Eq. (2) as^{23}

$$\hat{H}_{0} \simeq -\frac{\hbar^{2}}{2m^{*}}\bar{\nabla}^{2} + \frac{1}{2}m^{*}\omega_{y}^{2}y^{2} + \frac{1}{2}m^{*}(\omega_{z}^{2} + \omega_{c}^{2})z^{2} + \frac{1}{2}g\mu_{B}B\hat{\sigma}, \qquad (3)$$

where $\omega_c = |eB/m^*|$ is the cyclotron frequency. The corresponding wave functions and subband dispersions are

$$\Psi_{k,n,l}^{\sigma}(x,y,z) = e^{ikx}\varphi_n(y)\phi_l(z)\chi(\sigma), \qquad (4)$$

$$E_{n,l}^{\sigma}(k) = \epsilon(k) + \left(n + \frac{1}{2}\right)\hbar\omega_{y} + \left(l + \frac{1}{2}\right)\hbar\sqrt{\omega_{z}^{2} + \omega_{c}^{2}} + \frac{1}{2}g\mu_{B}B\sigma,$$
(5)

where $\epsilon(k) = \hbar^2 k^2/2m^*$ is the translational energy, $\varphi_n(y)$ and $\phi_l(z)$ are harmonic oscillator wave functions with n, l = 0, 1, 2, ... for frequencies ω_y and $\sqrt{\omega_z^2 + \omega_c^2}$, respectively; $\chi(\sigma)$ is the spin eigenfunction for up and down spins (\uparrow, \downarrow) . The *g* factor in the Zeeman term is set at 1.9,²² μ_B is the Bohr magneton. Given the differences in sublevels in the two directions, we assume that only the lowest mode $\varphi_0(z)$ is occupied (l = 0). Consequently, we may ignore the diamagnetic shift in energy and wave-function component in the *z* direction, leaving the simple effective Q1D Hamiltonian for the *x* and *y* components,

$$\hat{H}_0 \simeq -\frac{\hbar^2}{2m^*} \bar{\nabla}^2 + \frac{1}{2} m^* \omega_y^2 y^2 + \frac{1}{2} g \mu_B B \hat{\sigma}, \qquad (6)$$

$$E_n^{\sigma}(k) = \epsilon(k) + \left(n + \frac{1}{2}\right)\hbar\omega_y + \frac{1}{2}g\mu_B B\sigma.$$
(7)

From now on we refer to ω_y as ω . The choice of magnetic field and gauge made here thus only generates a Zeeman splitting and has virtually no other effects on the system. Any other choice of magnetic-field direction would shift the wave function sideways and thereby complicate the modeling significantly.

So far our wave functions have been found without considering any electron interactions. At low subband fillings these one-electron wave functions are expected to remain essentially unchanged, an assumption that makes further calculations easier. To include interactions among the particles and the possibility of spontaneous spin polarization, we introduce the spin-relaxed Hatree-Fock approximation. To use, for example, more sophisticated approaches like the local spin-density approximation (LSDA) would be quite cumbersome because the required exchange-correlation potential V_{xc} is not known for a biased wire for which the subband occupations may become strongly asymmetric. To simplify further at this exploratory stage, we follow the approach of Lassl *et al.*²⁴ who proposed that the Coulomb interactions may be represented by Dirac point interactions,

$$V_{\rm int}(\vec{r} - \vec{r}') = \gamma \,\delta(\vec{r} - \vec{r}'),\tag{8}$$

for two particles at \vec{r} and $\vec{r'}$. The interaction strength used here is $\gamma = \pi \hbar^2 / m^* .^{24}$ The Dirac contact potential may be seen as an approximation of a sufficiently screened Coulomb potential, and while experimental setups never reach quite that level of screening the approximation will reveal itself to be quite effective. Lassl *et al.* and more recently Wan *et al.*¹⁴ have used this approach with considerable success to simulate the transport in split-gate QPCs and it should also be accurate enough for our purposes.¹² To validate the present case we have also used Coulomb interactions for some typical cases.

With these simplifications we get the unrestricted Hartree-Fock equations for a wave function $\Psi_i^{\sigma}(\vec{r})$ for a particle with spin σ ,

$$[H_0 + \gamma n_{-\sigma}(\vec{r})]\Psi_i^{\sigma} = \bar{E}_i^{\sigma}\Psi_i^{\sigma}, \qquad (9)$$

where \hat{H}_0 is given by Eq. (6), \bar{E}_i^{σ} is a subband energy, and $n_{-\sigma}(\vec{r})$ is the density of particles with spin $-\sigma$, i.e., the sum over all occupied states $|\Psi_i^{-\sigma}|^2$, which are normalized to a unit length of the wire. Thus only particles with opposite spins interact. The total density is obviously $n(\vec{r}) = n_{\sigma}(\vec{r}) + n_{-\sigma}(\vec{r})$.

Index *i* in Eq. (9) is a shorthand for the different subband indices (n,k) where *n* refers to the *n*:th sublevel and *k* to the subband dispersion $\epsilon(k)$. Thus $\bar{E}_i^{\sigma} \equiv E(n,k,\sigma) = E_n^{\sigma} + \epsilon(k)$. Let us now assume that a source-drain bias potential V_{SD} is applied over the wire. The bias potential is modeled by shifting the chemical potential E_F up and down symmetrically depending on the direction of motion,²⁵

$$\mu_S = E_F + eV_{\rm SD}/2,\tag{10}$$

$$\mu_D = E_F - eV_{\rm SD}/2. \tag{11}$$

The potential is thus assumed to be constant within the wire because the applied voltage V_{SD} is assumed to drop at the two (distant) ends of the wire, i.e., a behavior that should be expected for long wires. Separating left and right motions we have the corresponding Fermi-Dirac occupation numbers for electrons with positive and negative *k* values,

$$f_{\rm FD}^+(n,k,\sigma) = \Theta(k)(e^{(E(n,k,\sigma)-\mu_S)/k_BT}+1)^{-1},$$
 (12)

$$f_{\rm FD}^{-}(k,n,\sigma) = \Theta(-k)(e^{(E(n,k,\sigma)-\mu_D)/k_BT}+1)^{-1},$$
 (13)

where we have omitted the diamagnetic shift; the step function $\Theta(x)$ equals 1 for $x \ge 0$ and 0 for x < 0; k_B is the Boltzmann constant and *T* is the temperature. With the notation $f_{\text{FD}} = f_{\text{FD}}^+ + f_{\text{FD}}^-$ we have for the density of σ electrons

$$n_{\sigma} = \sum_{n} \int \frac{dk}{2\pi} f_{\rm FD}(k,n,\sigma) \left| \Psi_n^{\sigma} \right|^2.$$
(14)

Transmission coefficients are tacitly assumed to be either 0 or 1, which applies to long QPCs and wires.

As mentioned, electrons interact only with electrons of opposite spin. Therefore if the electrons are spin polarized, the interaction energy is lowered. As also seen, the electrons will arrange themselves in parabolic bands along the *k* axis. Although the expressions above work for multiple bands with different values for *n*, we use in most cases only the first band n = 0 to make contact with measurements. We have performed self-consistent calculations using Eq. (9) under various settings for Fermi energy E_F , bias potential V_{SD} , magnetic field *B*, and temperature *T* in order to determine how the electron bands split and populate. The calculations are repeated until the results in two successive iterations become the same within a given tolerance level. The total current *I* is

afterwards calculated using the given population of electron states as

$$I = \frac{e\hbar^2}{m^*} \sum_n \sum_{\sigma} \int \frac{dk}{2\pi} k f_{\rm FD}(k, n, \sigma)$$
(15)

and the conductance as $G_{AC} = dI/dV_{SD}$ at fixed E_F ; $f_{FD}(k,n,\sigma)$ is divided into two parts depending on the sign of k as in Eqs. (12) and (13). In the non-Ohmic regime, $E(n,k,\sigma)$ may depend on V_{SD} in a complex way. For this reason it is convenient to use numerical differentiation as is actually done also for the measured data. Below we report on the differential conductance G_{AC} . The present model is indeed quite basic. In spite of that it predicts conduction features that are of principle interest, e.g., the 0.25 and 0.85 plateaux and an associated spin alignment. As mentioned above, we have also used Coulomb interactions for some typical cases to validate our approach. In this case the full computations become rather prohibitive and, as it seems, do not yield a deeper physical insight than the present model.

III. RESULTS OF SIMULATIONS

Subband levels and energy bands obtained from the selfconsistent calculations are shown in Fig. 1. As seen, a number of polarization regions will form, marked by A-F. The figure also shows schematically the population of the bands for the corresponding polarization states. When the wire begins to be populated the electrons will begin to polarize so that only one type of spin exists in the system. This happens in region A for zero bias. This case is similar to the spin splitting obtained previously for infinite wires^{5–7} and for finite QPCs and wires, ^{14,24,26–28} although the splitting often appears more gentle in these cases.

The effect of applying a finite V_{SD} on the splitting is profound.¹² As shown for regions C and D, a finite bias makes the polarization region in A split into more complex substructures. Since the interaction energy only depends on electrons of opposite spin, occupied spin states as in Fig. 1 become interaction free. Hence the sublevel is the same as for noninteracting particles except for only minor deviations due to the thermal population of the higher down-spin states. At T = 0 K the sublevel E_0^{\uparrow} would thus remain perfectly pinned at the bottom of the band with no down-spin electrons present. At the same time, E_0^{\downarrow} is raised above the chemical potential via the interactions with the up-spin electrons in the occupied band. However, since the interaction energy increases as $\sqrt{E_F}$, the upper level E_0^{\downarrow} will eventually have to become populated. As that happens, the electrons can no longer remain fully polarized—instead they turn spin degenerate. Generally, spin splitting only persists as long as at least one directional



FIG. 1. (Color online) Plots of the lowest subband levels E_0^{σ} and the corresponding population of the subbands as a function of the Fermi energy E_F for different bias potentials V_{SD} . Lower and upper curves refer to up- and down-spin electrons, respectively. The diagonal lines denote chemical potentials; solid line for E_F and dashed lines for μ_S and μ_D . The calculations have been performed for 100 mK, zero magnetic field, and $\hbar\omega = 1$ meV for the transverse confinement. The left upper panel refers to zero-bias potential, the top right to $eV_{\text{SD}} = 0.1$ meV, and the bottom left to $eV_{\text{SD}} = 0.14$ meV. A number of different polarization features are marked by A-G. Cases A, C, D, and E are partially or fully spin polarized while B, F, and G are spin degenerate. The right bottom panel shows schematically the band populations for four cases that are closely related to the 0.25 and 0.85 features.



FIG. 2. (Color online) Zeeman splitting of the subband levels E_0^{σ} at a magnetic field B = 0.5 T (the *g* factor is 1.9 (Ref. 22) and the remaining parameters are as in Fig. 1. Although there is now a Zeeman splitting, the system receives additional splitting through interactions into the same marked population regions as in Fig. 1. The left panel refers to zero bias V_{SD} and the right one to a bias of 0.3 meV.

subband may be fully spin polarized. We have found that this also takes place when multiple electron bands have been considered.

Results for a moderate magnetic field are shown in Fig. 2. Although there is now an additional Zeeman splitting that eliminates spin degeneracy there will still be an additional splitting due to electron-electron interactions. This additional splitting will cause the bands to form the same type polarization regions with the same subband populations as found for zero magnetic field. Because the populated subbands are still the same, and because the conductance features are defined by the subband populations, not the spin splitting itself, the same conductance plateaux will form regardless of the magnetic field, as illustrated by Fig. 3. The only difference is the extension of the different spin regions. Figure 3 also shows the impact of these polarization states on the conductance, without or with a magnetic field present.

State *C*, in which only positive-*k* electrons are present, is spin polarized according to Figs. 1 and 2. If the lowest subband level is pinned within *C* and assuming that we may evaluate the zero-temperature conductance from Eq. (15) as

$$G_{\rm AC} = \frac{d}{dV_{\rm SD}} \frac{e}{h} \left(E_F + \frac{eV_{\rm SD}}{2} - E_0^{\uparrow} \right) = 0.25 \frac{2e^2}{h}.$$
 (16)

The 0.25 conductance observed in experiments²⁰ may thus be explained in terms of spontaneous spin polarization of unidirectional electrons at high bias. If calculations were done with a Coulomb interaction potential, however, there would also be some interaction between spin electrons with parallel spins. As a consequence, the sublevel would be pinned less perfectly and some deviation from 0.25 should be expected. Finite temperatures also cause deviations, in our case $\sim 5\%$ at 100 mK.

At zero *B* field the spin-splitting in *C* collapses into degenerate states *G* on increasing E_F . The conductance varies between ~0.25 and 0.4, increasing slowly with E_F but still behaving much like a plateau. At 0 K the conductance is

$$G_{\rm AC} = \frac{d}{dV_{\rm SD}} \frac{e}{h} \left(E_F + \frac{eV_{\rm SD}}{2} - E_0^{\uparrow} + E_F + \frac{eV_{\rm SD}}{2} - E_0^{\downarrow} \right)$$
$$= (0.5 - \zeta) \frac{2e^2}{h}, \tag{17}$$

where $\zeta = dE_0^{\uparrow}/d(eV_{SD}) = dE_0^{\downarrow}/d(eV_{SD})$ because of degeneracy in region *G*. The sublevels are no longer constant in this region. For a given E_F the two degenerate sublevels will thus increase with V_{SD} which implies that $\zeta > 0$. The conductance should therefore be less than 0.5 in *G*; in fact,



FIG. 3. Gray-scale images of the magnitude of the transconductance $|dG_{AC}/dE_F|$. Bright regions are conductance plateaux and dark branches are regions in which the current and conductance change rapidly due to the rearrangements of electron states and band occupations. The regions C-F refer to the same polarization regions as in Figs. 1 and 2. The left panel refers to zero magnetic field (cf. Fig. 1) and the right one to an applied moderate magnetic field (cf. Fig. 2). The conductance (in units of $2e^2/h$) is 0.25 in C; in G it varies from 0.25 at low E_F to 0.4 at high E_F , and in E from 0.85 at low E_F to 1 at high E_F . In D and F the conductance equals 0.5 and 1, respectively.

in this degenerate region the model yields values between 0.25 and 0.4 at T = 0 (cf. the Hartree model²¹ referred to in the Introduction). At higher temperatures the C population region eventually disappears and, in that case, region Gpersists all the way from the onset of band population. In such cases the conductance begins at zero and increases slowly over the previous C region. Apart from a spike in the theoretical conductance just at the transition between Cand G it remains smooth at that point. Together the C and G regions correspond well to the entire 0.25 plateau as observed in experiments,²⁰ including the slight increase in conductance with increasing E_F for a given V_{SD} . However, there is no sign in experiments of the spike just at the crossover from C to G. This is not unexpected since our model refers to infinite wires with steplike transmission coefficients. Numerically the spiky region is hard to treat accurately and for this reason we prefer to focus on the magnitude of the transconductance $|dG_{AC}/dE_F|$ in Fig. 3. Similar "resonancelike" behavior at the border regions between different polarizations may also be noticed for finite QPCs with sharp geometric features.14,23,28

Region *E* is characterized by partial spin polarization as seen in Fig. 1. Although there is not a well-defined plateau, a conductance of approximately 0.85–1 is obtained, which corresponds well to the experimental observations of a 0.85 plateau.²⁰ Neglecting ζ in Eq. (17), simple analytical calculations¹² of the conductance in the *E* region gives a contribution of 0.5 from the up-spin electrons and 0.25 from the down-spin electrons, in total 0.75. Although roughly correct, it indicates the importance of including the dependence of the sublevels on the bias.

IV. SUMMARY AND CONCLUDING REMARKS

Using a very basic translational subband model frequently used in the analysis of experimental transport data¹² we have shown that the 0.25 and 0.85 plateaux may be associated with spontaneous spin splitting due to electron-electron interactions. The 0.25 plateau is, however, not one region but in fact two. One part that is spin split, the 0.25-lower case, and the other, the 0.25-upper case, is degenerate as in Ref. 21. Although not all of the plateau is related to spin splitting, it is the 0.25-lower region that defines the plateau by creating a steplike behavior as population sets in. Without such an onset the conductance would decrease gradually to pinchoff. Finally, transitions between the different regions and associated magnetizations in the non-Ohmic regime may, as indicated here, be controlled by electric means only. As noted in Refs. 14 and 29, in connection asymmetric QPCs with spin-orbit coupling this could be of potential interest for semiconductor spintronics in which there would be no need of an external magnetic field. This is, of course, highly speculative but certainly very challenging.

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- ¹D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Rotchie, and G. A. Jones, J. Phys. C **21**, L209 (1988).
- ²B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson,
 L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev.
 Lett. 60, 848 (1988).
- ³M. Büttiker, Phys. Rev. B 41, 7906 (1990).
- ⁴K. J. Thomas, J. T. Nicholls, M. Y. Simmons, M. Pepper,
- D. R. Mace, and D. A. Ritchie, Phys. Rev. Lett. 77, 135 (1996).
- ⁵C. K. Wang and K.-F. Berggren, Phys. Rev. B **54**, 14257 (1996); **57**, 4552 (1998).
- ⁶D. J. Reilly, T. M. Buehler, J. L. O'Brien, A. R. Hamilton, A. S. Dzurak, R. G. Clark, B. E. Kane, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **89**, 246801 (2002).
- ⁷D. J. Reilly, Phys. Rev. B **72**, 033309 (2005).
- ⁸S. M. Cronenwett, H. J. Lynch, D. Goldhaber-Gordon, L. P. Kouwenhoven, C. M. Marcus, K. Hirose, N. S. Wingreen, and V. Umansky, Phys. Rev. Lett. **88**, 226805 (2002).
- ⁹Y. Meir, K. Hirose, and N. S. Wingreen, Phys. Rev. Lett. **89**, 196802 (2002).
- ¹⁰T. Rejec and Y. Meir, Nature (London) **442**, 900 (2006).
- ¹¹M. Pepper and J. Bird, J. Phys.: Condens. Matter **20**, 160301 (2008) (Special issue).
- ¹²For a recent review, see K.-F. Berggren and M. Pepper, Philos. Trans. R. Soc. London, Sect. A 368, 1141 (2010).

- ¹³Y. Komijani, M. Csontos, I. Shorubalko, T. Ihn, K. Ensslin, Y. Meir, D. Reuter, and A. D. Wiek, Europhys. Lett. **91**, 67010 (2010).
- ¹⁴J. Wan, M. Cahay, P. Debray, and R. Newrock, Phys. Rev. B 80, 155440 (2009).
- ¹⁵R. Cortes-Huerto and P. Ballone, J. Phys.: Condens. Matter **22**, 295302 (2010).
- ¹⁶A. D. Güçlü, C. J. Umrigar, H. Jiang, and H. U. Baranger, Phys. Rev. B **80**, 201302(R) (2009).
- ¹⁷E. Welander, I. I. Yakimenko, and K.-F. Berggren, Phys. Rev. B 82, 073307 (2010).
- ¹⁸J. S. Meyer and K. A. Matveev, J. Phys.: Condens. Matter **21**, 023203 (2009).
- ¹⁹W. K. Hew, K. J. Thomas, M. Pepper, I. Farrer, D. Anderson, G. A. C. Jones, and D. A. Ritchie, Phys. Rev. Lett. **102**, 056804 (2009).
- ²⁰T.-M. Chen, A. C. Graham, M. Pepper, I. Farrer, D. A. Ritchie, and M. Y. Simmons, Appl. Phys. Lett. **93**, 032102 (2008).
- ²¹S. Ihnatsenka and I. V. Zozoulenko, Phys. Rev. B **79**, 235313 (2009).
- ²²A. C. Graham, K. J. Thomas, M. Pepper, N. R. Cooper, M. Y. Simmons, and D. A. Ritchie, Phys. Rev. Lett. **91**, 136404 (2003).
- ²³K.-F. Berggren, P. Jaksch, and I. I. Yakimenko, Phys. Rev. B 71, 115303 (2005).
- ²⁴A. Lassl, P. Schlagheck, and K. Richter, Phys. Rev. B 75, 045346 (2007).

- ²⁵L. Martin-Moreno, J. T. Nicholls, N. K. Patel, and M. Pepper, J. Phys.: Condens. Matter **4**, 1323 (1992).
- ²⁶A. A. Starikov, I. I. Yakimenko, and K.-F. Berggren, Phys. Rev. B **67**, 235319 (2003).
- ²⁷P. Jaksch, I. I. Yakimenko, and K.-F. Berggren, Phys. Rev. B **74**, 235320 (2006).
- ²⁸P. Havu, M. J. Puska, R. M. Nieminen, and V. Havu, Phys. Rev. B 70, 233308 (2004).
- ²⁹P. Debray, S. M. S. Rahman, J. Wan, R. S. Newrock, M. Cahay, A. T. Ngo, S. E. Ulloa, S. T. Herbert, M. Muhammad, and M. Johnson, Nat. Nanotechnol. 4, 759 (2009).